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Mathematical modeling of brown stock washing problems and their numerical solution using MATLAB

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ABSTRACT

The mechanism of the displacement washing of the bed of pulp fibers is mathematically modeled by the basic material balance equation. Non-linear Langmuir type adsorption isotherm is used to describe the relationship between the concentration of the solute in the liquor and concentration of the solute on the fibers. In the present study, the numerical solutions are obtained of the displacement washing model for multistage in counter current manner. For the numerical solution "pdepe" solver in MATLAB is applied on the axial domain of the system of governing partial differential equations. Numerical solutions thus obtained are in good agreement with the results of earlier workers. The technique used in the present investigation is simple, elegant and convenient for solving two point boundary value problems with varying range of parameters.

1. Introduction

Modeling of pulp washing is done mainly using three approaches namely: (a) process modeling, (b) physical modeling and (c) statistical modeling. In process modeling approach, each stage in pulp washing operation is treated as black box. Using material balances, process models express the efficiency of an individual washing stage in terms of some performance parameters such as displacement ratio, Norden efficiency factor, and equivalent displacement ratio. Although these models are useful for routine process design calculations, but provide little information as to how the design or operation of a washer improves its efficiency. A complete review of the various process models used so far describing the pulp washing process has been presented by Pekkanen & Norden (1985).

Physical models describe the washing operation in terms of fundamental fluid flow and mass transfer principles, occurring at microscopic level during displacement washing of a fibrous bed. These models involve parameters such as longitudinal dispersion coefficient and mass transfer coefficients. Physical models proposed by various investigators such as Lapidus & Amundson (1952), Brenner (1962), Sherman (1964), Pellett (1966), Kuo & Barret (1970), Grah (1975) and Perron & Lebeau (1977) has been classified based on mass transfer principles of two types (1) differential contact models (macroscopic) and (2) dispersion models (macroscopic). Lapidus & Amundson (1952) have studied the effect of longitudinal diffusion in ion-exchange and chromatographic columns and obtained differential equation for the wash liquor. Brenner (1962) studied the washing of filter cake by neglecting the accumulation capacity of fibers and assumed that the phenomena of longitudinal mixing and obtained model in terms of the differential equation. Sherman (1964) has described the overall movement of solute in the bed of non-porous granular material with the diffusion like differential equation by replacing molecular diffusion coefficient with longitudinal dispersion coefficient as molecular diffusion coefficient was found very small as compared to longitudinal dispersion coefficient. An additional term was used to account for the accumulation (or depletion) capacity of material sorbed by the solids. Pellett (1966) has studied the longitudinal dispersion of solute, intraparticle diffusion of solute and liquid-phase mass transfer for the particles of cylindrical and spherical geometry by using a modified step function input. Kuo & Barret (1970) neglected the longitudinal dispersion coefficient to study sodium chloride washing and obtained differential equation for the wash liquor. Grah (1975) has divided the packed bed of cellulose fibers into three different zones namely zone of flowing liquor, stagnant liquor and fibers. Longitudinal dispersion and mass transfer in the flowing liquor zone is characterized by the differential equation. Perron & Lebeau (1977) had taken the model equation without considering the effect of longitudinal dispersion coefficient.

Most of the researcher described the washing model by coupling the transport equation with various adsorption isotherms to describe the relationship between the concentration of the solute in the liquor and concentration of the solute on the fibers.

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Nomenclature

A'	surface area of bed (m^2)
c	concentration of the solute in the liquor (kg/m^3)
C_0	concentration of solute inside the vat (kg/m^3)
N_0	amount of solute accumulated on the fiber surface at the inlet ($kg/m^{3)}$
C_s	concentration of solute in the wash liquor (kg/m^3)
D_L	longitudinal dispersion coefficient (m^2/s)
D_V	molecular diffusion coefficient (m^2/s)
A, B	Langmuir constants (m^3/kg)
L	cake thickness (m)
n	concentration of solute on fibers (kg/m^3)
t	time (s)
C	dimensionless concentration of solute in the liquor
N	dimensionless concentration of solute in the fiber
Z	dimensionless distance
T	dimensionless time
τ	mean time
u	liquor speed in cake pores (m/s)
z	variable cake thickness (m)
Δz	small increment in cake thickness (m)
C_{yi}	inlet vat consistency of the pulp (kg of fiber/kg of liquor)
C_{yd}	discharged consistency of the pulp (kg of fiber/kg of liquor)
x_i	dissolves solids inside the vat (%)
x_s	dissolved solids in washing water (%)
x_f	dissolved solids in the filtrate (%)
x_d	dissolved solids in discharged pulp (%)
x_r	dissolved solids in recycle liquor (%)
L_i	amount of liquor inside the vat (kg of liquor/kg of pulp)
L_s	amount of wash water (kg of water/kg of pulp)
L_d	amount of liquor in discharge pulp (kg of liquor/kg of pulp)
L_f	amount of filtrate (kg of liquor/kg of pulp)
L_r	amount of liquor recycled to previous washer (kg of liquor/kg of pulp)

Sherman (1964), Pellett (1966), Neretnieks (1974), Grah (1975) and Viljakainen (1985) have considered the linear or non-linear adsorption isotherm equations along with the dispersion diffusion based transport equations. Towers & Scallan (1996) used Donnan equilibrium theory to characterize the distribution; a mathematical model has been developed to describe the partitioning of cations between fiber walls and surrounding liquor. Tervola & Rasanen (2006) described the cake washing of freely mobile ions by an advection-dispersion equation combined with Donnan equilibrium and an overall ion transfer model between the external liquid-phase and the fiber phase of the kraft pulp.

A typical washing system in industry consists of three or four washers in counter flow arrangement. Counter flow is an engineering technique where in two process streams interact as they move in opposite directions. As applied to pulp washing this means a series of washers is set up with the final wash being performed with clean water. The wastewater from the last washer is then used to wash the stock in the second from the last washer. Water from the second to last washer is used to wash stock in a third to last washer and so on for the total number of washers used. Tervola (2006) developed a Fourier series solution method for solving a multi-stage counter current cake washing problem, so that the solute concentration gradient inside a cake between washing stages was preserved.

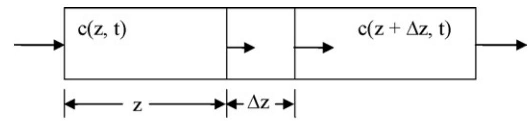

Fig. 1. A simple shell balance.

In the present study dispersion diffusion based transport equations of pulp washing are developed based on the assumptions of Sherman (1964) and Pellett (1966) coupled with the equation of mass transfer, i.e. Fick's second law of diffusion. The equations are coupled with many other fluid mechanical parameters and finally two models are obtained, differing in boundary conditions. Non-linear Langmuir type adsorption isotherm given by Fogelberg & Fugleberg (1963) is used for equilibrium between the concentration of the solute in the liquor and concentration of the solute on the fibers. The above mentioned mathematical models of two simultaneous partial differential equations with various boundary conditions are extremely intricate in nature and practically appear to be unsolvable for multistage washing system even by using sophisticated numerical techniques. It is important to mention that the problem for single washer with its simplified version has been solved analytically by Brenner (1962) and Kukreja (1996) using Laplace transform and numerically using orthogonal collocation by Grah (1975) and Arora, Dhaliwal, and Kukreja (2006a) and Arora, Dhaliwal, and Kukreja (2006b). Kumar (2002) attempted to solve the washing model using Finite difference method. All these methods are very complex and time consuming. Similar problem for a single stage advection-dispersion washing has been solved by Singh, Kumar, and Kumar (2008) by pdepe solver easily by using much less time and give the similar results as analytical solution given by Brenner (1962). The present work focuses on the four stages counter current washing system described by the mathematical model for washing zone of a rotary drum washer and washing theory based on Langmuir adsorption isotherm. Pdepe solver in MATLAB source code is used to solve the model of washing zone for multistage counter current cake washing.

2. Description of mathematical models for cake washing zone

The mat of pulp fibers can be assumed to be stationary packed bed of homogeneous symmetrical cylindrical fibers. Instantaneous behavior of any system of this type can only be expressed by an equation involving the variables and their partial derivatives. Using simple material balance for setting up a differential equation, consider a thin slice of a filter cake (pulp mat) as shown in Fig. 1, through which filtrate or wash water flows.

Material balance across the simple shell given in Fig. 1, in the z direction can be written as

$$\begin{aligned}&\text{rate of mass of solute in}\\&\quad+\text{rate of mass production by chemical reaction} = \text{rate of mass out}\\&\quad+\text{rate of mass accumulation in the liquid phase}\\&\quad+\text{rate of mass accumulation in the solid phase due to}\\&\qquad\text{adsorption-desorption.}\end{aligned}$$

If A' is the area of the bed, ε_t the total average porosity (sum of porosities in the displaceable liquid ε_d and in the immobile phase ε_s), u the velocity of the liquor in the mat, c the concentration in the liquid-phase, the equation in one dimension can be written as

$$(uc\varepsilon_t A')_{z,t} - (uc\varepsilon_t A')_{z+\Delta z,t} = \left[\frac{\partial}{\partial t}\{(c\varepsilon_t A'\Delta z + n(1-\varepsilon_t)A'\Delta z\}\right]_{\bar{z},t} \quad (1)$$

Table 1
Existing mathematical models for washing zone used in present investigation (dimensionless form).

S. no.	Transport equation	Adsorption isotherm	Boundary conditions
1.	$(\partial^2 C/\partial Z^2) = Pe(\partial C/\partial Z + \partial C/\partial T + \mu' \partial N/\partial T)$	$N = ABC_0C/(1+BC_0C)N_0$	$(\partial C/\partial Z) = Pe(C - C_s/C_0)$ for $(Z=0, T>0)$ and $(\partial C/\partial Z) = 0$ at $(Z=1)$
2.	$(\partial^2 C/\partial Z^2) = Pe(\partial C/\partial Z + \partial C/\partial T + \mu' \partial N/\partial T)$	$N = ABC_0C/(1+BC_0C)N_0$	$(C = C_s/C_0)$ for $(Z=0, T>0)$ and $(\partial C/\partial Z) = 0$ at $(Z=1)$

where $z < z^- < z + \Delta z$. Taking ε_t and A' as constant and taking the limit as $\Delta z \to 0$, one can obtain the following expression:

$$-\varepsilon_t c\left(\frac{\partial u}{\partial z}\right) = \varepsilon_t u\left(\frac{\partial c}{\partial z}\right) + \varepsilon_t\left(\frac{\partial c}{\partial t}\right) + (1-\varepsilon_t)\left(\frac{\partial n}{\partial t}\right) \quad (2)$$

The above equation contains principally two accumulation terms, one related to dispersion–diffusion and another related to adsorption–desorption. Other terms are velocity gradient and convective flow terms. Using Fick's second law of diffusion, i.e.:

$$-c\left(\frac{\partial u}{\partial z}\right) = (D_L + D_V)\left(\frac{\partial^2 c}{\partial z^2}\right) \quad (3)$$

the following equation is obtained:

$$(D_L + D_V)\left(\frac{\partial^2 c}{\partial z^2}\right) = u\left(\frac{\partial c}{\partial z}\right) + \left(\frac{\partial c}{\partial t}\right) + \frac{1-\varepsilon_t}{\varepsilon_t}\left(\frac{\partial n}{\partial t}\right) \quad (4)$$

According to Sherman (1964) the longitudinal dispersion coefficient D_L is a function of flow pattern within the bed (unless very low flow rates are used). The molecular diffusion coefficient D_V is very small compared to D_L and so may be neglected. Writing $(1-\varepsilon_t)/\varepsilon_t$ as μ for convenience, Eq. (4) may be written as

$$D_L\left(\frac{\partial^2 c}{\partial z^2}\right) = u\left(\frac{\partial c}{\partial z}\right) + \left(\frac{\partial c}{\partial t}\right) + \mu\left(\frac{\partial n}{\partial t}\right) \quad (5)$$

This is a non-homogeneous, non-linear, first degree, second order, parabolic, partial differential equation. Here u, ε_t and D_L are functions of z while c and n are functions of both z and t. As the lumen of the fiber is porous and the same is true with the wall of the fiber, the porosity values for these cases are different from the porosity of the interfiber mass. Therefore three porosity values are required to represent the pulp mat system. It is extremely difficult to distinguish precisely between the values of porosity at the lumen and at the wall. Therefore, for practical calculations these are assumed to be the same. Hence, to describe the system two porosity values are assumed, one for the interfibers ε_d and another for intrafibers ε_s, so that $\varepsilon_d + \varepsilon_s = \varepsilon_t$, the total porosity for the entire system. The model Eq. (5) is same as dispersion model for pulp washing given by Sherman (1964) and Pellett (1966).

2.1. Adsorption isotherms

The details of the adsorption isotherms which are used by some earlier workers are as follows.

Lapidus & Amundson (1952) used the adsorption isotherm given by

$$\frac{\partial n}{\partial t} = k_1 c - k_2 n \quad (6)$$

and assumed that the rate of adsorption is finite and plotted the effect of longitudinal diffusion for an infinite column in which equilibrium is established locally. Initial adsorbate concentration was assumed to be zero. Singh et al. (2008) used this linear isotherm successfully and give comparable results.

Sherman (1964) used the adsorption of diacetyl solution by porous viscous fibers with simple isotherm equation, i.e.:

$$n = kc \;\text{ or }\; \frac{\partial n}{\partial t} = k\frac{\partial c}{\partial t} \quad (7)$$

and assumed the liquid solid concentration inside the fibers and surrounding the fibers to be identical at any time and at any position within the bed, implying that diffusion, both within the fiber and between the fiber and the surrounding fluid is sufficiently rapid which does not affect the rate of the overall transport process.

Perron & Lebeau (1977) used the isotherm equation, i.e.:

$$\frac{\partial n}{\partial t} = k(c-n) \quad (8)$$

the diffusion of the solute within the fibers towards the washing liquor is described by a partial differential Eq. (8), which is solved assuming that the mass transfer rate through the stagnant film is finite.

Fogelberg & Fugleberg (1963) used non-linear Langmuir type adsorption isotherm to describe the relationship between the concentration of the solute in the liquor and concentration of the solute on the fibers as

$$n = \frac{ABc}{1+Bc} \quad (9)$$

where A and B are Langmuir constants.

Arora et al. (2006b) successfully used non-linear Langmuir type adsorption isotherm using one dimensional axial dispersion model for single stage washing, by using orthogonal collocation on finite elements. Thus in the present investigation for sodium species, non-linear Langmuir type adsorption isotherm is used for the solution of four stage counter current washing problem.

2.2. Initial and boundary conditions

For the solution of washing models for multistage the initial condition is $c(z,t) = n(z,t) = C_0$ for $0 < t < L/u$, where L/u corresponds to displacement time. Boundary conditions for Langmuir type isotherm are as follows:

$$uc - D_L\frac{\partial c}{\partial z} = uC_S \;\text{ at }\; z=0 \;\text{ and }\; t>0 \;\text{ and}$$

$$\left(\frac{\partial c}{\partial z}\right) = 0 \;\text{ at }\; z=L \;\text{ and }\; t>0$$

Perron & Lebeau (1977) give the boundary condition at the inlet of the bed $c = C_s$, at $z = 0$ and $t > 0$.

In the present investigation both the cases of inlet boundary conditions are used. Thus two sets of model are obtained for different boundary conditions. The initial condition is same for both the models, i.e. $C(Z,0) = 1 = N(Z,0)$ in dimensionless form. Thus possible models which are differing in boundary conditions only are summarized in Table 1 (dimensionless form).

2.3. Dimensionless models

The dimensionless form of the models is obtained by using certain dimensionless parameters like Peclet number (or Bodenstein number), dimensionless time, dimensionless thickness and dimensionless concentrations given below:

$$Pe = \frac{uL}{D_L}, T = \frac{ut}{L}, Z = \frac{z}{L}, C = \frac{c}{C_0}, N = \frac{n}{N_0} \;\text{ and }\; \mu' = \frac{\mu N_0}{C_0}$$

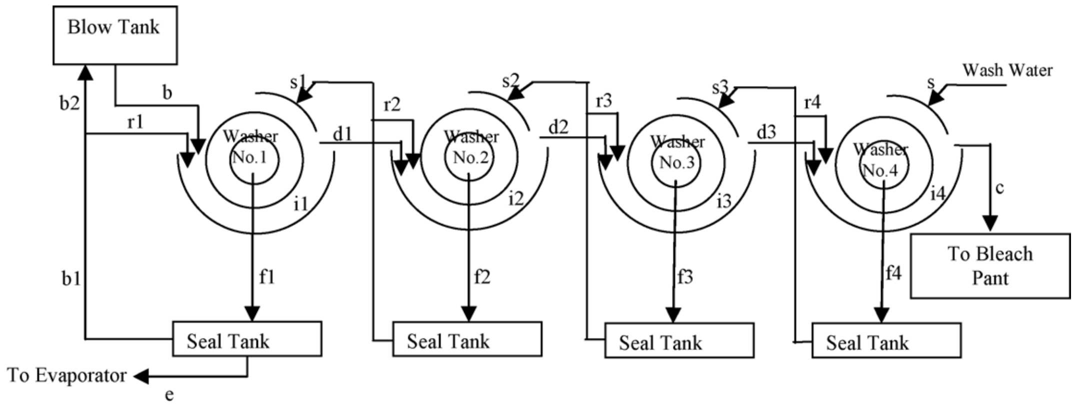

Fig. 2. Flow diagram of a counter current washing system with four drum washers.

Thus the dimensionless form of the transport equation, adsorption isotherm and boundary conditions are given in Table 1.

2.4. Steady state modeling for multistage washing process

In the present investigation a typical four stages counter current washing system is considered. A series of four washers are set up for pulp washing with the final wash being performed with clean water. The steady state material balance equations can be obtained for each washer, with the help of Fig. 2.

Steady state mass balance equations for washer 1 are given below.

First washer mass balance:

$$\text{Liquor}: L_b+L_{r1}+L_{s1}=L_{f1}+L_{d1}, L_{f1}=L_{b1}+L_e, L_{b1}=L_{b2}+L_{r1}, L_{i1}=L_{r1}+L_b \quad (10)$$

$$\begin{aligned}\text{Solids}: & L_b x_b + L_{r1}x_{r1} + L_{s1}x_{s1} = L_{f1}x_{f1} + L_{d1}x_{d1}, L_{f1}x_{f1} = L_{b1}x_{b1} \\ & + L_e x_e, L_{b1}x_{b1} = L_{b2}x_{b2} + L_{r1}x_{r1}, L_{i1}x_{i1} = L_{r1}x_{r1} + L_b x_b \end{aligned} \quad (11)$$

$$\text{Fiber}: L_b C_{yb} = L_{d1} C_{yd1} \quad (12)$$

$$\begin{aligned}\text{Water}: & L_b(1-C_{yb})(1-X_b) + L_{r1}(1-X_{r1}) + L_{s1}(1-X_{s1}) \\ & = L_{f1}(1-X_{f1}) + L_{d1}(1-Cy_{d1})(1-X_{d1}) \end{aligned} \quad (13)$$

In a similar manner mass balance equations are obtained for each washer and then solved, using actual data of a near by pulp and paper mill. The steady state operational data is given below:

(a) Pulp yield = 47%.
(b) Consistency of blown pulp = 13%.
(c) Solids in blown pulp = 22%.
(d) Liquor in blown pulp = 6.69 kg of liquor/kg of pulp.
(e) Standard consistency = 12%.

2.5. Algorithm to calculate approximate Peclet number for the displacement zone for second and subsequent washers

As varying flow conditions can be assumed in different parts of the pulp bed, an average value of the dispersion coefficient is required to be estimated. This is important as various authors simulated with various ranges of Peclet number though the pulp quality remains almost the same. As for example Potucek (1997) used very low *Pe* (1.0–11.3) for pulp fibers and high for glass fibers following Brenner (1962), whereas Grah (1974), Poirier, Crotogino, Trinh, and Douglas (1987) and Crotogino, Poirier, and Trinh (1987) used high ranges (generally 80–100). Poirier et al. (1987) categorically mentioned that displacement efficiency decreases below *Pe* number 20 and therefore one expects inconsistencies in the profiles. An order of estimate values of *Pe* therefore should be predicted. As washing efficiency has been expressed by Potucek (1997) as a function of *Pe*, it becomes more relevant to evaluate for washer performance study. The modified Peclet number, Pe_m and modified dispersion coefficient D_{Lm} are then calculated as

$$Pe_m = \frac{uL}{D_{Lm}}$$

The algorithmic procedure given by Kumar (2002) is used for estimation of Peclet number. According to Kumar (2002), first we obtain the *C* vs. *T* data from this present investigation for both the models by using the Peclet number 71.26 for washer no. 1 is obtained from the data of Grah (1974), which is used for the simulation. Then the Peclet number for displacement zone of second washer is estimated approximately avoiding iterative calculations and considering open vessel as used by Han & Edwards (1988), Edward, Peyron, and Minton (1986) and Potucek (1997).

First we use the data of Grah (1974) for the kappa no. 49.7 and for this data we obtain the Peclet number for the first washer of the series by using the formula, $Pe = uL/D_L$. After that we use the following algorithm given by Kumar (2002) for calculating the Peclet number for the subsequent washers.

- obtain *C* vs. *T* data for the first washer by solution of the model;
- find out the mean time $\tau_i = (\Sigma T_i C_i \Delta T_i)/(\Sigma C_i \Delta T_i)$;
- find the spread of the distribution, measured by the variance representing the spread of the distribution $\sigma^2 = (\sum T_i^2 C_i \Delta T_i)/(\sum C_i \Delta T_i) - \tau_i^2$;
- fit the dispersion model for the large extent of the dispersion for open vessel $\sigma_\theta^2 = (\sigma^2/\tau_i^2) = 10(D_L/uL) = (10/Pe)$.

3. Result and discussion

The mathematical model of pulp washing given by transport equation (Eq. (5)), combined with the corresponding equation of isotherm and various boundary conditions is given in Table 1. The model is extremely intricate in nature and practically appears to be unsolvable for multistage washing system, even by using sophisticated numerical techniques. As mentioned earlier that the problem for single washer with its simplified version has been solved analytically by Brenner (1962) and Kukreja (1996) using Laplace transform and numerically using orthogonal collocation by Grah (1975) and Arora et al. (2006a). Arora et al. (2006b) first discretized the partial differential equations into differential algebraic equations, which are then solved using ODE15s solver

Table 2
Process data of four stage brown stock washing system.

Input parameters	Washer no. 1	Washer no. 2	Washer no. 3	Washer no. 4
C_{yi} (%)	1.25	1.25	1.25	1.25
C_{yd} (%)	12.00	12.00	13.00	14.00
x_i (%)	15.59	7.29	2.62	0.96
x_s (%)	7.00	2.50	0.90	0.00
x_f (%)	15.00	7.00	2.50	0.90
x_d (%)	10.12	3.77	1.58	0.30
x_r (%)	15.00	7.00	2.50	0.90
L_i (kg/kg)	79.00	79.00	79.00	79.00
L_s (kg/kg)	10.33	10.33	9.69	9.14
L_f (kg/kg)	82.00	82.00	82.00	82.00
L_d (kg/kg)	7.33	7.33	6.69	6.14
L_r (kg/kg)	72.31	71.67	71.67	72.31

separately. Kumar (2002) attempted to solve the washing model using finite difference method. While Tervola (2006) developed a Fourier series solution method for solving a multistage counter current cake washing problem. All these methods are very complex and time consuming. Application of such solution techniques in control systems is not possible due to the more processing time and involvement of high mathematical skills at operator level.

For control purpose the transient behavior of the solute concentration in the black liquor is of more interest, rather than solute concentration in fiber. In the present work value of $\partial N/\partial T$ in terms of $\partial C/\partial T$, C, C_0, N and N_0 is obtained by differentiating the isotherm equation, and then substituted in the transport equation (given in Table 1).

For the solution of washing zone models for all four washers, steady state data is observed from a paper mill (given in Table 2) and values of various process parameters for washing zone of a washer is taken as given by Grah (1974) (given in Table 3) for sodium ion species. The washing liquor for the washers' 1, 2 and 3 contains the dissolved solid 7.00%, 2.50%, 0.9% and in the last washer, fresh water is used which also contains 0.5% dissolved solids. The incoming pulp from the digester has the consistency 12%.

In this investigation dimensionless bed depth as well as dimensionless time is divided into 21 equal parts and then the influence of Z and T on C is estimated. The behavior of exit solute concentration with respect to time as well as variable cake thickness is shown by (input curve at $Z=0$ to break through curves at $Z=1$) 3D graphs. The solution obtained for all washers' are given in Table 4, for both the models, respectively. The variations in the dimensionless solute concentration with respect to dimensionless time as well as dimensionless cake thickness are shown in Figs. 3–6 for model 1 and Figs. 7–10 for model 2 for all four washers', respectively.

In the present investigation Langmuir type non-linear isotherm is used to describe the relationship between the concentration of the solute in the liquor and concentration of the solute on the fibers. Dimensionless initial solute concentrations on fiber for all four washers' is obtained are 0.009, 0.002, 0.0002 and 0.00001 (kg/m^3), respectively. The solution of the washing zone models of a rotary vacuum washer is obtained for multistage in counter current manner by pdepe solver in MATLAB source code. Peclet number has

Table 3
Data for simulation for sodium species.

Parameters	Values	Unit
L	0.105	m
D_L	10.8×10^{-7}	m^2/s
ε_t	0.928	–
u	7.33×10^{-4}	m/s
C_0	0.570	kg/m^3
C_s	0.005	kg/m^3
A	0.01263	m^3/kg
B	3.955	m^3/kg

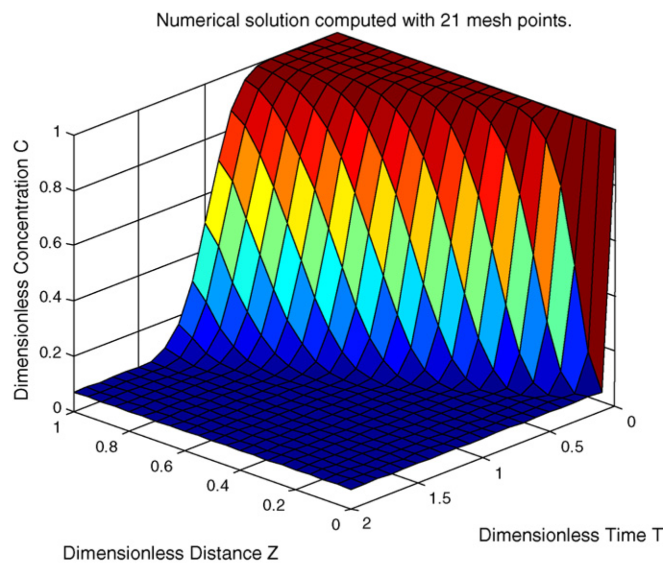

Fig. 3. Solution for washer 1 (Model 1).

been included as dimensionless parameters in the solution of the models. For the washer no. 1 we used the Peclet number $Pe=71.26$ based on the simulation data. Since the dimensionless time for the first and second washers is $10.33/7.33=1.41$, for the third washer

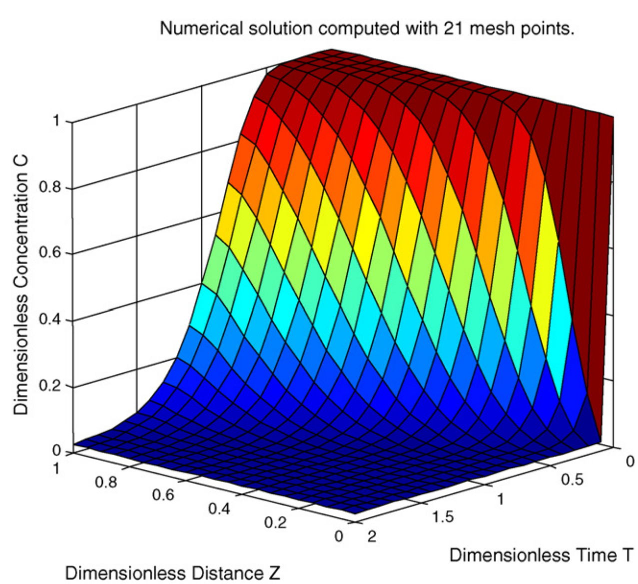

Fig. 4. Solution for washer 2 (Model 1).

Table 4
Dimensionless exit solute concentration in for washer for dimensionless time $T = 0$ to $T = 1.49$.

T	Model 1				Model 2			
	Washer no. 1	Washer no. 2	Washer no. 3	Washer no. 4	Washer no. 1	Washer no. 2	Washer no. 3	Washer no. 4
0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.3	1.0000	0.9998	0.9997	0.9997	1.0000	0.9998	0.9997	0.9996
0.4	1.0000	0.9964	0.9949	0.9946	1.0000	0.9963	0.9947	0.9944
0.5	0.9991	0.9769	0.9704	0.9695	0.9991	0.9764	0.9696	0.9686
0.6	0.9919	0.9211	0.9067	0.9046	0.9919	0.9297	0.9049	0.9027
0.7	0.9596	0.8195	0.7985	0.7954	0.9594	0.8172	0.7957	0.7925
0.8	0.8708	0.6832	0.6604	0.6568	0.8704	0.6803	0.6569	0.6533
0.9	0.7085	0.5357	0.5153	0.5118	0.7080	0.5327	0.5118	0.5082
1.0	0.4997	0.3990	0.3827	0.3795	0.4993	0.3963	0.3796	0.3762
1.1	0.3039	0.2856	0.2732	0.2702	0.3037	0.2834	0.2706	0.2675
1.2	0.1682	0.1993	0.1892	0.1863	0.1681	0.1976	0.1872	0.1842
1.3	0.0991	0.1377	0.1284	0.1254	0.0990	0.1365	0.1269	0.1239
1.4	–	–	–	0.0831	–	–	–	0.0820
1.41	0.0733	0.0925	–	–	0.0733	0.0918	–	–
1.45	–	–	0.0706	–	–	–	0.0697	–
1.49	–	–	–	0.0570	–	–	–	0.0563

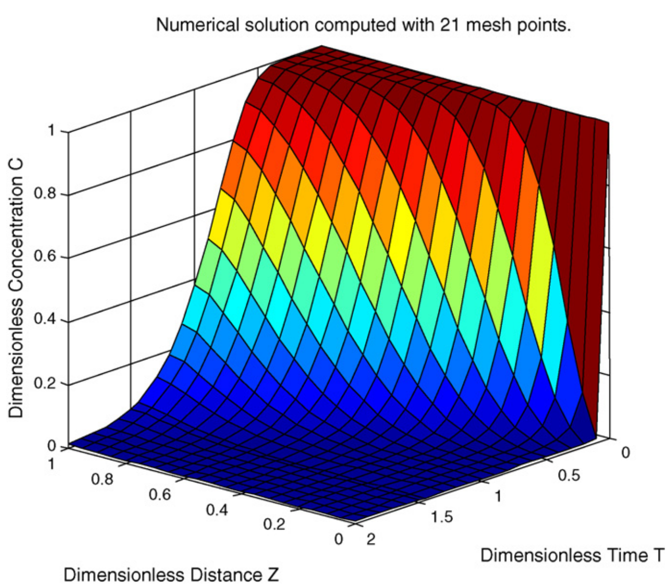

Fig. 5. Solution for washer 3 (Model 1).

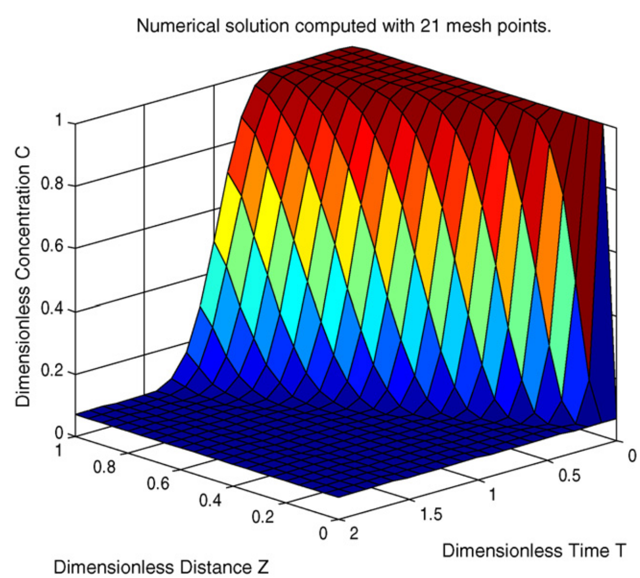

Fig. 7. Solution for washer 1 (Model 2).

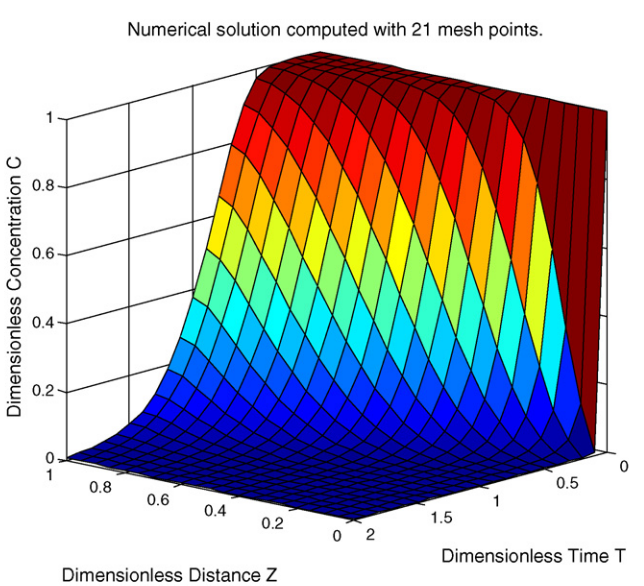

Fig. 6. Solution for washer 4 (Model 1).

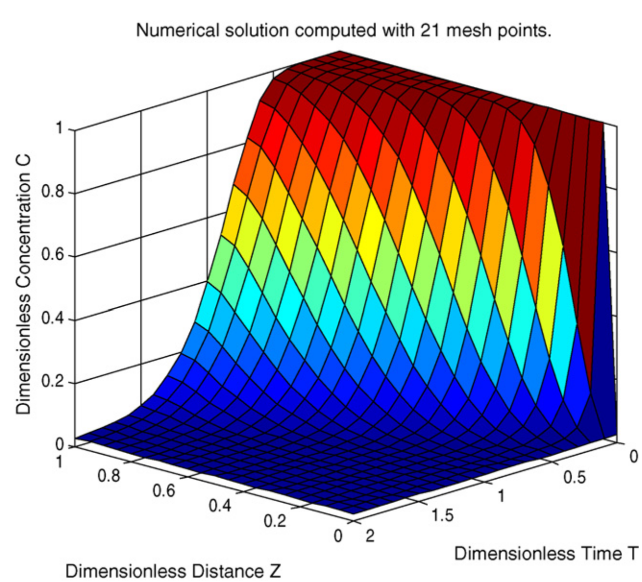

Fig. 8. Solution for washer 2 (Model 2).

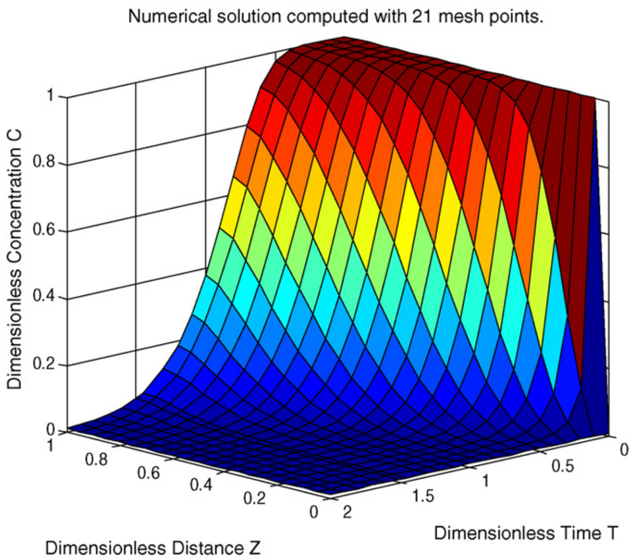

Fig. 9. Solution for washer 3 (Model 2).

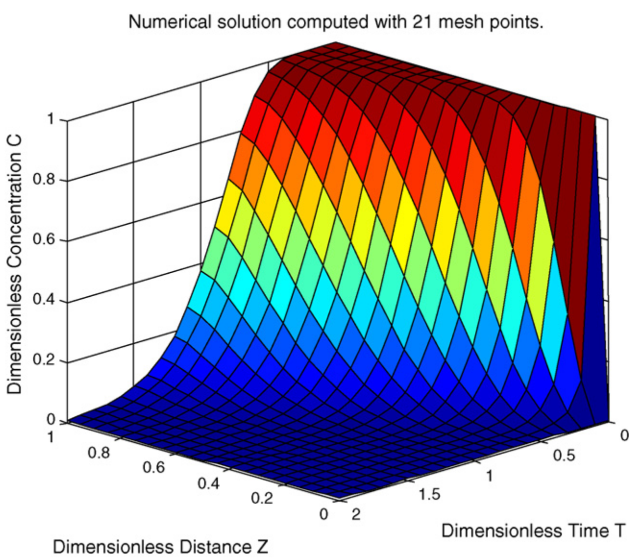

Fig. 10. Solution for washer 4 (Model 2).

is 9.69/6.69 = 1.45 and finally for the last washer is 9.14/6.14 = 1.49. Thus the exit dimensionless solute concentration for washer no. 1 and 2 is taken at T = 1.41 and similarly for third and fourth washers at T = 1.45 and 1.49, respectively. The exit dimensionless solute concentrations for all four washers with respect to dimensionless time are shown in Table 4, for both the models, respectively. Exit dimensionless solute concentrations for each washers' are converted into absolute concentrations. The exit absolute concentration of previous washer is use to calculate the various parameters like N_0, μ', etc. for the subsequent washer. Based on C vs. T data of previous washer the Peclet number of subsequent washer is obtained by using algorithm given by Kumar (2002). The Peclet number obtained by this algorithm for washers nos. 2, 3 and 4 are 21.94, 19.90 and 19.70 for model 1 and 21.93, 19.90 and 19.67 for model 2, respectively.

The washing results for both the models are slightly different at three or four decimal places, so it may conclude that the boundary conditions do not influence much on the washing results. It is clear from figures that the dimensionless concentration of the solute in the liquor decreases with the increase of the dimensionless time, whereas increases with the increase in the dimensionless distance which is same as obtained by earlier workers such as Brenner (1962), Grah (1975), Kumar (2002) and Arora et al. (2006a).

4. Conclusion

The present investigation is for multistage counter current washing system based on mathematical models derived for washing zone of a rotary vacuum washer in paper industry. Langmuir adsorption isotherm is used to describe the relationship between the concentration of the solute in the liquor and concentration of the solute on the fibers. The numerical solution are obtained for four stages in counter current manner by using "pdepe" solver in MATLAB source code and taking Pe = 71.26 for first washer based on the simulation data. Peclet numbers for second and subsequent washers are obtained by using the algorithm given by Kumar (2002). The following conclusions may be drawn from the present study:

1. Boundary conditions do not influence significantly on the washing results.
2. Pdepe solver can be used successfully for the solution of multistage pulp washing model.
3. The pdepe solver used in the present investigation is simple, elegant and convenient for solving two point boundary value problems with varying range of parameters and show a comparable performance with QUICK method in terms of CPU time and average numerical errors.
4. The algorithms in this solver are easy to set up, and so the method represents an advantage and good alternative to the available techniques for such type problems.

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