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Virtual power plant-based distributed control strategy for multiple distributed generators

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Abstract: A distributed control strategy is developed to control the output of multiple distributed generators (DGs) in a coordinated fashion such that these generators develop into a virtual power plant (VPP) in a distribution network. To this end, cooperative control methodology from network control theory is used to make the VPP converge and operate at an optimal output, which is determined by the DGs' costs and the necessary service assigned by the distribution network. For each DG, the strategy only requires information from its neighbouring units, making communication networks (CNs) among the DGs simple and robust. A set of sufficient conditions under which the proposed method is valid are provided. It is shown that the proposed strategy has the advantages that the corresponding CNs are local and there is no central station collecting global information from the DGs. These features enable the VPP to have both self-organising and adaptive coordination properties. The proposed method is simulated using the IEEE standard 34-bus distribution network.

1 Introduction

In recent years, a large number of distributed generators (DGs) have been integrated into modern distribution networks because of their clean and renewable characteristics. However, the high penetration of renewable DGs introduces many technical challenges to the industry [1, 2]. Thus, it is often necessary to control the output of DGs according to certain grid codes.

For a large number of DGs, it is difficult to control their output in a coordinated fashion, especially when the output of the generators varies rapidly and randomly. One of the viable solutions to this difficult problem is to aggregate the DGs such that they appear as a whole like a conventional generator with more stable behaviour. Several aggregation approaches have been reported in the literature, for example, active distribution networks, micro-grids, virtual utilities and virtual power plants (VPPs) [3, 4] etc.

The concept of VPPs has been introduced and studied systematically in recent years [1, 3–5]. It is also under investigation in many ongoing projects such as FENIX and the virtual fuel cell power plant [6]. The basic idea is that several DGs or other controllable elements are considered together and their total output is controlled to resemble a traditional power plant. Therefore a VPP can provide some ancillary services [5, 7], such as voltage regulation and frequency regulation and so on.

To realise a VPP, one of the most important issues is to fairly control the output of the DGs within the VPP via a certain communication topology, similar to a resource allocation problem [8]. In general, a centralised control mode with system-wide information sharing can be used. For example, optimal power flow based approaches can be used to dispatch the DGs when they are few in number [9, 10]. However, for a distribution network with numerous and geographically dispersed DGs, such centralised controls are prohibitively expensive to be implemented and the resulting system would not be robust or efficient, since the abnormal operation of a single communication channel may affect the overall system.

Instead, the almost possible solution is a distributed control configuration which can inherit the best features of both centralised and decentralised controls while limiting their disadvantages [2, 11, 12]. The distinctive feature of an appropriately designed distributed control is that it only requires local communication networks (CNs) and is robust to the CNs [13, 14]. Specifically, a DG should incorporate information available from its neighbouring generators into its control law. In fact, such a control configuration is in line with the concept that a future smart grid would include communication among network elements [15].

The distributed control has been used in many engineering areas [12, 13, 15–17], such as aircraft control, Robert control, wind turbine control and so on. It can also be used in power systems, in which the potential applications have been investigated in [11, 18–20]. In [11], a distributed control is provided for the power output coordination problem of multiple photovoltaic units such that the total power output makes active power flow across a certain transmission line constant. Similarly, a distributed algorithm is studied in [18] for output regulation of multiple DGs. In those methods, the total aggregated power output needs can be measured from a busbar where information of the total active power can be acquired. Consequently, these methods are good for some special power networks such as a radial power network, in which the total power output can be easily measured; otherwise the output of every DG must be measured directly and it needs a centre collecting global information. In other words, these strategies are not purely distributed since some global information is required in the control laws.

In this paper, a purely distributed control strategy is proposed for multiple DGs such that the DGs can easily constitute a VPP. The proposed method can control several DGs which exchange information among their neighbouring units. Moreover, in the VPP the cost function of the DGs is considered in the control law. The distinct characteristics of the proposed control strategy are that it only uses local CNs and the total power output of the DGs need not be measured directly, but the active power output of the VPP is self-optimising and converges to expected operational points. Therefore the strategy provides a possible way for the DGs to realise the plug-and-play property. Typical communication setups are discussed and sufficient conditions under which the proposed method is valid are presented as well.

The remainder of the paper is organised as follows. In Section 2, the problem to be solved is discussed. A distributed control strategy for the VPP is given in Section 3. In Section 4, the simulation based on the IEEE 34-bus benchmark distribution network is introduced to illustrate the validity of the proposed strategy. Conclusions are drawn in Section 5.

2 Problem formulation

In this paper, the concept of the VPP means that several geographically dispersed DGs are considered together and their total output is controlled to mimic that of a traditional power plant, which satisfies the requirements of the distributed energy management system (DEMS). Fig. 1 shows a VPP including four DGs.

Generally, suppose the VPP includes n DGs and the dynamics for the ith DG are

$$\begin{gathered} T_i \dot{P}_{Gi} = -P_{Gi} + P_{Gi}^{\text{ref}} \\ i = 1, 2, \ldots, n \end{gathered} \tag{1}$$

where P_{Gi} is the active power output of the ith DG, and P_{Gi}^{ref} is the input to be designed, T_i is the time constant. To realise the VPP with communication and information as the enabling technology, a distributed control which uses local CNs is a good choice. The main goal of this paper is to develop a distributed control strategy for system (1) and design the communication topology among the DGs such that the output of the VPP can be self-optimising and converge to an expected operational point. The expected operational point confirms that the output of the DGs is optimal with respect to the cost functions and provides necessary service(s) required by the DEMS.

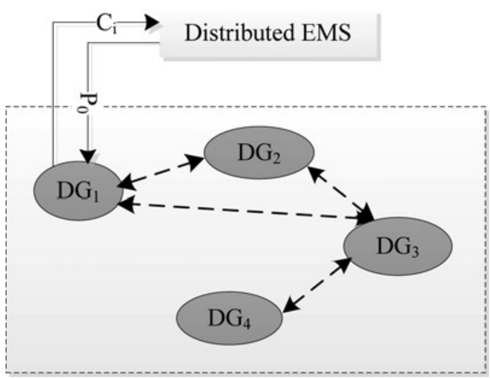

Fig. 1 *VPP including four DGs connected by distributed CNs*

The distributed control strategy to be studied is shown in Fig. 1, where the DGs within the VPP share information using the distributed CNs. Moreover, to maintain the total power output P_0 required by the DEMS, no centre station collecting global information from the DGs (e.g. every DG's output and its cost function) is needed. Thus, the expected strategy has the possibility to realise the plug-and-play property when there are a large number of DGs.

In summary, the expected distributed control algorithm can be formulated as the following problem:

Problem 1: Consider a VPP whose dynamics are described by (1). A distributed control strategy which avoids global information from all the DGs is to be designed for all the P_{Gi}^{ref} such that the trajectories of system (1) converge to the optimal solution of the following problem

$$\begin{aligned} &\min_{P_{Gi}} C(P_{G1}, P_{G2}, \ldots, P_{Gn}) := \sum_{i=1}^{n} C_i(P_{Gi}) \\ &\text{s.t.} \quad P_{G1} + P_{G2} + \cdots + P_{Gn} = P_0 \end{aligned} \tag{2}$$

where $C_i(P_{Gi})$ ($i = 1, 2, \ldots, n$) are the DGs' cost functions, which are assumed to be smooth and convex, and P_0 is the total required power usually determined by the DEMS in a distribution network.

Once the control strategy is implemented, the CNs are also distributed and the CNs with long distances can be avoided. Moreover, the CNs can be designed with abundance, so the proposed strategy can be much robust with respect to the CNs.

In the subsequent analysis, the following conditions are assumed to be satisfied.

Assumption 1: For all i, all the functions $C_i(\cdot)$ are smooth and convex as $d^2 C_i(x_i)/\mathrm{d}x_i^2 \geq q_{\min} > 0$.

Remark: The assumption implies that $C_i(\cdot)$ are strict convex functions. This property is usually assumed in distributed optimisation (e.g. [8, 21] and the references therein). This condition is commonly assumed in the literature. Moreover, Assumption 1 also implies that optimisation of (2) has a unique optimal solution.

Clearly, Problem 1 can be solved by a centralised mode based approach, where all the information of the DGs is acquired by a centre controller. However, in the future smart grid there may be too many objects to be handled, making the CNs very large and expensive to be implemented. In addition, even if the CNs can be realised, they would not be robust when the number of communication channels becomes large.

Motivated by the advantages of network control [12], we will use a distributed algorithm to solve Problem 1. Precisely speaking, we want to alleviate the disadvantage of the centralised mode which requires global information

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within the VPP, thus a distributed control which only requires neighbouring information is preferred. The proposed control strategy has the following form

$$P_{Gi}^{\text{ref}} = w_i(s_{i0}P_0, s_{i1}P_{G1}, s_{i2}P_{G2}, \ldots, s_{in}P_{Gn}),\quad i = 1, 2, \ldots, n$$

where P_0 denotes the total required power; P_{Gi} represents the output of the ith DG; $S=(s_{ij})$ is the matrix of the communication topology and is defined by (3), where $s_{ii}=1$ is satisfied for all i; $s_{ij}=1$ if the output of the jth DG is known to the ith DG at time t; and $s_{ij}=0$ if otherwise. The information of the total required power P_0 is considered to be the 0th DG.

The communication matrix determines the information that is used in the control law. It should be noted that the control law in Problem 1 will avoid global information from all the DGs. That is to say, in the control law of the ith DG, there exists some j such that $s_{ij}=0$ is satisfied. In fact, in the distributed algorithm to be given later, only the neighbouring information is used for every DG

$$S = \begin{bmatrix} s_{10} & 1 & s_{12}(t) & \cdots & s_{1n}(t) \\ s_{20} & s_{21}(t) & 1 & \cdots & s_{2n}(t) \\ \vdots & \vdots & & 1 & \vdots \\ s_{n0} & s_{n1}(t) & s_{n2}(t) & \cdots & 1 \end{bmatrix} \in R^{n\times(n+1)} \tag{3}$$

3 Distributed control strategy

To yield a distributed control, we consider the following transformation by introducing some auxiliary variables, say u_i $(i=1, 2,\ldots,n)$

$$P_{Gi} = -u_i + \sum_{j=1}^{n} d_{ij}u_j + d_{i0}P_0 \tag{4}$$

that is

$$\mathbf{P}_G = (-\mathbf{I} + \mathbf{D})\mathbf{u} + P_0\mathbf{d}_0 \tag{5}$$

where $\mathbf{I}$ is the identity matrix of appropriate dimensions; $\mathbf{P}_G=[P_{G1}, P_{G1},\ldots,P_{Gn}]^{\mathrm{T}}$, $\mathbf{d}_0=[d_{10}, d_{20},\ldots,d_{n0}]^{\mathrm{T}}$, d_{ij} is the ith row, jth column entry of the matrix $\mathbf{D}\in R^{n\times n}$

$$d_{i0} = s_{i0}/\left(\sum_{i=1,2,\ldots,n} s_{i0}\right),\quad d_{ij} = s_{ij}/\left(\sum_{j=1,2\ldots,n} s_{ij}\right) \tag{6}$$

The definition of d_{ij} in (6) implies that the matrix $\mathbf{D}$ is row-stochastic and the summation of d_{i0} is 1, so it can follow from expression (4) or (5) that

$$\mathbf{1}^{\mathrm{T}}\mathbf{P}_G = \mathbf{1}^{\mathrm{T}}(-\mathbf{I} + \mathbf{D})\mathbf{u} + P_0\mathbf{1}^{\mathrm{T}}\mathbf{d}_0 = P_0 \tag{7}$$

which implies that the constraint of (2) is satisfied. Hence, the optimisation problem of (2) is changed into the following unconstrained optimisation

$$\min_{\mathbf{u}} C(\mathbf{u}) = \sum_{i=1}^{n} C_i\left(-u_i + \sum_{j=1}^{n} d_{ij}u_j + d_{i0}P_0\right) \tag{8}$$

where $\mathbf{u}=[u_1, u_2,\ldots,u_n]^{\mathrm{T}}$.

The following lemma shows that the optimisation of (8) is equivalent to the optimisation of (2).

Lemma 1: Suppose that the following conditions are satisfied:

1. The communication matrix $\mathbf{S}$ is irreducible and symmetrical.
2. Assumption 1 is satisfied.

Then the optimal solution to problem (2) is the same as that of problem (8), where the map between their decision variables is given by (4).

Proof: It follows from the conditions that both the optimisation problems of (2) and (8) are convex and thus they have unique optimal solutions, say C_{PG}^* and C_u^*, respectively. In addition, it follows from the first given condition and formula (6) that D is non-negative, irreducible, symmetrical and doubly stochastic, that is, there are $D=D^{\mathrm{T}}$, $D\mathbf{1}=\mathbf{1}$ and $\mathbf{1}^{\mathrm{T}}\mathbf{d}_0=1$.

Since $\mathbf{D}$ is a row-stochastic and symmetrical matrix, it follows from (4) and (7) that there is

$$\sum_{i=1}^{n} P_{Gi} = P_0\sum_{i=1}^{n} d_{i0} - \sum_{i=1}^{n} u_i + \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}u_j$$

$$= P_0 + \sum_{i=1}^{n} u_i - \sum_{j=1}^{n} u_j = P_0 \tag{9}$$

Therefore for a solution to (8), we can obtain a feasible solution to (2) [induced by the map shown in (4)], which implies that the optimal value of (8) is not larger than that of (2), that is

$$C_u^* \le C_{PG}^* \tag{10}$$

On the other hand, for a feasible solution to (2), say, P_G^*, the irreducibility of $\mathbf{S}$ implies that $I - D$ is rank $n-1$ [12], so there exists an $\mathbf{u}$ satisfying (4) in accordance with P_G^*. Thus, there is

$$C_{PG}^* \le C_u^* \tag{11}$$

Therefore (10) and (11) lead to the conclusion that the optimal solution to (2) is the same as that of (8). □

Remark: In the lemma, the term of the irreducibility of a communication matrix is used to denote the connectivity of the CNs. This term is equivalent to that of the term 'strongly connected' in graph theory and an easy method to verify the irreducibility of a CN is provided in [12].

Lemma 1 shows that the constrained optimisation problem (2) can be solved by solving the unconstrained problem (8). Moreover, the unconstrained problem (8) is a convex problem and every $C_i(\cdot)$ is only related to the neighbouring information determined by the CNs, hence it is possible to use a distributed gradient algorithm to solve the problem. To arrive at this goal, we use the following distributed

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strategy

$$\begin{cases} \dot{u}_i = k_0\left(\lambda_i - \sum_{j=1'}^{n} d_{ij}\lambda_j\right) \\ \lambda_i = \partial C_i(P)/\partial P_{Gi}\big|_{P=P_{Gi}} \\ P_{Gi}^{\text{ref}} = -u_i + \sum_{j=1}^{n} d_{ij}u_j + d_{i0}P_0 \end{cases} \tag{12}$$

$$i = 1, 2, \ldots n$$

where k_0 is a given positive gain, λ_i is an intermediate variable, which can be considered to be the price of the ith DGs power.

Clearly, to implement the proposed strategy, the CNs used by the ith DGs are the same as those used in (4). Intuitively, it follows from the network control theory that if the gains are chosen deliberately, the trajectories introduced by system (1) and (12) will be convergent to the equilibrium. Moreover, (12) implies that all the λ_i are the same value at the equilibrium, that is, the marginal cost of every DG is the same, thus the trajectories indeed converge to the optimal solution of (2).

To prove these results, we consider that the renewable DGs constituting a VPP are connected to the power network via PWM inverters, so the dynamics of the DG can be generally ignored [i.e. T_i is very small in (1)] compared with the time scale of the control strategy (12) [in fact, (12) is a secondary control in the power system and the DGs' dynamics are much faster than it]. Thus, we first ignore the dynamics of the DGs. Namely, $P_{Gi}^{\text{ref}} = P_{Gi}$ are satisfied for all $i = 1, 2, \ldots, n$.

Lemma 2: Assume that communication matrix $\mathbf{S}$ is irreducible, then the equilibrium of (12) satisfies $\boldsymbol{\lambda} = c\mathbf{1}$, where c is a constant.

Proof: This conclusion can be directly obtained from the results in [12] (in Section 5.1.1). □

Lemma 3: Suppose Assumption 1 is satisfied, then there exists a $\beta_0 > 0$ such that

$$||\nabla C(\mathbf{u})||^2 \geq \beta_0\big(C(\mathbf{u}) - C(\mathbf{u}^*)\big) \tag{13}$$

is satisfied, where $\mathbf{u}^*$ denotes the optimal solution to (8).

Theorem 1: Assume the conditions in Lemma 1 are satisfied, then the trajectories of (12) exponentially converge to the optimal solution to (8).

The proofs of Lemma 3 and Theorem 1 are included in the appendix.

It follows from Theorem 1 that the proposed approach guarantees that the expected solution of Problem 1 can be obtained. Note that the auxiliary variables in accordance with the optimal solution are not unique. For instance, suppose that $\mathbf{u}_0 = [u_{10}, u_{20}, \ldots, u_{n0}]^{\mathrm{T}}$ is an equilibrium of (12), then $\mathbf{u}_0 + c\mathbf{1}$ is also an equilibrium for every $c \in R$. In fact, c is determined by the initial states of (12), similar to the results in cooperative control theory, where the consensus is satisfied but the value is purely determined by the initial states [12].

To further explain the proposed approach, we give the following remarks:

1. Each DG only uses its neighbouring units' information and a centralised controller is not necessary to collect global feedback information (i.e. there is no explicit information on total DGs' output in the proposed algorithm). Therefore the strategy is distributed and only local CNs are required. Moreover, if some DGs join or leave the VPP, only the CNs corresponding to these DGs need to be activated or deactivated, hence the proposed strategy has both self-organising and adaptive coordination properties. In other words, the VPP-based distributed control mode provides a possible way to make the DGs supply power in a plug-and-play manner.
2. The equality constraint can be satisfied quickly when the required power P_0 of the VPP is changed, hence the consensus rate is almost determined by the CNs (in fact, the rate is determined by the Fiedler value of the CNs [22]). However, if the algorithms proposed in [11, 18] are used, the gradient of the total power is used to update the units' output, hence the convergent rate depends on not only the Fiedler value but also the leader's dynamics. As is well known in power systems it is of utmost importance to keep power balance and economics as a secondary issue. Therefore in a practical power system, quick response of total power is an advantage of the proposed method.
3. Control saturation is ignored for all DGs' output. Indeed, the proposed method can be extended to the saturation case by adding some minor revisions. For example, if the ith generator's output goes to the upper (or lower) bound, we can revise its cost function by adding a sufficiently positive (or negative) function as a penalty and keep the revised cost function to be smooth and convex, thus convergence can be guaranteed.
4. There are many choices for the CN satisfying the proposed algorithm as long as the topology of the CN is connected. Therefore it is important to design the topology of CNs such that information can be transferred economically and correctly. However, the robustness and the cost are generally two contrary indices in practical systems, thus a possible way is to use the optimisation method with the convergence rate being an additional constraint. However, discussion on the optimisation model of the CN is out of the scope of our paper.

As previously mentioned, a VPP is generally composed of renewable generators, which are connected to the power system by very fast PWM inverters, so the dynamics of the DGs can be ignored. However, if the dynamics of the DGs are not ignored, the distributed control strategy can be considered to be a high-level control (e.g. secondary control), such that if the gain in (12) is a reasonable value then the closed-loop system is asymptotically stable. This leads to the following corollary.

Corollary 1: Assume the conditions given in Theorem 1 are satisfied, then there exists a constant $k^* > 0$ such that for every $k \in (0, k^*)$ the dynamic system made up of (1) and (12) is asymptotically stable and its trajectories will converge to the optimal solution to problem (2).

Proof: From the results of Theorem 1, we know that the trajectories introduced by the proposed algorithm are exponentially stable when the inner dynamics of the DGs are ignored. Clearly, when k^* is small, the closed-loop system including (1) and (12) can be considered to be a singularly perturbed system where (12) is much slower than (1). The remaining part of the proof is very similar to the stability analysis of a singularly perturbed system [23], similar steps can be found in [20]. □

Remark: In Corollary 1, k^* is related to the response rate of the DGs' dynamics, that is, the inertial time constant T_i in (1). If T_i is quite large for some i, then k^* must be small in order to make the closed-loop system asymptotically stable.

4 Simulation study

The benchmark IEEE 34-bus distribution network is used to illustrate the effectiveness of the proposed control strategy. The network has a voltage level of 24.9 kV and 16 DGs are connected to it. The topology is shown in Fig. 2.

The parameters of the system can be found at the link http://ewh.ieee.org/soc/pes/dsacom/testfeeders.pdf. Basic operational conditions and dynamic models are:

External grid: An infinite bus (1.0 p.u.).

Loads: 1.45 MW + j0.85 MVAR. The composite model is considered for all the loads, in which a 50% dynamic part and a 50% constant impedance part are used. The details can be found in [11].

We assume that the VPP includes all the DGs and the CN among the DGs is denoted by matrix **S** shown in (14), where G_i denotes the ith DG ($i = 1, 2, \ldots, 16$).

$$S = \begin{array}{c} \begin{matrix} G_0 & G_1 & G_2 & G_3 & G_4 & G_5 & \cdots & G_{16} \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & \vdots & \vdots & \vdots & \vdots & 1 & \ddots & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{matrix} \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ \vdots \\ G_{16} \end{matrix} \tag{14}$$

The initial output of the VPP is 2.0 MW + j0.12 MVAR and satisfies the optimisation of (2). It is assumed that the 1st, 3rd, 7th and 16th DGs receive the command of total power from the DEMS and their corresponding coefficients are all 0.25 (i.e. $d_{10} = d_{20} = d_{70} = d_{16,0} = 0.25$). In addition, the DGs are classified into four groups in which the following quadratic cost functions are considered

$$C_i(P_{Gi}) = 0.5a_i P_{Gi}^2 + P_{Gi}$$

where the coefficients are shown in Table 1.

The dynamic responses of the VPP under some disturbances will be studied when the proposed strategy is implemented. Three kinds of disturbances are considered: (i) the specified power output of the VPP is changed; (ii) the cost functions of some DGs are changed; and (iii) a short-circuit fault occurs and is cleared in 0.1 s. Specifically, the total active power requirement assigned to the VPP is changed from the initial value 2.0–3.0 MW at 2.0 s and changed to 1.6 MW at 12.0 s. The cost functions of DGs in group 2 are changed to $C_i(P_{Gi}) = 1.2P_{Gi}^2 + P_{Gi}$ at 22.0 s; the fault occurs at bus (a) as shown in Fig. 2.

4.1 Responses of the VPP under changed specified power and cost functions

When the specified VPP's power output required by the distribution network is changed, it follows from Section 3 that the P_0 is changed and thus the information will be received by the 1st, 3rd, 7th and 16th DGs. Other DGs need not obtain the signal. Clearly, global information is avoided and only local CNs are used for transmission of the signal.

The responses of the DGs under the distributed control strategy are shown in Figs. 3 and 4, which shows that the proposed control strategy can make the VPP's output converge asymptotically. Clearly, it follows from Fig. 4*a* that the total power generated by the VPP is immediately changed from the initial value 2.0 MW to the given value. Fig. 4*b* shows that the same increment cost is realised by

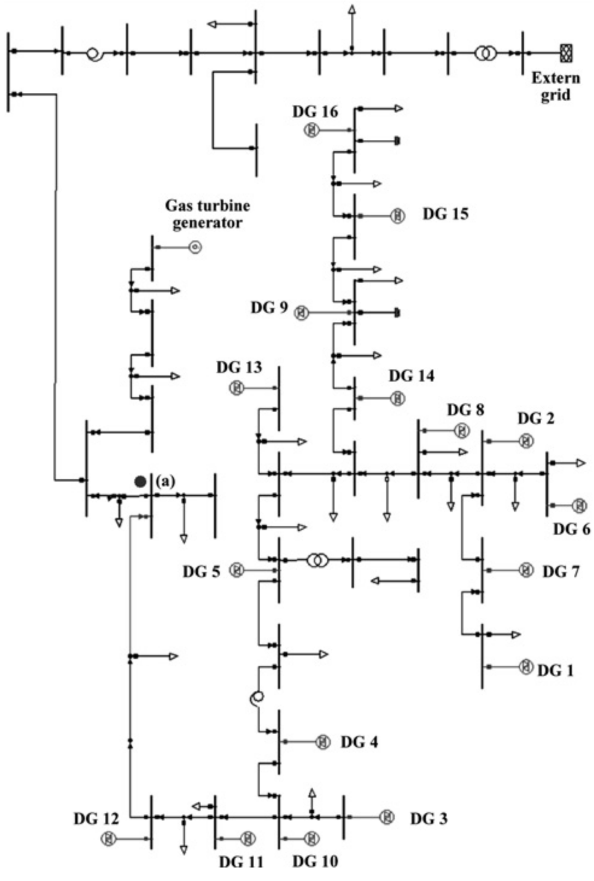

Fig. 2 *Diagram of the IEEE 34-bus network*

Table 1 Coefficients of the cost functions and inertial time constants for DGs

DGs	i	a_i	T_i, s
group 1	1, 5, 9, 13	1.0	0.1
group 2	2, 6, 10, 14	1.5	0.1
group 3	3, 7, 11, 15	2.0	0.1
group 4	4, 8, 12, 16	2.5	0.1

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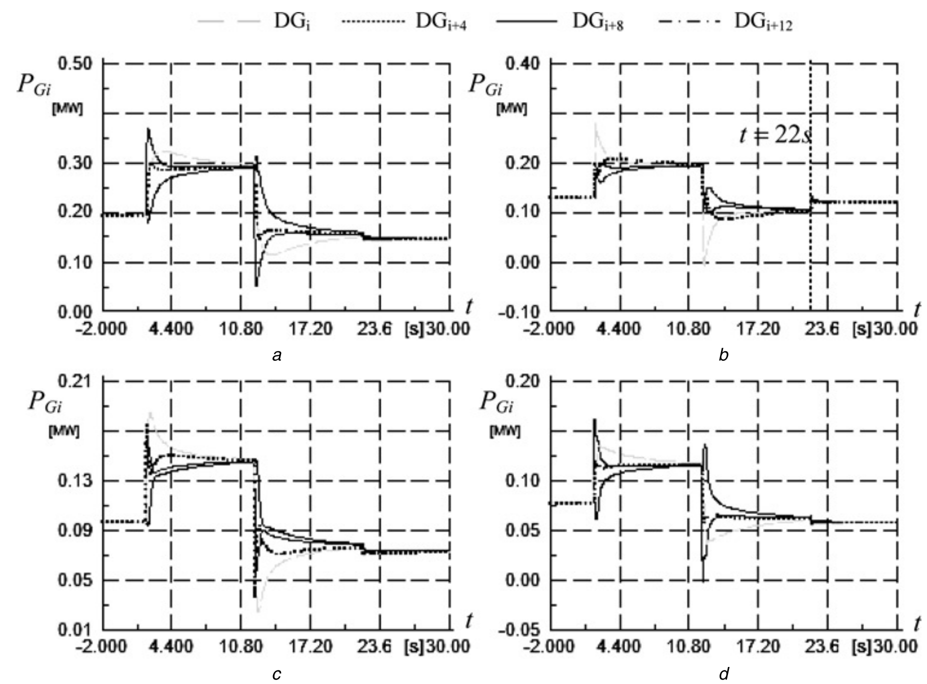

Fig. 3 *Power output of group 1 under change of the VPPs specified power*

a Active power of the DGs in group 1
b Active power of the DGs in group 2
c Active power of the DGs in group 3
d Active power of the DGs in group 4

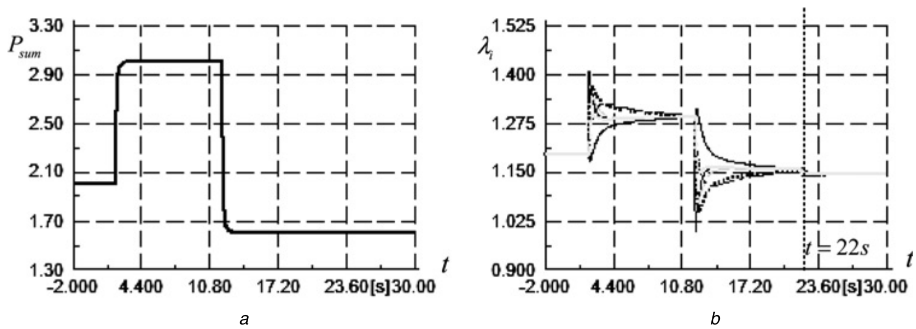

Fig. 4 *Responses of total power and λ under change of total power requirement*

a Total power of all DGs
b Response of the consensus variables of DG_1–DG_8

the proposed strategy, making the DGs' power output converge to the unique optimal solution of (2).

It should be noted that the inertial time constants are considered in the DGs (i.e. $T_i > 0$ as shown in Table 1), but the stability of the trajectories induced by the proposed control strategy is still guaranteed, which is consistent with the conclusion in Corollary 1.

Also note from Fig. 3 that the DGs' output contains some transients induced by the proposed strategy; the transients induced by the local dynamics of DGs are small since the inertial time constants are small in this example. However, as discussed in Section 3, the total power does not contain transients in theory, which is illustrated by Fig. 4*a*. Thus, the proposed method fits to make the DGs provide some ancillary service requiring quick response of total power (e.g. frequency adjustment during some severe conditions). Clearly, this property is also important in a micro-grid since

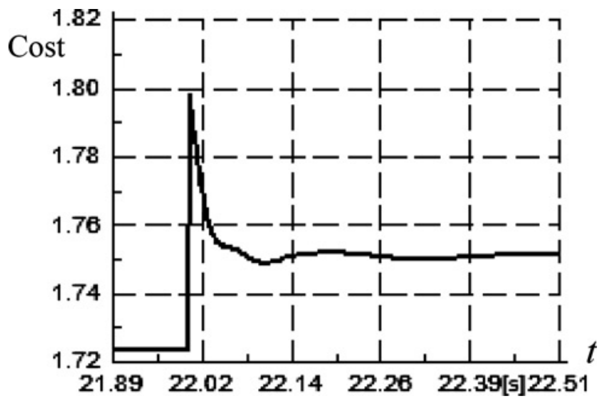

Fig. 5 *Total costs of all DGs when the cost functions of the DGs in group 2 are changed*

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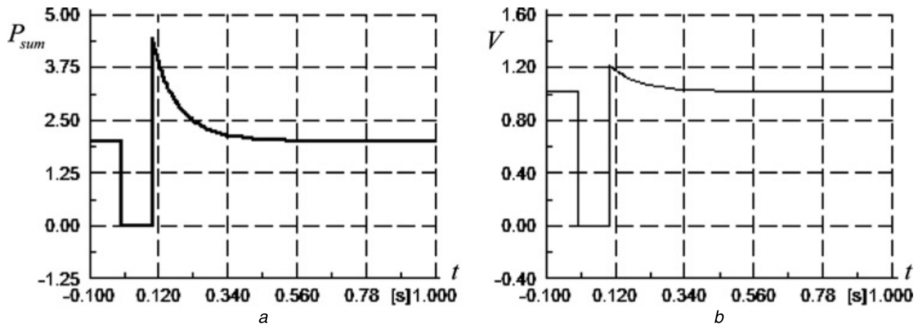

Fig. 6 *Responses of total cost and voltage under a short-circuit fault*

a Total power of all DGs
b Voltage response

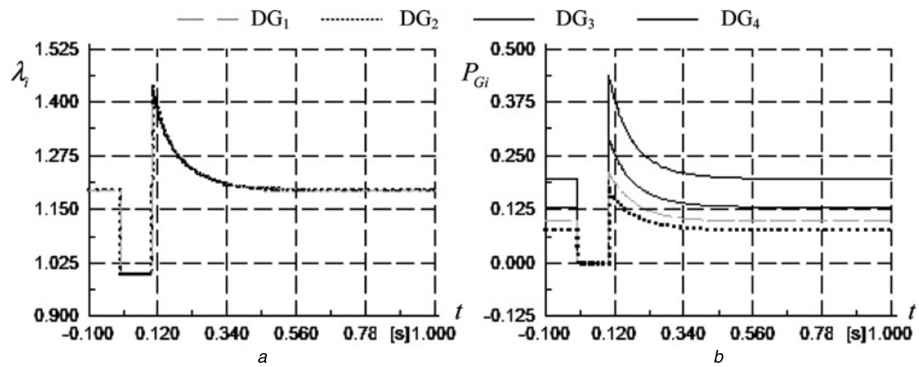

Fig. 7 *Responses of DGs 1, 2, 3 and 4 under a short-circuit fault*

a Response of λ_i
b Response of power P_{Gi}

it is difficult to keep energy balance when there are many DGs of variable output.

Figs. 3 and 4 also plot the results of the VPPs responses when the cost functions of DGs in group 2 are changed at 22.0 s. It follows from Fig. 3 that when the cost functions of some DGs decrease then the output of the corresponding DGs increases. Meanwhile, a new optimal solution of (2) can be obtained along with the trajectories induced by the proposed strategy. The dynamic process is shown in Fig. 5, in which the total cost of the VPP increases immediately when the cost functions of the DGs in group 2 increase and the total cost can converge to a new value in about 2.5 s.

Note that the optimal power output of all the DGs can be obtained by an iterative way and the dynamics are good. However, if the DGs can be controlled by a centralised mode, the responses of a changed command can be ignored. From this viewpoint, the distributed algorithm is worse than the centralised one. However, the figures show that the final optimal values are the same, hence the distributed algorithm is better for the case because the centralised one is not applicable or is not economical when robustness of the CNs is required.

4.2 *Responses of the VPP under short-circuit fault*

To observe the dynamics of the proposed method under an extreme condition, we set a short-circuit fault at bus (a), as shown in Fig. 2. The responses of the VPPs output and the voltage of the faulted bus are plotted in Fig. 6. The power outputs of the representative DGs (one from each group) are shown in Fig. 7.

Clearly, the results in Figs. 6 and 7 show that there are transient processes when a disturbance occurs, but the fluctuation of the VPP' output decays quickly when the disturbance is cleared. Similar results on λ_i are shown in Fig. 7. Therefore the simulation shows that the proposed method can make the VPP return to the initial state even under a severe fault.

5 Conclusion

A centre-free algorithm is developed to coordinately control the power output of the DGs in a VPP. The proposed strategy uses local CNs among the DGs, and it can autonomously adjust the VPP's output to converge to an optimal operational point. In such a fashion, the DGs can provide necessary service assigned by the distribution control centre. As long as the designed CNs meet sufficient conditions on information exchange, the proposed strategy ensures convergence. Thus, the proposed VPP-based distributed control mode provides a possible way for numerous DGs to supply power in a plug-and-play manner. Simulations of the IEEE benchmark distribution network are used to validate the features and effectiveness of the proposed method. Future work should be conducted on the design of topology among the DGs.

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7 References

1 Mashhour, E., Moghaddas-Tafreshi, S.M.: 'Bidding strategy of virtual power plant for participating in energy and spinning reserve markets – part I: problem formulation', *IEEE Trans. Power Syst.*, 2011, **26**, (2), pp. 949–956

2 Keane, A., Ochoa, L.F., Vittal, E., Dent, C.J., Harrison, G.P.: 'Enhanced utilization of voltage control resources with distributed generation', *IEEE Trans. Power Syst.*, 2011, **26**, (1), pp. 252–260

3 Ruiz, N., Cobelo, I., Oyarzabal, J.: 'A direct load control model for virtual power plant management', *IEEE Trans. Power Syst.*, 2009, **24**, (2), pp. 959–966

4 Pudjianto, D., Ramsay, C., Strbac, G.: 'Virtual power plant and system integration of distributed energy resources', *IET Renew. Power Gener.*, 2007, **1**, (1), pp. 10–16

5 Mashhour, E., Moghaddas-Tafreshi, S.M.: 'Bidding strategy of virtual power plant for participating in energy and spinning reserve markets – part I: problem formulation', *IEEE Trans. Power Syst.*, 2011, **26**, (2), pp. 949–956

6 Braun, M., Strauss, P.: 'A review on aggregation approaches of controllable distributed energy units in electrical power systems', *Int. J. Distrib. Energy Resour.*, 2008, **4**, (4), pp. 297–317

7 Baran, M.E., El-Markabi, I.M.: 'A multiagent-based dispatching scheme for distributed generators for voltage support on distribution feeders', *IEEE Trans. Power Syst.*, 2007, **22**, (1), pp. 52–59

8 Xiao, L., Boyd, S.: 'Optimal scaling of a gradient method for distributed resource allocation', *J. Optim Theory Appl.*, 2006, **129**, (3), pp. 469–488

9 Tsikalakis, A., Hatziargyriou, N.: 'Centralized control for optimizing microgrids operation', *IEEE Trans. Energy Convers.*, 2008, **23**, (1), pp. 241–248

10 Dent, C., Ochoa, L., Harrison, G.: 'Network distributed generation capacity analysis using OPF with voltage step constraints', *IEEE Trans. Power Syst.*, 2010, **25**, (1), pp. 296–304

11 Xin, H., Qu, Z., Seuss, J., Maknouninejad, A.: 'A self organizing strategy for power flow control of photovoltaic generators in a distribution network', *IEEE Trans. Power Syst.*, 2011, **26**, (3), pp. 1462–1473

12 Qu, Z.: 'Cooperative control of dynamical systems' (Springer, London, 2009)

13 Yang, T.C.: 'Networked control system: a brief survey', *IET Control Theory Appl.*, 2006, **153**, (4), pp. 403–412

14 Fang, X., Wang, J.: 'Stochastic observer-based guaranteed cost control for networked control systems with packet dropouts', *IET Control Theory Appl.*, 2008, **2**, (11), pp. 980–989

15 Lopes, J., Hatziargyriou, N., Mutale, J., Djapic, P., Jenkins, N.: 'Integrating distributed generation into electric power systems: a review of drivers, challenges and opportunities', *Electr. Power Syst. Res.*, 2007, **77**, (9), pp. 1189–1203

16 Rabello, A., Bhaya, A.: 'Stability of asynchronous dynamical systems with rate constraints and applications', *IET Control Theory Appl.*, 2003, **150**, (5), pp. 546–50

17 Madjidian, D., Martensson, K., Rantzer, A.: 'A distributed power coordination scheme for fatigue load reduction in wind farms'. Presented at the ACC, 2011

18 Zhang, Z., Yuen, C.M.: 'Incremental cost consensus algorithm in a smart grid environment'. IEEE PES, San Francisco, 2011

19 Dominguez-Garcia, A., Hadjicostis, C.: 'Coordination and control of distributed energy resources for provision of ancillary services'. Proc. IEEE SmartGridComm, Gaithersburg, 2010

20 Xin, H., Lu, Z., Qu, Z., Gan, D., Qi, D.: 'Cooperative control strategy for multiple photovoltaic generators in distribution networks', *IET Control Theory Appl.*, 2011, **5**, (14), pp. 1617–1629

21 Ghadimi, E., Johansson, M., Shames, I.: 'Accelerated gradient methods for networked optimization' (ACC, 2011)

22 Li, C., Qu, Z., Das, A., Lewis, F.: 'Cooperative control with improved network connectivity'. American Control Conf., Baltimore, 2010

23 Khalil, H.K.: 'Nonlinear systems' (Prentice Hall, New Jersey, 1996, 2nd edn.)

24 Bazaraa, M.S., Sherali, H.D., Shetty, C.M.: 'Nonlinear programming – theory and algorithms' (John Wiley & Sons, 1993, 2nd edn.)

8 Appendix

8.1 Proof of Lemma 3

Since u^* is an optimal point of $C(\mathbf{u})$, it follows from the Taylor's series and Assumption 1 that there are

$$\nabla C(\mathbf{u}^*) = 0 \tag{15}$$

$$0 \geq C(\mathbf{u}^*) - C(\mathbf{u}) \geq \nabla^{\mathrm{T}} C(\mathbf{u})(\mathbf{u}^* - \mathbf{u}) \tag{16}$$

$$C(\mathbf{u}) - C(\mathbf{u}^*) = 0.5(\mathbf{u} - \mathbf{u}^*)^{\mathrm{T}} \nabla^2 C(\mathbf{u}')(\mathbf{u} - \mathbf{u}^*) \tag{17}$$

where $\mathbf{u}' = \mathbf{u}^* + \theta(\mathbf{u} - \mathbf{u}^*)$ and $\theta \in (0, 1)$.

From the given condition, we know that matrix $\mathbf{D}$ is symmetrical, irreducible and row-stochastic, so there are n real eigenvalues, say 1, $\gamma_2, \ldots, \gamma_n$, satisfying $|\gamma_i| < 1$. Let $\mathbf{1}$, $\zeta_2, \ldots, \zeta_n$ be the eigenvalues of $\mathbf{D}$ corresponding to the eigenvalues 1, $\gamma_2, \ldots, \gamma_n$ and satisfy $\|\zeta_i\| = 1$. Clearly, there exist some constants, say σ_i, such that there is

$$\mathbf{u}^* - \mathbf{u} = \sigma_1 \mathbf{1} + \boldsymbol{\zeta} \tag{18}$$

where $\boldsymbol{\zeta} = \sigma_2 \zeta_2 + \ldots + \sigma_n \zeta_n$.

Since $\mathbf{D}$ is symmetrical, the eigenvectors $\mathbf{1}$, $\zeta_2, \ldots, \zeta_n$ are orthogonal, thus we have

$$\mathbf{1}^{\mathrm{T}} \boldsymbol{\zeta}_i = 0 \tag{19}$$

$$||\boldsymbol{\zeta}||^2 = \sigma_2^2 + \cdots + \sigma_n^2 \tag{20}$$

It follows from (16) that there is

$$C(\mathbf{u}) - C(\mathbf{u}^*) \leq \nabla^{\mathrm{T}} C(\mathbf{u})(\mathbf{u} - \mathbf{u}^*)|| \leq ||\nabla C(\mathbf{u})||||\boldsymbol{\zeta}|| \tag{21}$$

On the other hand, there is

$$\nabla^2 C(\mathbf{u}) = (-\mathbf{I} + \mathbf{D}) \nabla^2 C(P_G)(-\mathbf{I} + \mathbf{D}) \tag{22}$$

and it follows from (17) that

$$C(\mathbf{u}) - C(\mathbf{u}^*) = 0.5(\mathbf{u} - \mathbf{u}^*)^{\mathrm{T}} \nabla^2 C(\mathbf{u}')(\mathbf{u} - \mathbf{u}^*) \tag{23}$$

Thus from (22) and (23) there is

$$\begin{aligned}
& C(\mathbf{u}) - C(\mathbf{u}^*) \\
& \quad = 0.5\big[(-\mathbf{I} + \mathbf{D})(\mathbf{u} - \mathbf{u}^*)\big]^{\mathrm{T}} \nabla^2 C(P'_G)\big[(-\mathbf{I} + \mathbf{D})(\mathbf{u} - \mathbf{u}^*)\big] \\
& \quad \geq 0.5 q_{\min} \boldsymbol{\xi}^{\mathrm{T}} (\mathbf{D} - \mathbf{I})^2 \boldsymbol{\xi} = 0.5 q_{\min} \sum_{i=2}^{n} \sigma_i^2 (1 - \gamma_i)^2 ||\boldsymbol{\xi}_i||^2 \\
& \quad = 0.5 q_{\min} \sum_{i=2}^{n} (1 - \gamma_i)^2 \sigma_i^2 \geq 0.5 q_{\min} (1 - \bar{\gamma})^2 \sum_{i=2}^{n} \sigma_i^2 \\
& \quad = 0.5 q_{\min} (1 - \bar{\gamma})^2 ||\boldsymbol{\xi}||^2
\end{aligned} \tag{24}$$

where $q_{\min}$ is the minimum eigenvalue of $\nabla^2 C(P_G)$ and $\bar{\gamma}$ is the second largest eigenvalue of D.

It follows from the condition that the communication matrix $\mathbf{S}$ is fixed, irreducible and symmetrical that $\mathbf{D}$ is non-negative and irreducible. Thus, 1 is a simple eigenvalue of **D** and $\bar{\gamma} < 1$ is satisfied. Therefore it follows from (21) and (24) that there is

$$||\nabla C(\mathbf{u})||^2 \geq \beta_0\big(C(\mathbf{u}) - C(\mathbf{u}^*)\big) \tag{25}$$

where $\beta_0 = q_{\min}(1 - \bar{\gamma}) > 0$.

8.2 Proof of Theorem 1

From (12), we have

$$\dot{u}_i = k_0\left[(1 - d_{ii})\lambda_i - \sum_{j=1, j\neq i'}^{n} d_{ji}\lambda_j\right] \tag{26}$$

It follows from (4) that there is

$$\frac{\partial P_{Gi}}{\partial u_j} = \begin{cases} -1 + d_{ij}, & j = i \\ d_{ij}, & j \neq i \end{cases} \tag{27}$$

Thus, by considering that $d_{ij} = d_{ji}$ there is

$$\frac{\partial C_j}{\partial u_i} = \frac{\partial C_j}{\partial P_{Gj}}\frac{\partial P_{Gj}}{\partial u_i} = \begin{cases} (-1 + d_{ii})\lambda_i, & j = i \\ d_{ij}\lambda_j, & j \neq i \end{cases} \tag{28}$$

Consequently, from (26) and (28), there is

$$\dot{u}_i = -k_0\partial C_i/\partial u_i - k_0\sum_{j=1, j\neq i'}^{n}\partial C_j/\partial u_i = -k_0\partial C/\partial u_i \tag{29}$$

that is

$$\dot{u} = -k_0\nabla C(u) \tag{30}$$

Hence, the derivative of $C(u)$ satisfies

$$\mathrm{d}C(u)/\mathrm{d}t = -k_0\big[\nabla C(u)\big]^{\mathrm{T}}\nabla C(u) = -k_0\big\|\nabla C(u)\big\|^2 \tag{31}$$

Assumption 1 implies that the solution to (2) has a lower bound, hence $C(u) \geq C^*$ is satisfied. Thus, (29) and (31) can lead to $\nabla C(u) \to 0$. Consequently, there is

$$\dot{u} = -k_0\nabla C(u) \to 0 \tag{32}$$

In addition, (32) implies that u is uniformly continuous with respect to t, hence there exists a u^* such that $u \to u^*$ and $\nabla C(u^*) = 0$ are satisfied. Since **D** is irreducible, by Lemma 2 $\nabla C(u^*) = 0$ implies that all the λ_i converge to the same value. Moreover, it follows from (12) that the constraint of (2) is satisfied. Therefore when $u = u^*$, the $K - T$ necessary conditions of (2) are satisfied [24], which in turn leads to the observation that u^* is the optimal solution by considering that problem (2) is convex.

In addition, it follows from Lemma 3 and (31) that the convergence rate is exponential. □

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