Adaptive Parallel Processing Algorithm in Digital Signal Series

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Abstract: During vector predictive coding of digital signal series, the vector signal series, obtained by grouping adjacent samples of sources signal series, can approximate to a vector autoregressive series with stable covariance. This paper, applying the orthogonal projection principle of Hilbert space, attempts to formulate a vector predictive coding strategy highly capable of parallel processing and to deduce from this strategy an adaptive parallel processing algorithm, which, compared with traditional lattice algorithms, has improved remarkably in calculation complexity and storage space.

Keywords: data signal series; vector prediction; adaptive algorithm; parallel processing.

1 Introduction

Lattice adaptive filter based on the Recursive Least Squares (RLS) is of good astringency and track ability, the best prediction can be sought according to a group of observation data, so it is widely used[1]. Peter Strobach, a Germanic scholar puts forward the modified adaptive algorithm on the basis of classical algorithms of Levinson and Schur[2]. However, this lattice algorithm can't adapt to the parallel processing, because its calculating complexity and storage are both very large when used to process vector series. The vector linear prediction(VLP) is a new technology in the digital signal processing like speech and image processing, which has important application value in the Vector Prediction Coding(VPC) of digital signal. In VLP, the predictive residual error vector, produced in coding structure, can be measured by the vector quantizer, which can get a better performance than that of the traditional scalar quantizer. One of the key problems in developing high efficiency of VLP technology is to seek the realized way of the optimization vector predictor, reducing the calculating complexity to realize the real-time processing, heighten the distinguishing precision to obtain perfect signal quality and low transmission bit error rate.

2 Optimal Vector Linear Prediction

2.1 Definition and lemma

Definition: two order scalar series $\{x(t), t \in I\}$, if $t, h \in I$ and exist positive whole numbers m,

E[$x(t+m)$]=E[$x(t)$];

$Cov[x(t+m) , x(h+m)]=Cov[x(t) , x(h)]$

Then we call the $\{x(t), t \in I\}$ as a period-correlated series whose period is m and order is two. if this series satisfies the following condition:

$$x(t)+\sum_{l=1}^{p_t} a_t(l)x(t-l)=e(t) \qquad (1)$$

Among them: $E[e(t)]=0$, $Cov[e(t),e(h)]=\delta_t^2\delta_{t,h}$

Then we call the $\{x(t), t \in \mathrm{I}\}$ as a period-correlated multi-autoregressive series whose period is m and order is $\{p_1, p_2, \Lambda, p_m\}$. It can describe the statistical characteristic of a large classes unstable series.

Lemma: Under the above-mentioned definition, command: [3]

$$\begin{cases} y_k(t)=x[m(t-1)+k]=x(mn+k) \\ Y(t)=[y_1(t), y_2(t), \Lambda, y_m(t)]^T \end{cases} \qquad (2)$$

where k=1, 2,⋯, m; $n= t-1$. Then the vector series $\{\mathrm{Y}(t), t \in I\}$ is a AR (p) series with stable convariance:

$$Y(t)+\sum_{j=1}^{p} A(j)Y(t-j)=W(t) \qquad (3)$$

where $E[W(t)]=0$; $Cov[W(t),W(h)]=\omega^2 \cdot \delta_{t,h}$;
$\omega^2=Var[W(t)]>0$; j=1,2,⋯, p

2.2 Description of Recursive Least Squares

Considering the vector series $\{Y(t)$, $t \in \mathrm{I}\}$, come from signal series $\{x(t), t \in I\}$ based on formula (2), its p order vector linear predictor is:

$$\hat{Y}(t)=-\sum_{j=1}^{p} A(j)Y(t-j) \qquad (4)$$

We called A(j) as predictor matrix. Predictive error

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vector is:

$$W(t) = Y(t) - \hat{Y}(t) = Y(t) + \sum_{j=1}^{p} A(j)Y(t-j)$$

Supposing the length of analytical frames is N, then VLP is to select a predictor matrix $A(j)$, which minimizes the performance index J.

$$J = \sum_{t=1}^{L} W^T(t)W(t) = min \qquad (5)$$

Where $L=N$ corresponds to the convariance method and $L=N+p$ corresponds to the autocorrelation one. The predictor obtained from above is called the optimal vector predictor.

Supposing $W_k(t)$ is the kth heft of the m dimension error vector $W(t)$, $a_k(j)$ is the kth row of predictor vector $A(j)$, and command:

$$b_k = [a_k^T(1), a_k^T(2), \Lambda, a_k^T(p)]^T \qquad (6)$$

Then the kth channel predictor is:

$$y_k(t) + \sum_{j=1}^{p} a_k(j)Y(t-j) = W_k(t) \qquad (7)$$

namely, $y_k(t) + [Y^T(t-1), \Lambda, Y^T(t-p)]b_k = W_k(t)$ .This is a hybird regressive equation. When t =1,2,⋯, N, we can get:

$$W_k = Y_k + Zb_k \qquad (8)$$

In the formula: $W_k = [W_k(1), W_k(2), \Lambda, W_k(N)]^T$

$Y_k = [y_k(1), y_k(2), \Lambda, y_k(N)]^T$

And then the performance index can be written as follows:

$$J = \sum_{t=1}^{L} W^T(t)W(t) = \sum_{t=1}^{L}\sum_{k=1}^{m} W_k^2(t) = \sum_{k=1}^{m}\sum_{t=1}^{L} W_k^2(t) = \sum_{k=1}^{m} J_k \qquad (9)$$

where $J_k = W_k^T W_k = \sum_{t=1}^{L} W_k^2(t)$

From this, we can get the optional vector predictor through parallel evaluate the b_k and make the J_k minimum. But we can get the regular equation base on formula (6):

$$Z^T Z b_k = Z^T Y_k \qquad (10)$$

Its condition number is $k(Z^T Z) = k^2(Z)$, so the regular equation has its proper morbidity. Therefore it is impossible for us to obtain a optional vector predictor by solving the channels hybrid regressive equation (7).

However, $J_k = W_k^T W_k = \sum_{t=1}^{L} W_k^2(t)$ is exactly the performance index of the kth channel optimal linear predictor. Because of $J = \sum_{k=1}^{m} J_k$, the problem of the optimal VLP can be changed to a RLS estimating question about m scalars. $\{y_k(t);\ t\in \mathrm{I}\}$ can be expressed by the kth channel p_k order autoregressive series of the source series $\{x(t);t\in \mathrm{I}\}$,

$$x(mn+k) + \sum_{i=1}^{p_k} a_k(p_k, i)x(mn+k-i) = e(mn+k)$$

where $E[e(mn+k)]=0$; $E[e^2(mn+k)]=\sigma_k^2$; While the corresponding optimal vector linear predictor is

$$\hat{y}_k(t) = \hat{x}(mn+k) = -\sum_{i=1}^{p_k} a_k(p_k, i)x(mn+k-i)$$

Its performance index is:

$$J_k = E[e^2(mn+k)] = \sigma_k^2 = min \qquad (11)$$

From here, we can know that the m dimension optimal vector predictor can be obtained by solving the m scalar optimal predictor. And to optimize each channel leads necessarily to optimizing the whole prediction, which must simplify the process of the optimal solution of the VLP.

3 Adaptive Parallel Processing Algorithm

Supposing H is Hilbert space, defining its l dimensions observation subspace as: [4]

$$H_l(k,n) = span\{x(mn+k), x(mn+k-1), \Lambda, x(mn+k-l+1)\} \qquad (12)$$

According to the orthogonal projection principle, the l order forward and backward linear predictive error filter of the kth channel of the source series $\{x(t);t\in \mathrm{I}\}$ at the time n can be given by

$$\begin{cases} f_l(mn+k) = x(mn+k) - \hat{E}[x(mn+k) \mid H_l(k-1,n)] \\ g_l(mn+k) = x(mn+k-l) - \hat{E}[x(mn+k-l) \mid H_l(k,n)] \end{cases} \qquad (13)$$

where $\hat{E}[x(\cdot)|H_l(\cdot)]$ denotes the orthogonal projection of $x(\cdot)$ on $H_l(\cdot)$.After the orthogonalizing analysis of Gram-Schmidt we can get

$$f_{l+1}(mn+k) = f_l(mn+k) - \\ -\frac{E[f_l(mn+k)g_l(mn+k-1)]}{E[g_l^2(mn+k-1)]} \cdot g_l(mn+k-1)$$

$$g_{l+1}(mn+k) = g_l(mn+k) - \\ -\frac{E[f_l(mn+k)g_l(mn+k-1)]}{E[f_l^2(mn+k)]} \cdot f_l(mn+k)$$

And: $f_0(mn+k) = g_0(mn+k) = x(mn+k)$

Command: $R_l(k,n) = E[f_l(mn+k)g_l(mn+k-1)]$;

$E_l(k,n) = E[f_l^2(mn+k)]$; $G_l(k,n) = E[g_l^2(mn+k)]$;

$\alpha_k(l+1) = -\frac{R_l(k,n)}{G_l(k-1,n)}$; $\beta_k(l+1) = -\frac{R_l(k,n)}{E_l(k,n)}$

Then the order recurrence equation of forward and backward predictive errors of the kth channel of the source series at the time n is

$$\begin{cases} f_{l+1}(mn+k) = f_l(mn+k) + \alpha_k(l+1)g_l(mn+k-1) \\ g_{l+1}(mn+k) = g_l(mn+k-1) + \beta_k(l+1)f_l(mn+k) \end{cases} \quad (14)$$

$\alpha_k(l+1)$, $\beta_k(l+1)$ are respectively called the forward and the backward reflection coefficient of the kth channel and the (l+1)th lattice segment; $E_l(k,n)$, $G_l(k,n)$ present respectively the energy of the corresponding forward residual error and the backward; $R_l(k,n)$ stands for the energy of the corresponding cross residual error.

From (14), the forward predictive residual error of the kth channel $f_{l+1}(mn+k)$ can be corrected by using the backward predictive residual error of then (k−1)th channel $g_l(mn+k-1)$, the backward predictive residual error of the kth channel $g_{l+1}(mn+k)$'s correction also base on the backward predictive residual error of the (k−1)th channel $g_l(mn+k-1)$. This is an important difference from the traditional lattice filter. Especially the first channel's predictive residual error must be corrected by $g_l(mn+0)$:

$$g_l(mn+0) = g_l(m(n-1)+m) = z^{-1}g_l(mn+m)$$

In the formula, z^{-1} is the single-step time shift operator，which operates when the flowing time is n; That is to say, when we predict the first channel's residual error recurrence on the time n , we must use the channel m's backward predict residual error $z^{-1}g_l(mn+m)$ to corrected it. The predict error's order correction process is carried on circularly based on the channel's series number 1→2→Λ→m→1.

Derived from this, we can get the process of the adaptive parallel processing algorithm:

$$k = 1,2,\Lambda\ ,m$$
$$l = 1,2,\Lambda\ ,p_k-1$$
$$E_l(k,n) = E_{l-1}(k,n) + \alpha_k(l)\xi_{l-1,l}(k,n)$$
$$G_l(k,n) = G_{l-1}(k-1,n) + \beta_k(l)\xi_{l-1,l}(k,n)$$
$$i = l+1, l+2, \Lambda\ , p_k$$
$$\begin{bmatrix} \xi_{l,i}(k,n) \\ \eta_{l,i}(k,n) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_k(l) \\ \beta_k(l) & 1 \end{bmatrix} \begin{bmatrix} \xi_{l-1,i}(k,n) \\ \eta_{l-1,i-1}(k-1,n) \end{bmatrix}$$
$$\alpha_k(l+1) = -\xi_{l,l+1}(k,n)/G_l(k-1,n)$$
$$\beta_k(l+1) = -\xi_{l,l+1}(k,n)/E_l(k,n)$$

4 Signal Series Simulation Test

4.1 Stable signal series simulation

For testing adaptive parallel processing algorithm's capabilities in estimating the predicting parameters, we adopt a data series generated by a four order autoregressive model to simulate a frame signal series. We can suppose that the frame length T=300 , the channel number m=3 and AR(4) model is [5]

$$x(t)-2.7607x(t-1)+3.8106x(t-2)-$$
$$-2.6535x(t-3)+0.9238x(t-4)=e(t) \quad (15)$$

In the formula, $e(t)$ is the normal white noise series of the average μ=0 and variance σ^2=1. Using this algorithm to proceed the computer simulation of the signal samples data series $\{x_1,x_2,x_3,\cdots,x_T\}$ simulated by the formula (15). The Comparison of the results of the simulation can be seen at Tab.1 and the distinguishing process curve of the prediction coefficient on Fig.1.

From Tab.1 and Fig.1, higher predictive precision. is obtained with adaptive parallel processing algorithm to predict parameters of stable signal series. All the relative errors between the predict estimations and the true values are less than 3.8%. The speed of convergence is fast, too. The prediction rapidly approaches to the true value when n is about to 30.

Tab.1 The simulated result of stable signal series

Parameters	a_1	a_2	a_3	a_4
True value	-2.7607	3.8106	-2.6535	0.9238
Forward prediction	-2.7699	3.7935	-2.6187	0.8970
Backward prediction	-2.7640	3.7761	-2.6004	0.8889

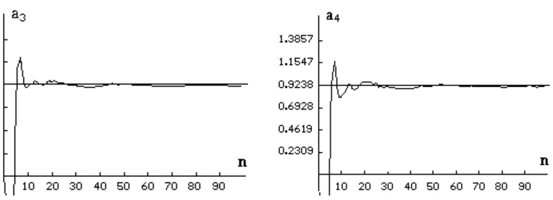

Fig. 1 The simulated result of stable signal series

4.2 Unstable signal series simulation

Actually, speech and image series is a slow time-varying unstable series. There is some errors when using stable series to simulate the signal series. We can use a slow time-varying unstable series to generate signal sample series, and then using the algorithm to test it. Unstable signal sample series can be generated by a four order model:

$$x(t)+a_1x(t-1)+a_2x(t-2)+$$
$$+a_3x(t-3)+a_4x(t-4)=e(t) \quad (16)$$

Different from formula (15), parameters a_1, a_2, a_3 and a_4 vary with time slowly. Supposing parameters a_1, a_2, a_3 and a_4 change as follows:

① The series is stable when t is between 0 and 200, the model parameters are:

a_1= −2.7613; a_2=3.8125; a_3= −2.6520; a_4=0.9224

② The series is unstable when t is between 200 and 500, the model parameters change slowly:

$$\begin{cases} a_1 = -[1.2599 + 2r_1 \cos \phi_1(t)] \\ a_2 = r_1^2 + 2.5198\, r_1 \cos \phi_1(t) + 0.9604 \\ a_3 = -[1.2599\, r_1^2 + 1.9208\, r_1 \cos \phi(t)] \\ a_4 = 0.9604\, r_1^2 \end{cases}$$

③ The series is stable again when t is between 500 and 700, the model parameters are:

$a_1 = -2.2499$; $a_2 = 3.1878$; $a_3 = -2.1856$; $a_4 = 0.9413$

Next, we operate the simulate computing with the index weighting and affiliation Hamming form separately.

***a.* Index weighting** index weighting is the simplest and common-used weighting method. And its error rule is

$$\sum_{i=0}^{t} \lambda^i e_{p,i}^2 = e_{p,t}^T \Lambda e_{p,t}$$

λ is index weighting factor and its value between 0 and 1.

$$\Lambda = diag\{1, \lambda, \lambda^2, \Lambda\Lambda, \lambda^{L-1}, \lambda^L\}$$

After index weighting, the recursive formula of the sample convariance series estimation is:

$$\phi_{0,j}(k,n) = \lambda \phi_{0,j}(k,n-1) + \psi_j(k,n) \qquad (17)$$

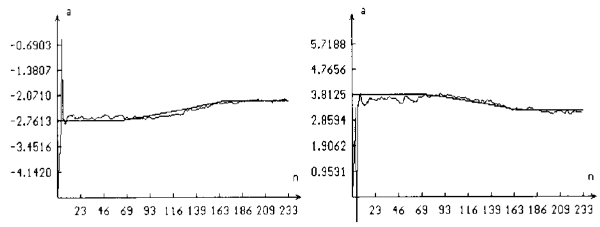

Fig. 2 The simulated result after the index weighting

After index weighting, the simulated result shows in Fig.2, which explains that this argorithm has better track ability to unstable signal series.

***b.* Import window function** Window function is a common weight function in digital signals processing. Hamming window, which contain a higher frequency resolution and lower frequency spectrum leakiness. so we add the Hamming window in the algorithm.

$$w(i) = \begin{cases} 0.54 - 0.46\cos(2\pi i/T - 1) & 0 \le i \le T-1 \\ 0 & else \end{cases}$$

the sample convariance series estimation is:

$$\phi_{0,j}(k,n) = 0.54 H_j(k,n) - 0.46 F_j(k,n) \qquad (18)$$

where $H_j(k,n) = \sum_{i=0}^{T-1} \psi_j(k,n-i)$;

$$F_j(k,n) = \sum_{i=0}^{T-1} \cos\frac{2\pi i}{T-1} \psi_j(k,n-i)$$

The simulated result are showed in Fig. 3, which explains the algorithm with Hamming window contain a better capability of tracting the unstable signal series. Its track result is similar to index weighting method.

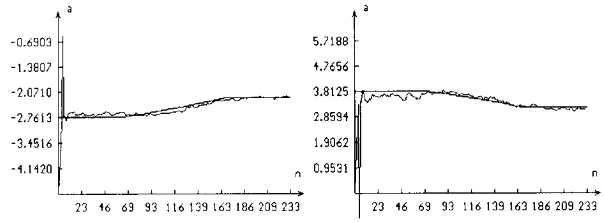

Fig. 3 The simulated result after import Hamming window

5 Conclusion

After the above simulated computing of the stable and unstable signal series, we can conclude that, the adaptive parallel processing algorithm is an efficient estimating method on the speech and image signal series parameters. The signal series parameters from this algorithm have lower error possibilities and higher precision. The algorithm gives us a way to realize the parallel processing through the scalar operation, and to solve the tough problem that the estimation of the parameters of AR model must carry on the calculation of the matrixes in traditional lattice algorithm. In this way, we can reduce the calculating quantity consumedly.

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