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Risk and quality control in a supply chain: competitive and collaborative approaches

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This paper provides a quantitative and comparative economic and risk approach to strategic quality control in a supply chain, consisting of one supplier and one producer, using a random payoff game. Such a game is first solved in a risk-neutral framework by assuming that both parties are competing with each other. We show in this case that there may be an interior solution to the inspection game. A similar analysis under a collaborative framework is shown to be trivial and not practical, with a solution to the inspection game being an 'all or nothing' solution to one or both the parties involved. For these reasons, the sampling random payoff game is transformed into a Neyman–Pearson risk constraints game, where the parties minimize the expected costs subject to a set of Neyman–Pearson risk (type I and type II) constraints. In this case, the number of potential equilibria can be large. A number of such solutions are developed and a practical (convex) approach is suggested by providing an interior (partial sampling) solution for the collaborative case. Numerical examples are developed to demonstrate the procedure used. Thus, unlike theoretical approaches to the solution of strategic quality control random payoff games, the approach we construct is both practical and consistent with the statistical risk Neyman–Pearson approach.

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1. Introduction

Supply chains are essentially organizational frameworks based on exchange and dependence between firms, each with its own objectives and motivations and drawing a payoff, whose risks it must also sustain and manage, in as many ways as it may be able to measure and conjure. Supply chains are therefore 'network of firms' that profit by collaborating. Collaboration is not always possible however, for agreements may be difficult to self-enforce and as a result, dependence risks of various sort may lead some firms to take advantage of their position in the supply chain network, either because of power or information asymmetries (Corbett and Tang, 1999; Agrawal and Seshadri, 2000; Cachon, 2002). Further, profit from collaboration must also be justified for parties involved if the supply chain collaborate, in fact. The control of quality (or rather non-quality) has assumed, however, that the underlying uncertainty faced by firms, individually and collectively, is neutral. In other words, the risk consequences measured by non-conforming quality are not motivated and therefore, the traditional approach to quality and its control has ignored the strategic and competitive effects of managing quality in an environment where firms act for their self-interest (Starbird, 1994; Reyniers and Tapiero, 1995a,b; Lim, 2001).

The implications of such an environment to the control of quality in supply chains are of course inherent in the assumptions we are willing to make, regarding the supply chain organization on the one hand and the quality contract engaging the parties on the other. For example, are there incentives to deliver conforming quality between the parties? What are the risk and economic consequences of delivering poor quality? Is quality controlled across the supply chain and what are the pre-posterior controls that allow both a monitoring-control and a choice of actions by the parties. Typically, in supply chains, uncertainty arises not only due to the uncertainties in the underlying processes producing quality, but also due to the motivations and preferences of each of the parties (Reyniers and Tapiero, 1995a,b; Tapiero, 1995, 1996, 2001). In this sense, in addition to statistical uncertainty, the management of quality may include strategic uncertainty arising due to conflicts latent between the supply chain firms. As a result, in such an environment, games of strategies and the control *ex ante* and *ex post* of quality might lead to quality control strategies that are 'mixed', with both strategic (threats and menaces) and statistical (information and assurance based) considerations. The purpose of this paper is to consider the control of quality contracts from a number of perspectives, emphasizing both competition and collaboration.

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A contract (for quality delivery, for example) is usually defined as a bilateral binding agreement by which agreed-upon exchange terms between two or more parties are used as substitutes to market mechanisms (Tagaras and Lee, 1996; Tsay *et al*, 1998). This may involve contracts to deliver parts or products of 'acceptable' quality defined by the contract on the one hand and on the other, by the economic consequences for each of the parties in case the terms of the contract are not met. The essential advantage resulting from a contract is therefore, to protect both parties, reduce the uncertainty they may face and thereby stabilize their respective operating environments. In a producer–supplier environment, a producer could assure (through inspection sampling) that special care be given by the supplier to materials and parts. Pre-contract negotiations, which vary from situation to situation, provide an opportunity to clarify future terms of exchange and provide protection for each of the parties once the contract is signed. A poorly designed contract may be disastrous for the supplier and the producer alike, since post-contract disagreements can lead to litigations which are usually very costly. For example, if delivery of quality products is not specifically stated in special clauses, suppliers may be tempted to supply substandard products (eg, see Friedman, 1986; Fudenberg and Tirole, 1991; Moulin, 1995; von Neumann and Morgenstern, 1944 for economic foundations).

The purpose of this paper is to provide a comparative and economic approach to strategic quality control in a supply chain setting which is compatible with the Neyman–Pearson statistical risk framework and based on economic considerations (for related studies see, Reyniers and Tapiero, 1995a,b as well as Reyniers, 1992; Tapiero, 1995, 2001; Tapiero 2005a,b). To articulate the focus of the paper, we consider some examples and calculate the risks and the control associated with specific supply chain organizations. Although the problems we formulate can be analysed analytically in a very limited number of cases, numerical calculations can be made with relative ease. To keep matters tractable, however, some simplifications are made.

In a lone-firm framework, control-sample selection consists in minimizing a consumer risk (or a type II risk $\beta_c(n,k)$ in a Neyman–Pearson statistical framework) which consists in accepting a lot, which is 'not conforming', subject to a Producer risk $\alpha_c(n,k)$ (or type I error) which consists in rejecting a 'good lot'. These probabilities are usually and explicitly defined in terms of control inspection parameters, for example (n,k) and can be formulated as follows (see Wetherhill, 1977 and Tapiero, 1996 for statistical control foundations):

$$\underset{(n\geqslant 0, 0\leqslant k\leqslant n)}{\text{Min}} [\beta_c(n,k)] \quad \text{s.t: } \alpha_c(n,k)\leqslant\bar{\alpha}_c \tag{1}$$

The parameter $\bar{\alpha}_c$ is usually specified by the test. In a producer–supplier environment, both the statistical risks of the supplier and the producer are to be considered and the economic consequences, negotiated, resulting from a game that both parties engage in. We consider such a game in this paper, by considering a number of situations co-existing in supply chains.

2. The risk-neutral game

Consider for simplicity, the strategic quality control game between a producer and a supplier engaged in an exchange with outcomes defined by the bimatrix random payoff game defined in (2), by $[\tilde{A},\tilde{B}]$. The strategies that each of the parties can choose consist in selecting a quality control (sampling) strategy for product assurance and supply controls. Such strategies assume many forms, although we shall focus our attention on the selection of elementary sampling strategies (for example, apply a specific sampling strategy, or do nothing). The consequences of such choices by the parties in the supply chain (for example, a supplier and a producer) are statistical and are denoted by the entries in the random payoff matrix (2), where $\sim$ denotes a random variable.

$$[\tilde{A},\tilde{B}] = \begin{bmatrix} \tilde{a}_{00} & \tilde{a}_{01}; & \tilde{b}_{00} & \tilde{b}_{01} \\ \tilde{a}_{10} & \tilde{a}_{11}; & \tilde{b}_{10} & \tilde{b}_{11} \end{bmatrix} \tag{2}$$

In such a game, there are two essential considerations faced by the supply chains parties—economic and risk, which are embedded in the bi-matrix random payoff entries. For example, let us say that, the sampling strategies that each of the parties can follow are: Use a binomial control sample $(n_j,k_j),\ j=0,1$ or do nothing. Here, the index $j=0$ denotes for example a supplier, while the index $j=1$ denotes a downstream producer. Of course, generally, we can consider a finite set of alternative control strategies that each of the parties can pursue. In this sense, sampling control by the producer acts as both a quality control and a 'threat' to the producer, expressing a 'lack of trust' in the supplier's quality. As commonly practiced in sampling control, we let (θ_1,θ_2) to be the proportions of defective parts in a lot where $\theta_1<\theta_2$ denotes a conforming lot (also called the AQL in statistical quality control) and the latter proportion, θ_2, denoting a non-conforming lot (also called LTFD in statistical quality control). In such a case, the 'probability risks' associated with each of these sampling strategies, coined type I and type II errors in a Neyman–Pearson statistical control framework are defined for both the supplier and the producer by (Wetherhill, 1977; Tapiero, 1996):

$$1-P(j\leqslant k_p|n_p,\theta_1)=\alpha_p; \qquad 1-P(j\leqslant k_s|n_s,\theta_1)=\alpha_s$$
$$P(j\leqslant k_p|n_p,\theta_2)=\beta_p; \qquad P(j\leqslant k_s|n_s,\theta_2)=\beta_s \tag{3}$$

In this approach, the parameters (θ_1,θ_2) are the negotiated contract quality terms which we assume as given, while the statistical control strategies $(n_j,k_j),\ j=0,1$ can be parameters defined by each of the parties together with their decision as to whether to apply such controls or not. Further, the production technology used by the supplier is assumed to be defined by its propensity to produce confirming lots, given by:

$$\tilde{\theta} = \begin{cases} \theta_1 & \text{with probability } 1 - v \\ \theta_2 & \text{with probability } v \end{cases} \tag{4}$$

In this sense, a supplier can both improve his process reliability by decreasing v (but of course, production might be costlier) or augment the amount of quality inspection controls and apply more stringent control rules. Given these risks and the parties strategic behaviour, the economic consequences, are necessarily random, expressed as a function of the sampling results and the uncertain consequences, due to the fact that the process of producing non-quality is also random (since non-conforming lots are produced in a random manner that the parties seek to control). For demonstration purposes, we assume that the following costs are defined for both the producer and the supplier: I_j denotes sampling inspection costs; E_j denotes consumers' costs borne by the party in case a bad lot is accepted; D_j denotes the cost, if both the parties sample while the second party (producer) detects the non-conforming lot; and finally, C_j, denotes the cost if a good lot is rejected. In this case, the bi-matrix random payoff game between the producer and the supplier on the basis of which we shall pursue our analyses are given by Equation (5) for the producer and Equation (6) for the supplier. Note that in this formulation, we have a random costs matrix, a function of the risk probabilities, each of the parties assume and a function of the organizational process (in this case, a single supplier and a single producer):

$$[\tilde{A}] = \begin{bmatrix} I_p + \begin{cases} E_p & wp & \beta_p\beta_s v \\ D_p & wp & (1-\beta_p)\beta_s v \\ C_p & wp & \alpha_p(1-v) \\ 0 & wp & 1-\alpha_p(1-v) \end{cases} & I_p + \begin{cases} E_p & wp & \beta_p v \\ D_p & wp & (1-\beta_p)v \\ C_p & wp & \alpha_p(1-v) \\ 0 & wp & 1-\alpha_p(1-v) \end{cases} \\ \begin{cases} E_p & wp & \beta_s v \\ 0 & wp & 1-\beta_s v \end{cases} & \begin{cases} E_p & wp & \beta_s v \\ 0 & wp & 1-\beta_s v \end{cases} \end{bmatrix} \tag{5}$$

$$[\tilde{B}] = \begin{bmatrix} I_s + \begin{cases} E_s & wp & \beta_p\beta_s v \\ D_s & wp & (1-\beta_p)\beta_s v \\ C_s & wp & \alpha_s(1-v) \\ 0 & wp & 1-\alpha_s(1-v) \end{cases} & \begin{cases} E_s & wp & \beta_p v \\ D_s & wp & (1-\beta_p)v \\ & & \\ 0 & wp & 1-(1-\beta_p)v \end{cases} \\ I_s + \begin{cases} E_s & wp & \beta_s v \\ & & \\ C_s & wp & \alpha_s(1-v) \\ 0 & wp & 1-\alpha_s(1-v) \end{cases} & \begin{cases} E_s & wp & v \\ & & \\ 0 & wp & 1-v \end{cases} \end{bmatrix} \tag{6}$$

While these economic costs are self-explanatory, we shall briefly discuss them. Assume that both the supplier and the producer apply a statistical control procedure and consider the first entry in the producer bi-matrix game. The cost C_p is the cost incurred if the producer rejects a good lot received from the supplier and produced by the supplier with a technology whose characteristic is defined by the probability parameter $(1 - v)$. As the risk probability of such an event, in case the producer applies his statistical control sample is α_p, the probability of such an event is $\alpha_p(1 - v)$. By the same token, D_p is the cost that the producer assumes if he rejects a bad lot (with risk probability $1 - \beta_p$) produced by the supplier with probability v. To do so however, the supplier must have accepted such a bad lot which he would with probability β_s. As a result, we obtain the appropriate entry in the producer payoff (costs) matrix. This cost may also be shared or passed back to the supplier, as specified by the contract drafted between these parties. Consider next the cost E_p which the producer sustains because of his accepting a bad lot passed on to consumers, who, unavoidably will detect its non-conforming quality. The risk probability of such a cost would necessarily be $\beta_p\beta_s v$. Further, a commensurate cost would be passed on to the supplier such that the total end-customer cost is $E_p + E_s$. A similar interpretation is associated to each of the terms in the bi-matrix game.

The strategic quality control random payoff (costs) game can provide some insights on the amount of controls, the parties will exercise. To resolve the problems associated with the solution of this random payoff game, we shall maintain the Neyman–Pearson risk framework and associate type I and type II risks to each strategy, the parties adopt and are explicitly given below. First, let us define by $P_{I,s}(i, j)$ the probability of the supplier accepting a good lot when applying a strategy i and the producer applying strategy j and let $P_{II,s}(i, j)$, be the probability that the supplier accepts a bad lot, although it is good and each of the parties follow sampling control strategies *i* and *j*. Let $(x, y),\ 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1$, be the probabilities that the producer and the supplier sample, then the risk probabilities assumed by the parties are in expectation, given for the supplier by:

$$\begin{aligned} P_{\mathrm{I,s}} &= (1-\alpha_{\mathrm{s}})(1-v)xy + (1-\alpha_{\mathrm{s}})(1-v)(1-x)y \\ &= (1-\alpha_{\mathrm{s}})(1-v)y \\ P_{\mathrm{II,s}} &= (1-y)v + xy\beta_{\mathrm{s}}v + (1-x)y\beta_{\mathrm{s}}v \\ &= (1-y)v + y\beta_{\mathrm{s}}v \end{aligned} \tag{7}$$

And for the producer (who receives lots from the supplier), by:

$$\begin{aligned} P_{\mathrm{I,p}} &= x(1-\alpha_{\mathrm{p}})(1-v) + (1-x)(1-v) \\ P_{\mathrm{II,p}} &= \beta_{\mathrm{p}}\beta_{\mathrm{s}}xyv + x(1-y)\beta_{\mathrm{p}}v \\ &\quad + (1-x)y\beta_{\mathrm{s}}v + (1-x)(1-y)v \\ &= v(x\beta_{\mathrm{p}} + 1 - x)(\beta_{\mathrm{s}}y + 1 - y) \end{aligned} \tag{8}$$

Note that in the first case, when calculating the probability of accepting a good lot, if a lot is properly produced, the prior actions taken by the supplier are not relevant. Therefore, the probability of accepting a good lot is essentially determined by the probability that it has been manufactured properly. In the second case, the probability is based on the strategies adopted by the supplier and the producer, based on sample results. Now, say that we impose (based on negotiations and agreements between the parties) the following expected acceptable risk parameters$(A_{\mathrm{s}}, A_{\mathrm{p}})$, consisting in the probability of rejecting a good lot for both the supplier and the producer. That is:

$$1 - P_{\mathrm{I,s}} \leqslant A_{\mathrm{s}} \quad \text{and} \quad 1 - P_{\mathrm{I,p}} \leqslant A_{\mathrm{p}} \tag{9}$$

By the same token, we define the risk parameters $(B_{\mathrm{s}}, B_{\mathrm{p}})$ such that:

$$P_{\mathrm{II,s}} \leqslant B_{\mathrm{s}} \quad \text{and} \quad P_{\mathrm{II,p}} \leqslant B_{\mathrm{p}} \tag{10}$$

Equations (7)–(10), thus provide a set of risk constraints which will be helpful in determining a solution to our strategic collaborative and competitive quality control games, faced by the supplier and the producer. We shall consider first a number of results, that provide some theoretical insights on the effects of strategic games on sampling control (and in fact contract controls) in supply chains. First, we consider the risk-neutral game, in which only the expected costs are minimized. Subsequently, we shall consider a collaborative and risk control game and provide an alternative approach to obtaining collaborative controls in supply chains. For simplicity, some of our results (when they are based on straightforward analysis of the underlying games) are summarized by propositions. First, as stated above, say that the supplier and the producer are risk-neutral. In this case, the expected costs for the producer and the supplier are:

$$[\hat{A}] = \begin{bmatrix} I_{\mathrm{p}} + E_{\mathrm{p}}\beta_{\mathrm{p}}\beta_{\mathrm{s}}v + D_{\mathrm{p}}(1-\beta_{\mathrm{p}})\beta_{\mathrm{s}}v & I_{\mathrm{p}} + E_{\mathrm{p}}\beta_{\mathrm{p}}v + D_{\mathrm{p}}(1-\beta_{\mathrm{p}})v \\ +C_{\mathrm{p}}\alpha_{\mathrm{p}}(1-v) & +C_{\mathrm{p}}\alpha_{\mathrm{p}}(1-v) \\ E_{\mathrm{p}}\beta_{\mathrm{s}}v & E_{\mathrm{p}}v \end{bmatrix} \tag{11}$$

$$[\hat{B}] = \begin{bmatrix} I_{\mathrm{s}} + E_{\mathrm{s}}\beta_{\mathrm{p}}\beta_{\mathrm{s}}v + D_{\mathrm{s}}(1-\beta_{\mathrm{p}})\beta_{\mathrm{s}}v & E_{\mathrm{s}}\beta_{\mathrm{p}}v + D_{\mathrm{s}}(1-\beta_{\mathrm{p}})v \\ +C_{\mathrm{s}}\alpha_{\mathrm{s}}(1-v) & \\ I_{\mathrm{s}} + E_{\mathrm{s}}\beta_{\mathrm{s}}v + C_{\mathrm{s}}\alpha_{\mathrm{s}}(1-v) & E_{\mathrm{s}}v \end{bmatrix} \tag{12}$$

These two matrices, define a 2-persons non-zero sum game whose solution can be found by an application of the well-known Nash equilibrium (Nash, 1950; Moulin, 1995). The following sampling strategies will result.

Proposition 1 *Define $d_k = D_k/E_k$; $c_k = C_k/E_k$; $i_k = I_k/E_k$, then the supplier and the producer Nash equilibrium sampling policies are defined by*:

$$x = \begin{cases} 1 & \text{if } v \leqslant \dfrac{i_{\mathrm{s}} + c_{\mathrm{s}}\alpha_{\mathrm{s}}}{(1-\beta_{\mathrm{s}}) + c_{\mathrm{s}}\alpha_{\mathrm{s}}} \\ 0 & \text{if } v \geqslant \dfrac{i_{\mathrm{s}} + c_{\mathrm{s}}\alpha_{\mathrm{s}}}{(d_{\mathrm{s}}(1-\beta_{\mathrm{p}}) - \beta_{\mathrm{p}})(1-\beta_{\mathrm{s}}) + c_{\mathrm{s}}\alpha_{\mathrm{s}}} \\ x^* & \text{otherwise} \end{cases}$$

$$x^* = \frac{v(1-\beta_{\mathrm{p}})(1-d_{\mathrm{p}}) - i_{\mathrm{p}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}(1-v)}{v(1-\beta_{\mathrm{s}})(1-\beta_{\mathrm{p}})(1-d_{\mathrm{p}})} \tag{13}$$

and

$$y = \begin{cases} 1 & \text{if } v \geqslant \dfrac{i_{\mathrm{p}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}}{(1-d_{\mathrm{p}})(1-\beta_{\mathrm{p}})\beta_{\mathrm{s}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}} \\ 0 & \text{if } v \leqslant \dfrac{i_{\mathrm{p}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}}{(1-d_{\mathrm{p}})(1-\beta_{\mathrm{p}}) + c_{\mathrm{p}}\alpha_{\mathrm{p}}} \\ y^* & \text{otherwise} \end{cases} \tag{14}$$

$$y^* = \frac{v(1-\beta_{\mathrm{s}}) - i_{\mathrm{s}} + c_{\mathrm{s}}\alpha_{\mathrm{s}}(1-v)}{v(1-\beta_{\mathrm{s}})(1-\beta_{\mathrm{p}})(1-d_{\mathrm{s}})}$$

Proof The proof is a straightforward application of Nash equilibrium to non-zero sum games (see also Tapiero, 1995).

In this solution, a number of insights are obtained. First, note that the larger the production technology reliability, the smaller the incentive to sample and vice versa. In this sense, production technology and statistical sampling controls are substitutes. If the propensity to produce non-conforming lots is larger than $i_{\mathrm{p}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}/(1-d_{\mathrm{p}})(1-\beta_{\mathrm{p}})\beta_{\mathrm{s}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}$, then the supplier will fully sample, while the producer will sample fully only if that same propensity is smaller than $i_{\mathrm{s}} + c_{\mathrm{s}}\alpha_{\mathrm{s}}/(1-\beta_{\mathrm{s}}) + c_{\mathrm{s}}\alpha_{\mathrm{s}}$. This is the case, because the producer will presume that it would be in the best interest of the supplier to fully sample (and therefore there would be no need for him to do so as well). By the same token, if the propensity to produce non-conforming units is smaller than $i_{\mathrm{p}} + c_{\mathrm{p}}\alpha_{\mathrm{p}}/(1-d_{\mathrm{p}})(1-\beta_{\mathrm{p}}) + c_{\mathrm{p}}\alpha_{\mathrm{p}}$, then the supplier presuming that his technology is reliable, will not sample at all. Interestingly, when the production technology is unreliable with

$$v \leqslant \frac{i_s + c_s\alpha_s}{(1-\beta_s) + c_s\alpha_s} \quad (15)$$

then, the producer will sample fully. For all other regions, there will be partial sampling as indicated in the proposition. The value for each of the parties in such a situation is given from Equation (2) as:

$$V_p(x, y) = \hat{a}_{00}xy + \hat{a}_{01}x(1-y) + \hat{a}_{10}(1-x)y + \hat{a}_{11}(1-x)(1-y)$$

$$V_s(x, y) = \hat{b}_{00}xy + \hat{b}_{01}x(1-y) + \hat{b}_{10}(1-x)y + \hat{b}_{11}(1-x)(1-y) \quad (16)$$

Thus, for an interior solution, we have (as calculated explicitly in Proposition 1) the following probabilities of sampling:

$$y^* = \frac{\hat{a}_{11} - \hat{a}_{01}}{\hat{a}_{00} - \hat{a}_{10} + \hat{a}_{11} - \hat{a}_{01}}, \quad x^* = \frac{\hat{b}_{11} - \hat{b}_{10}}{\hat{b}_{00} - \hat{b}_{10} + \hat{b}_{11} - \hat{b}_{01}} \quad (17)$$

which leads to the following Nash values:

$$V_p^N(x^*, y^*) = \frac{\hat{a}_{11}\hat{a}_{00} - \hat{a}_{10}\hat{a}_{01}}{\hat{a}_{00} - \hat{a}_{10} + \hat{a}_{11} - \hat{a}_{01}}$$

or

$$V_p^N(x^*, y^*) = E_p v \frac{i_p + c_p\alpha_p(1-v)}{v(1-\beta_p)(1-d_p) - c_p\alpha_p(1-v)} \quad (18)$$

and

$$V_s^N(x^*, y^*) = \frac{\hat{b}_{00}\hat{b}_{11} - \hat{b}_{10}\hat{b}_{01}}{\hat{b}_{00} - \hat{b}_{10} + \hat{b}_{11} - \hat{b}_{01}}$$

$$\text{or} \quad V_s^N(x^*, y^*) = E_s \frac{i_s + c_s\alpha_s(1-v)}{1-\beta_s} \quad (19)$$

Of course, all cases $(x, y=0, 1)$ ought to be analysed as well, corresponding to all the situations we have indicated in our proposition. From (18) and (19), we clearly see the effects of the *ex post* (customers) quality costs on both the supplier and the producer alike. The larger these costs, the larger the costs for the producer, while, for the supplier, it seems that the Nash equilibria costs given by

$$V_s^N(x^*, y^*) = \frac{I_s + C_s\alpha_s(1-v)}{1-\beta_s}$$

are only functions of the amount of inspection carried and the expected cost of rejecting good lots, augmented by $1/(1-\beta_s)$, which is the inverse of the probability of rejecting a good lot. For example, for the following parameters, $v = 0.1$, $\theta_2 = 0.3$, $\theta_1 = 0.01$ with the following specified risks $\alpha_p = 0.10$, $\beta_p = 0.05$; $\alpha_s = 0.05$, $\beta_s = 0.05$, arising from the choice of sampling techniques of the supplier and the producer and the following costs parameters for the producer and the supplier, $E_p = 30$, $D_p = 10$, $I_p = 0.75$, $C_p = 2$; $E_s = 20$, $D_s = 4$, $I_s = 0.5$, $C_s = 4$, we note an interior solution to sampling by both the producer and the supplier given by: $x^* = 0.8448$, $y^* = 0.6259$.

When the supplier and the producer collaborate by setting up a centralized control over the chain to minimize the overall supply chain cost, the resulting system-wide cost is:

$$V_p^C(x, y) + V_s^C(x, y) = (\hat{a}_{00} + \hat{b}_{00})xy + (\hat{a}_{01} + \hat{b}_{01})x(1-y) + (\hat{a}_{10} + \hat{b}_{10})(1-x)y + (\hat{a}_{11} + \hat{b}_{11})(1-x)(1-y) \quad (20)$$

The Hessian matrix of function (20) is indefinite. Therefore, the sampling solution in such a case is a corner solution, in which case, the supplier will always fully sample or not, and similarly for the producer. In this case, both the supplier and the producer disregard their own costs and risks with four potential solutions to be compared, including:

$$V_p^C(1,1) + V_s^C(1,1); \quad V_p^C(1,0) + V_s^C(1,0);$$
$$V_p^C(0,1) + V_s^C(0,1); \quad V_p^C(0,0) + V_s^C(0,0) \quad (21)$$

This approach, however, is neither interesting nor practical because it negates the existence of the risks that both the supplier and the producer seek to manage. Thus, in a collaborative framework, both the expected costs for both parties and the risks implied by both the producer and the supplier are to be accounted for. In this case, an appropriate formulation of the random payoff game, in terms of expected costs and the controlling Neyman–Pearson constraints defined by Equations (7)-(8) are given by:

$$\min_{0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant 1} \quad V_p^{CR}(x, y) + V_s^{CR}(x, y)$$
$$\text{s.t.} \quad 1 - P_{I,s} \leqslant A_s \quad \text{and} \quad 1 - P_{I,p} \leqslant A_p$$
$$P_{II,s} \leqslant B_s \quad \text{and} \quad P_{II,p} \leqslant B_p \quad (22)$$

This is a straightforward non-linear optimization problem whose solution can be reached by standard numerical methods. The disadvantage of this formulation is that it still assumes full collaboration or vertical integration of the supply chain, which is rarely possible and ignores the individual costs and costs transfers between the parties.

Alternatively, we can obtain a collaborative binary as well as interior solutions that are sensitive to both the risk constraints and individual costs of the supplier and producer by assuming that the producer's propensity to control quality, is proportional to that of the supplier, denoted for convenience by $x = ky$. With such an assumption, a number of possibilities are neglected and can be verified separately. These possibilities include the following six sampling strategies:

$$(x, 1), (x, 0), (1, y), (0, y), (1, 0), (0, 1) \quad (23)$$

For example, for a sampling strategy $(x, 0)$, we have the following (using Equations (7), (8) and the objectives stated above):

$$1 - P_{\mathrm{I,s}} = 1,$$

$$P_{\mathrm{I,p}} = x(1-\alpha_{\mathrm{p}})(1-v) + (1-x)(1-v) \leqslant A_{\mathrm{p}}$$

$$P_{\mathrm{II,s}} = v,$$

$$P_{\mathrm{I,p}} = x(1-\alpha_{\mathrm{p}})(1-v) + (1-x)(1-v) \leqslant A_{\mathrm{p}},$$

$$P_{\mathrm{II,p}} = v(x\beta_{\mathrm{p}} + 1 - x) \leqslant B_{\mathrm{p}} \tag{24}$$

and therefore the risk constraints are reduced to:

$$\frac{1-(B_{\mathrm{p}}/v)}{(1-\beta_{\mathrm{p}})} \leqslant x \leqslant \frac{A_{\mathrm{p}} - v}{(1-v)\alpha_{\mathrm{p}}}, \tag{25}$$

while the joint objective of the collaborating supply chain is:

$$V_{\mathrm{p}}^{\mathrm{CR}}(x, y) + V_{\mathrm{s}}^{\mathrm{CR}}(x, y) = \hat{a}_{11} + \hat{b}_{11} + (\hat{a}_{01} - \hat{a}_{11} + \hat{b}_{01} - \hat{b}_{11})x \tag{26}$$

A solution is then necessarily determined by the risk constraints. Namely, $x = 0$ if $\hat{a}_{01} + \hat{b}_{01} > \hat{a}_{11} + \hat{b}_{11}$, violating the risk constraint (25) and therefore $x \neq 0$ necessarily. When the inequality is reversed, we obtain also, a sampling program determined by the upper constraint imposed by the type I risk of the producer. As a result:

$$x = \begin{cases} \dfrac{1-(B_{\mathrm{p}}/v)}{(1-\beta_{\mathrm{p}})} & \text{if } \hat{a}_{01} + \hat{b}_{01} \leqslant \hat{a}_{11} + \hat{b}_{11} \\ \dfrac{A_{\mathrm{p}} - v}{(1-v)\alpha_{\mathrm{p}}} & \text{if } \hat{a}_{01} + \hat{b}_{01} > \hat{a}_{11} + \hat{b}_{11} \end{cases} \tag{27}$$

Similarly, other constraints can be treated in a similar manner.

For convenience, consider interior solutions, by letting $x = ky$. The collaborative objectives of the supplier and the producer are then convex and therefore a global solution can be found, summarized by the following proposition.

Proposition 2 Let the risk constraints not be binding and define:

$$\xi = \frac{2\hat{a}_{00} + 2\hat{b}_{00} - \hat{a}_{01} - \hat{a}_{10} - \hat{b}_{01} - \hat{b}_{10}}{\hat{a}_{11} + \hat{b}_{11} - \hat{a}_{10} - \hat{b}_{10}} \tag{28}$$

If $k > \frac{1}{1+\xi}$ then (23) has a unique interior optimal solution.

Proof The proof is straightforward and can be obtained by verifying the second-order optimality condition along with binary constraints.

The advantage of this collaborative approach is that, once a solution for sampling is determined in terms of the parameter k, we can employ k for fine tuning the supply chain to prevent violations of the risk constraints. Specifically, substituting $x = ky$ into the objective function (20), we have the collaborative cost:

$$y^{**} = \frac{(\hat{a}_{11} + \hat{b}_{11})(1+k) - (\hat{a}_{01} + \hat{b}_{01})k - \hat{a}_{10} - \hat{b}_{10}}{2k(\hat{a}_{00} - \hat{a}_{01} + \hat{a}_{11} - \hat{a}_{10} + \hat{b}_{00} - \hat{b}_{01} + \hat{b}_{11} - \hat{b}_{10})},$$
$$x^{**} = ky^{*} \tag{29}$$

While the collaborative cost is:

$$\begin{aligned} V^{\mathrm{CR}} = {} & ky^2(\hat{a}_{00} - \hat{a}_{01} + \hat{a}_{11} - \hat{a}_{10} + \hat{b}_{00} - \hat{b}_{01} + \hat{b}_{11} - \hat{b}_{10}) \\ & - y((\hat{a}_{11} + \hat{b}_{11})(1+k) - (\hat{a}_{01} + \hat{b}_{01})k - \hat{a}_{10} - \hat{b}_{10}) \\ & + \hat{a}_{11} + \hat{b}_{11} \end{aligned} \tag{30}$$

To obtain a feasible solution, satisfying the producer and the supplier risk constraints, we thus solve the following problem:

$$\begin{aligned} & \text{Find } 0 \leqslant k \text{ such that:} \\ & 1 - P_{\mathrm{I,s}}(k) \leqslant A_{\mathrm{s}} \quad \text{and} \quad 1 - P_{\mathrm{I,p}}(k) \leqslant A_{\mathrm{p}} \\ & P_{\mathrm{II,s}}(k) \leqslant B_{\mathrm{s}} \quad \text{and} \quad P_{\mathrm{II,p}}(k) \leqslant B_{\mathrm{p}} \end{aligned} \tag{31}$$

Explicitly, this is given by:

$$\begin{aligned} & \text{Find } 0 \leqslant k \text{ such that:} \\ & 1 - (1-\alpha_{\mathrm{s}})(1-v)y \leqslant A_{\mathrm{s}} \\ & x(1-\alpha_{\mathrm{p}})(1-v) + (1-x)(1-v) \leqslant A_{\mathrm{p}} \\ & (1-y)v + y\beta_{\mathrm{s}}v \leqslant B_{\mathrm{s}} \\ & v(x\beta_{\mathrm{p}} + 1 - x)(\beta_{\mathrm{s}}y + 1 - y) \leqslant B_{\mathrm{p}} \end{aligned} \tag{32}$$

If the risk constraints are not binding, then there is a non-empty interval defined by (k_1, k_2) where k turns out to be a potentially negotiating parameter, *defining both the economic costs sustained by the producer and the supplier and the type I and type II risks, (both a function of k).* This is illustrated in Figure 1. Explicitly, assume the following parameters $E_{\mathrm{p}} = 30$, $D_{\mathrm{p}} = 10$, $I_{\mathrm{p}} = 0.75$, $C_{\mathrm{p}} = 2$, $E_{\mathrm{s}} = 20$, $D_{\mathrm{s}} = 4$, $I_{\mathrm{s}} = 0.5$, $C_{\mathrm{s}} = 4$ with risk parameters $\alpha_{\mathrm{s}} = 0.1$, $\alpha_{\mathrm{p}} = 0.05$, $\beta_{\mathrm{p}} = 0.1$, $\beta_{\mathrm{s}} = 0.1$, $v = 0.055$ (a function of the sampling plans adopted). For this parameter, we see the effects of parameters k on the costs sustained by each of the parties. Clearly, the sum of the Nash equilibrium costs for both parties is much larger than the sum of collaborative costs. In addition, we see also that collaborative costs are increasing in k as stated in Proposition 3.

Proposition 3 If the probability of non conforming production lots satisfies the condition below:

$$v \leqslant \frac{I_{\mathrm{p}}}{(E_{\mathrm{p}} + E_{\mathrm{s}})} + \frac{(D_{\mathrm{p}} + D_{\mathrm{s}})(1-\beta_{\mathrm{p}})v}{(E_{\mathrm{p}} + E_{\mathrm{s}})} + \frac{C_{\mathrm{p}}\alpha_{\mathrm{p}}(1-v)}{(E_{\mathrm{p}} + E_{\mathrm{s}})}$$

then, the smaller the collaboration parameter k, the lower is the collaborative supply chain cost.

Proof The proof is obtained by differentiating the cost function (30) with respect to k, resulting in the condition $\hat{a}_{11} + \hat{b}_{11} \leqslant \hat{a}_{01} + \hat{b}_{01}$ which requires such a result to be positive. □

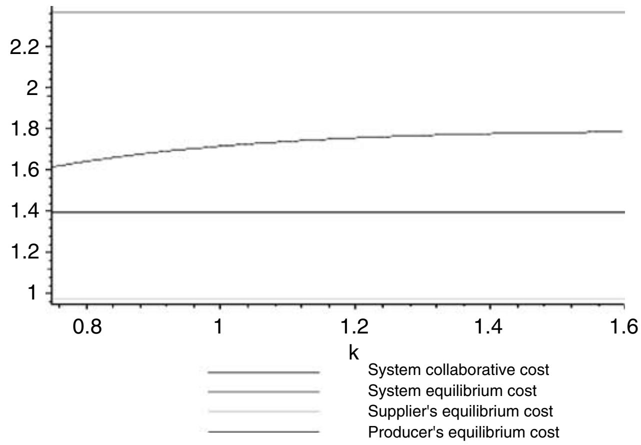


Figure 1 Supply chain costs as a function of the negotiating parameter k.

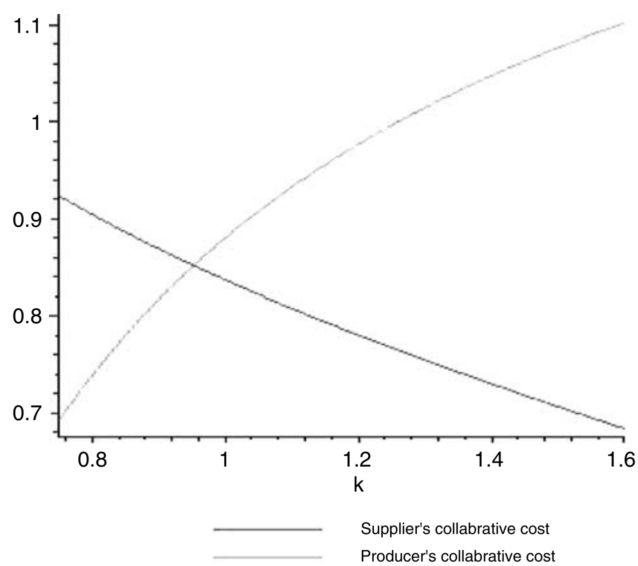


Figure 2 Individual costs of the supplier and producer as a function of the negotiating parameter k.

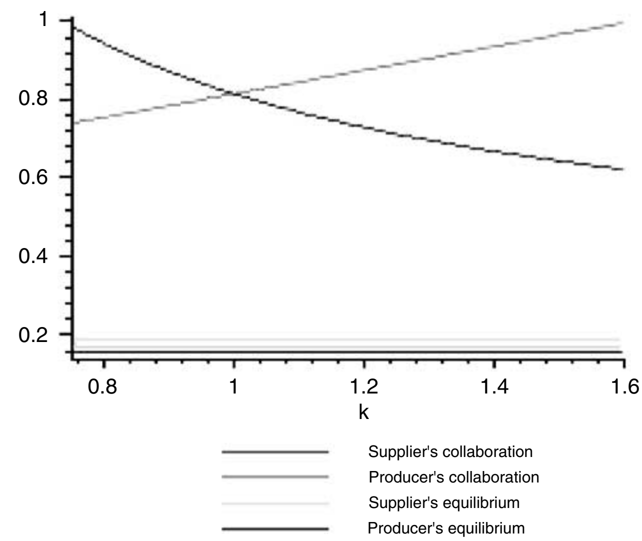


Figure 3 Inspection effort: collaborative *versus* Nash competing solution.

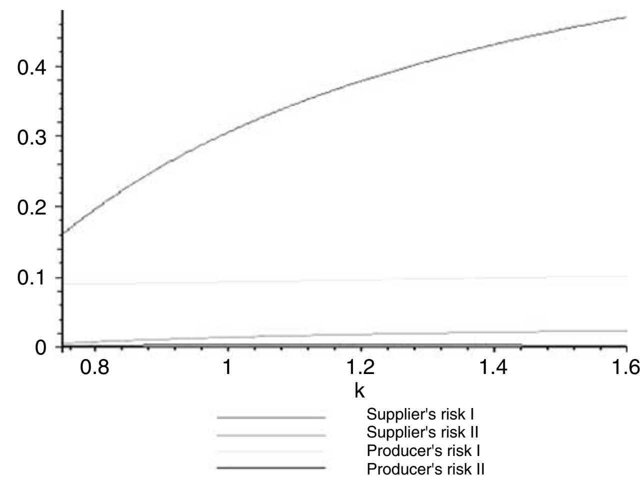


Figure 4 Risk constraints as a function of the negotiating parameter k.

The implication of this proposition is that the party with larger inspection costs will reduce the amount of inspection and thus the associated cost while transferring some of the inspection effort and cost to the other party. This is observed in Figures 2 and 3. Specifically, the supplier's cost (Figure 2) and inspection effort (Figure 3) decrease, while the producer's cost and inspection effort increase as k increases. At the intersection point of the two cost curves in Figure 2, both the parties incur identical costs. At this point, $k<1$, pointing out to unequal inspection efforts exercised by the parties (see Figure 3). Furthermore, at this point, the parties attain equal individual costs, but do not minimize the system-wide cost of the collaborative supply chain (see Figure 1).

Finally, Figure 3 outlines the effects of the parameter k on the risk constraints. For the parameters selected, we note that the maximum errors tolerated by the producer and the supplier are as defined in the figure. The conclusion to be drawn from such a numerical analysis confirms the intuition that, having the supplier augment the control of quality (meaning a smaller k), relative to that of the producer, will result in smaller risks for both the producer and the supplier. In this sense, the conventional wisdom that sampling upstream the supply chain is efficient is verified here as well. Further, as stated earlier, the parameter k, is shown to be a parameter with which both the costs and risks substitution can be determined (Figure 4).

3. Conclusion

The purpose of this paper was two-fold: to provide a quantitative and a comparative economic and risk approach to strategic quality control in a supply chain using a random payoff game. Such a game was first solved in a risk-neutral framework. In this case, we have shown that in a competitive state, there may be an interior solution to the inspection game as stated in Proposition 1. The decision to control or not, for the supplier is then a function of the underlying process reliability. For the producer, a reliable process may require some inspection as well (of course, we are not considering in this case the extreme case of zero default production). The propensity to inspect by the producer is then merely a result of the parties motivations and the mutual distrust implied in the Nash solution. A similar analysis under a collaborative framework turns out to be trivial, with a solution to the inspection game turning out to be an 'all or nothing' solution for one or both the parties. Of course, such a solution is not realistic and neither does it confirm to the observed behaviour of industrial firms operating in a supply chain. This is the case because we have neglected the risk effects that are particularly important in the control of quality. In this sense, the risk-neutral problem has a limited interest while the random-payoff inspection game is difficult to resolve in a practical sense. A potential approach entertained was to assume a risk attitude (Munier and Tapiero, 2007) by both the producer and the supplier and thereby transform the competitive game into a deterministic non-zero sum game. Such an approach is not appropriate, however, as it introduces risk attitude parameters that are only implicit in decision makers' actions rather than known explicitly. Further, when studying the risk attitude sensitive problem, we also reached the same conclusion for a collaborative supply chain, neglecting again, the implied risk constraints that underlie the decision to control quality or not. For these reasons, following an approach set by Tapiero (2005a,b), the random-payoff strategic quality control game was transformed into a Neyman–Pearson risk constraints game. In other words, while maintaining the risk-neutral valuation of economic costs, this paper has appended to the parties decision processes the risk qualifications (type I and type II risks in the Neyman–Pearson statistical framework). Explicitly, as parties strategies are defined in terms of both the choice of sampling plans and the randomized strategies applied in selecting these plans, we have introduced a concept of 'expected' type I and type II risks to be sustained by both the producer and the supplier. Such an approach leads to a broad number of new and potential equilibria, when combined with sampling plan selections. Further, in a collaborative framework, the model assessed will also lead to results that might not be practical due to the producer's and suppliers' focus on specific parameters and selecting the relationships that they ought to maintain one with respect to each other. In this sense, assuming that there is an interior solution to the game where such a relationship is maintained, defined by parameter k, we demonstrated (Proposition 2) that in the collaborative game there can be an interior solution to the sampling random-payoff game, meeting the parties risk constraints that can be used to select an optimal sampling strategy by each of the parties on one hand and selecting the compatible optimal sampling plan on the other. In this sense, our approach, unlike a number of theoretical approaches to the solution of strategic random payoff games is both practical and applicable to strategic quality control as defined in this paper.

Of course, further research as well as numerous situations, based on an explicit definition of the sampling plans and their associated type I and II risks as well as a broader variety of supply chains organizations can be addressed and resolved based on the approach formulated in this paper. For example, for binomial sampling programs for the producer and the supplier, we could define the following type I and II risks for each (Tapiero, 2007):

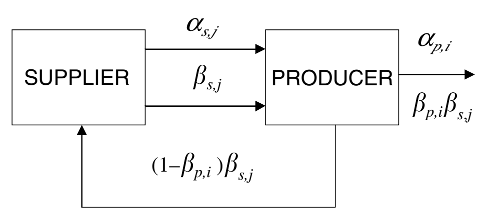
$$\alpha_{p,i} = 1 - \sum_{1=0}^{k_{p,i}} \binom{n_{p,i}}{1} (\theta_1)^1 (1-\theta_1)^{n_{p,i}-1};$$

$$\beta_{p,i} = \sum_{1=0}^{k_{p,i}} \binom{n_{p,i}}{1} (\theta_2)^1 (1-\theta_2)^{n_{p,i}-1}$$

$$\alpha_{s,j} = 1 - \sum_{1=0}^{k_{p,j}} \binom{n_{s,i}}{1} (\theta_1)^1 (1-\theta_1)^{n_{s,j}-1};$$

$$\beta_{s,j} = \sum_{1=0}^{k_{p,j}} \binom{n_{s,j}}{1} (\theta_2)^1 (1-\theta_2)^{n_{s,j}-1}$$

Note that we can accommodate other sampling programs that need not be the same, but rather 'compete' one with another. In this sense, our approach provides a risk-based approach to sampling selections that can be both economical and risk-sensitive in a conflicting or collaborative environment. For the following organizational risk transfers model, for a supplier and a producer (see also Tapiero, 2007),



then the expected risk constraints for the producer and the supplier would be:

$$\sum_{j=1}^{m}\sum_{i=1}^{n} x_i y_j \alpha_{p,i}(1-\alpha_{s,j}) \leqslant \bar{\alpha}_p; \quad \sum_{j=1}^{m}\sum_{i=1}^{n} x_i y_j \beta_{p,i}\beta_{s,j} \leqslant \bar{\beta}_p$$

$$\sum_{j=1}^{m} y_j \alpha_{s,j} \leqslant \bar{\alpha}_s; \quad \sum_{j=1}^{m} y_j \beta_{s,j} \leqslant \bar{\beta}_s$$

Expected economic costs will then be defined as an expected consequences implied by the risk probabilities of this game. While such problems are analytically intractable and computationally difficult to solve, their analysis is feasible as pointed out in this paper through examples.

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