Direct Optimized Probabilistic Calculation

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*Abstract:*Various calculation methods based on the theory of probability and statistics are used not when designing and assessing reliability of elements and systems in load-carrying structures so that the required reliability could be reached.Those methods have been becoming very popular recently.Using the probabilistic method, it is possible to analyse a reliability margin defined in a computational model where at least some input characters are random.New methods which are being developed now include the Direct Optimized Probabilistic Method("DOProC"). This is a purely numerical method which uses no simulation techniques. Results of the probabilistic tasks are more accurate and, often, more fast to reach.

*Key-Words:*DOProC, Direct Optimized Probabilistic Calculation, ProbCalc, HistAn, HistOp, probabilistic methods, reliability assessment, random variable, probability of failure

1 Introduction

Probability methods have started penetrating as standard designing methods into the construction assessment and designing recently only [1, 7, 16, 17].The probabilistic method can be used only if enough input quantities are available and if certain experience in practice has been reached [12, 13, 18] as many input data cannot be just modeled and measures in laboratories. Source information from standards is essential too (for instance, CSN EN 1990).

Monte Carlo [14, 20] is the group of calculation methods which is used most frequently and includes most techniques which are based on standard simulation. Some advanced or stratified methods are available as well (such as LHS, Importance Sampling) [15].Eurocodes which are in force now mention the use of approximation FORM and SORM methods which are employed typically for calibration of partial factors.

The approximation methods include Response Surface [9].

This paper deals with one of new probabilistic methods which are under development now: Direct Optimized Probabilistic Calculation ("DOProC") which is a purely numerical method without any simulation techniques. Results are more accurate-there is only a numerical error and inaccuracies resulting from discretizing of input and output quantities. In many cases, the calculation is faster.DOProC should be used in probabilistic tasks and probabilistic reliability assessments where at least some input quantities are of a random nature and can be described by probabilistic non-parametric (empiric) or parametric distribution.This is frequently a method efficient method of calculation.A disadvantage is much machine time needed for calculations in tasks with many random quantities which were discretized using rather many intervals (classes).A solution would be often the optimising techniques which considerably reduce the machine time needed for the calculation and maintain correctness of the results.

If DOProC is used for the probabilistic reliability assessment, theprobability of failure, p_f can be directly expressed and compared with the designed probability,p_d, which is defined in applicable standards.If the probability of failure is zero (the structure is very reliable) or one (each combination of input quantities results in a failure), DOProC seems to be very suitable and efficient because the result is known immediately and no other probabilistic calculations are needed.

2 Principles of the method

DOProC has been developed since 2002 and theoretical backgrounds have been described in many publications [3, 4].The DOProC probabilistic calculation is clearly determined for the task by its algorithm, while in Monte Carlo simulation methods

the calculation data for the simulation are randomly generated.
This basic feature of the method gave originally the name to this method - Direct Determined Fully Probabilistic Method – DDFPM. The word “determined" was, however, misleading and that is why the name was altered.

Reasons for the term "optimized" in the final naming of the method are as follows:

The number of random quantities which are used in the calculation of the probability of failure is limited by the ability to manage the task numerically.If there are too many random quantities, the tasks require too much time even if the advanced computational facilities are used.Therefore, optimizing methods have been developed and searched for which would reduce the number of operations and maintain correctness of the calculation (see Chapter 2.2).

2.1 Basic algorithm of calculations

The calculation algorithm of DOProC results from basic terms and techniques used in the theory of probability.When DOProC is used, computational operations are performed with histograms of random input quantities which might be expressed by discrete or purely discrete distribution of probabilities.

The resulting histogram, *B*, is any function, *f* (e.g., the function of reliability for the probabilistic reliability assessment) for the input histograms A_j, where *j* is from 1 to *n*.This means:

$$B = f\left(A_1, A_2, A_3, ..., A_j, ..., A_n\right) . \tag{1}$$

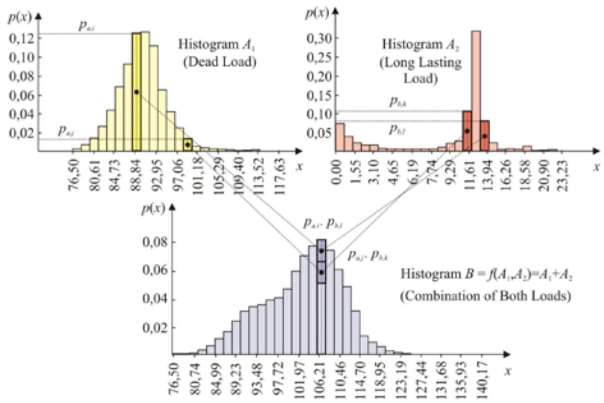

Fig. 1:Principles of numerical operations with two histograms (combination of two load components)

Each histogram A_j has got i_j classes (intervals).The number of classes,i_j, in each histogram, A_j, can be different. The number of intervals, *i*, in the resulting histogram, *B*, can differ as well.The number of intervals is the decisive factor for the number of computational operations and time needed for the calculation.The number of interval influences also considerably accuracy of the probabilistic calculation,Fig. 1 shows the principles of numerical operations in the probabilistic calculation with two random quantities expressed in a histogram.This case combines two load components – this means, the sum of two histograms.

In case of the probabilistic reliability assessment of the construction or a structural element [2],*f* in the formula (1) is given by the computational model which defines the reliability function, *RF*, as follows:

$$RF = R - S \ , \tag{2}$$

where*R* is resistance of the construction and *S*is the load effect.Thebasic algorithm of the probabilistic task processed byDOProC is that shown inFig.2.

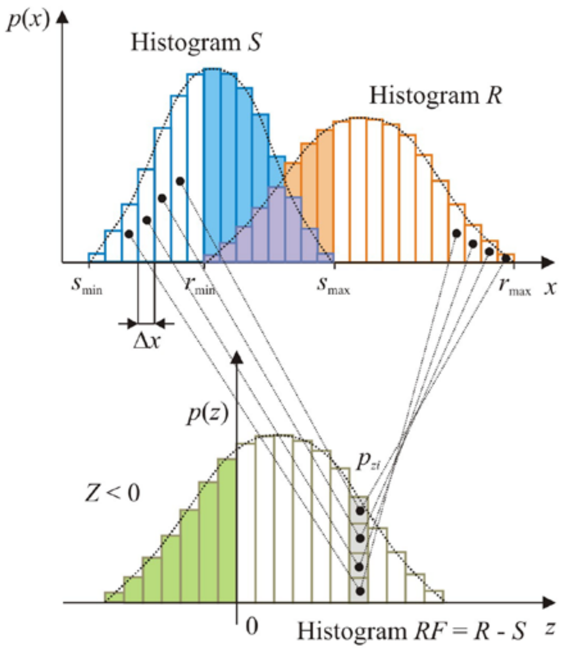

Fig. 2:Basic calculation of the reliability function histogram, *RF*, of two random variables using DOProC

2.2 Optimizing techniques

The goal of the optimizing techniques which were developed in DOProC is to minimize the time of calculation because the basic algorithm which is mentioned in Chapter 2.1 is limited to a certain extent, the being in particular, the case of extensive tasks where the number of computational tasks is very high.If the optimizing techniques are applied in

DOProC, the result of the probabilistic task can be determined in a real time, maintaining the necessary correctness and sufficient accuracy even for relatively complex probabilistic tasks.

Below are the available optimizing actions:

(a) **Grouping of input variables:** This technique uses only input variables (such as the load components) which can enter the calculation jointly and for which a joint histogram can be prepared.This technique is well suited for situations when the combined load consists of several components of random variables with the same point of action. Then, only one joint histogram can be used.This technique is applicable to similar situations with other input quantities.

(b) **Interval optimizing:** This techniquereduces the number of classes of the input variables in the histograms, which the total extent of each random input variable is maintained. This method which accelerates the calculation is usedin such a way so that the result and correctness of the solution would not be affected too much.First, the influence of the number of interval of each random variable on the solution is tested. Then, the number of intervals is minimized.

(c) **Zone optimizing:** The calculation uses only those intervals which are involved in the searched value, for instance, the probability of failure of the construction, p_f .

(d) **Trend optimizing:** This technique considers the suitable direction (trend) in the calculation probability algorithm.

(e) **Grouping of partial results of the calculation:** Partial results are grouped – for instance, when preparing the resulting reliability function,*RF*.

(f) **Parallelizing the calculation:** several processors or cores are calculating the processors or cores at the same time.

(g) Combination of the mentioned optimizing techniques.

Theoretical background of the optimizing techniques was described in detail, for instance, in[3, 4].

3 ProbCalc Software

DOProC can be used in ProbCalc [5] – this software consists of several computational modules and is still under development.

3.1 HistAn software

HistAn (Fig.3) is used for a detailed analysis of input and resulting histograms.HistAn can be used for obtaining basic characteristics of the histogram and for performing simple calculations: to determine the functional value with a corresponding quantille or for inverse operations (determination of a quantille for the specified functional value of the variable).HistAn can help determining combination of several input histograms and the summary histogram which is used for wind rose calculations.In HistAn it is also possible to prepare histograms with parametric distribution of probability for entering of necessary parameters which depend on the probability distribution.The user can choose from among twenty frequently used types of parametric distribution.The histograms with parametric or nonparametric (empirical) distribution can be created on the basis of measured data which are assessed in terms of statistics and grouped into classes.In case of the parametric distributions, the best type of the parametric distribution is selected depending on the coefficient of determination.This computational operation can be performed accordingly in other computational modules of ProbCalc.

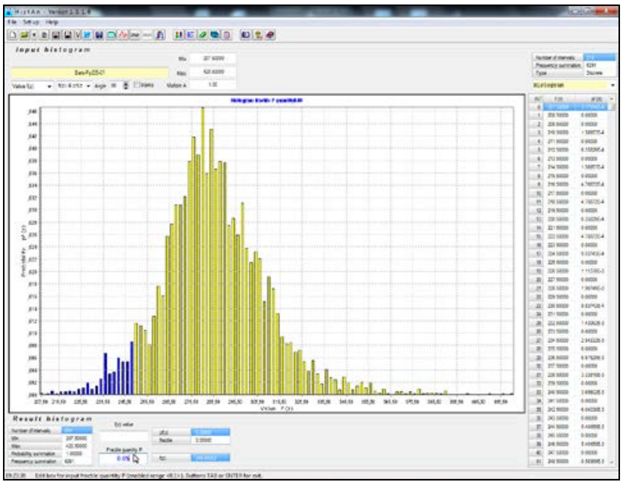

Fig. 3:HistAn desktop with the nonparametric (empirical) tension histogram at the yield stress - steel, S235

3.2 HistOp software

The histograms can be processed in standard mathematical operations.For instance, in case of the combined load most frequently the used mathematical operations are the addition of the histograms for individual types of load (see Fig. 1).A software tool HistOp (Fig.4) was developed for basic arithmetic operations. Following arithmetic operations can be done with a pair of the histograms:the addition, subtraction, multiplication and division of the both histograms, and the square and absolute value of the first histogram.

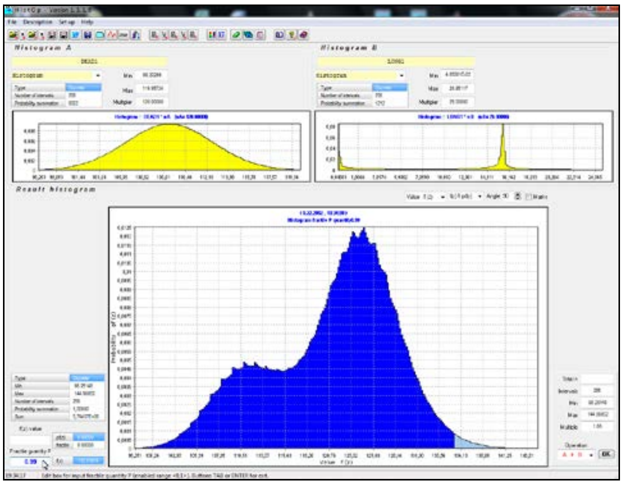

Fig. 4:HistOp desktop with the final histogram which combines two load components -Fig.1

3.3 ProbCalc software

The main computational utility for ProbCalc is the utility of the same name. It is possible to implement generally an analytical transformation model of the probabilistic tasks, using a text-oriented editor, into this utility (in a similar way as in [19]).In case of rather complex numerical models it is possible to use a dynamic library (with the DLL extension) in a user-defined procedure.The person who enters the probabilistic tasks in ProbCalc should be a well-trained user.It is necessary to know, at least in basic principles, the essence of the algorithm - these influences the way of defining the computational model and selection of a good optimizing technique.

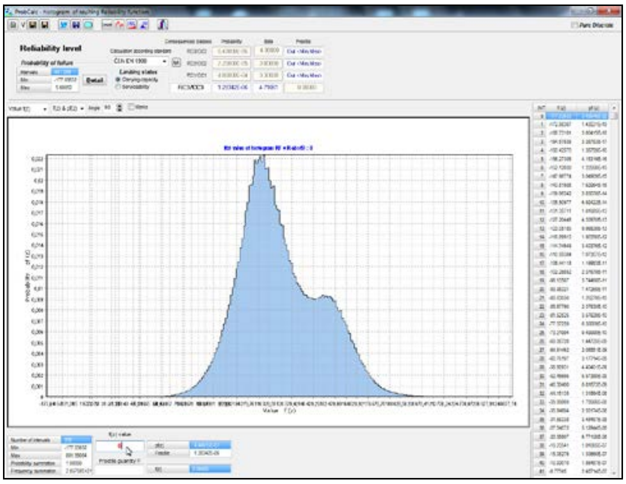

Fig. 5:ProbCalc desktop with results of the reliability assessment - the histogram of the reliability function, *RF*, and the calculated probability of failure,p_f, which is compared with the designed probability, p_d , pursuant to EC

If the goal of the probabilistic calculation is the reliability assessment of a construction, the probability of failure, p_f, as well as assessment pursuant to EC can be performed using the histogram of the analyzed reliability function,*RF* (Fig.5).In this software, a 3D view of the analyzed*RF* function (Fig. 6) is possible. This means, it is possible to analyze the tasks in detail.

The software which is being still developed includes the aforementioned optimizing techniques (the interval, zone and trend optimizing) which make it possible to reduce considerably the number of computational steps and, in turn, the machine time needed for the calculation.It is possible to combine the optimizing techniques mutually.

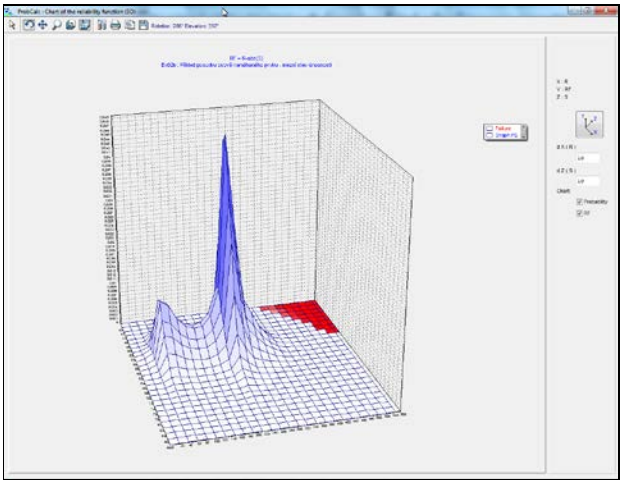

Fig. 6:ProbCalc desktop with the resulting reliability function,*RF*, in 3D

Another possibility for reducing the time needed for the calculation of the probabilistic tasks is to calculate the tasks parallel. The computational operations in complex tasks in DOProC can be easily adapted to run parallel and this feature has been also implemented into ProbCalc.If there are two or more processors or cores in the computer, it is possible to reduce considerably the machine time thanks to the parallel computations(similarly as with[8]).

3.4 Other software tools

As ProbCalc is intended for general purposes, its interface might not be sufficient of an experienced user. For that purpose, it is possible to use application software which has been customizedfor specified probabilistic tasks. It is not necessary then for the user to define a complex computational model or choose a type of the optimizing technique.Then user just enters the input quantities, starts the calculation and analyses results of the probabilistic task.

This is, for instance, the case ofAnchor which can be efficiently used for the probabilistic design and reliability assessment of the anchoring reinforcement of long mine workings and underground constructions [6] or FCProbCalc (see below).

4 Using DOProC in practice

DOProC is used mostly for the probabilistic assessment of reliability of the load-carrying structures.This method can be also used for the probabilistic design of elements in the construction with the requiredreliability.

In past, DOProC was used for the probabilistic assessment of the combined load, for the probabilistic assessment of reliability of cross sections and systems with statically (un)certain load-carrying structures, for the probabilistic assessment of features of concrete and fibre concrete mixtures and for the assessment of reliability of load-carrying structures exposed to impact and arch supports in long mining works with a particular focus on anti-slippage features.

DOProC has been successfully applied in the probabilistic calculation of propagating fatigue cracks in steel structures and bridges which are exposed to cyclic loads. For this purpose, FCProbCalc [10] was used – in this software, it is possible to monitor efficiently and flexibly development of fatigue damage in the construction, to specify times for inspections and decide whether the construction should not be put out of operation because of the fatigue damage.The available methodology [11] and application of the methodology in practice may considerably improve quality of estimates costs for maintenance of the constructions and bridges which are subject to load cycles.

5 Conclusion

This paper discussed development of the probabilistic methods and use of those methods for the reliability assessment of the structure. A particular attention was paid to the new probabilistic method, DOProC, which is being developed now.

DOProC has proved to be suitable not only for the reliability assessment tasks but also for other probabilistic calculations where ProbCalc can be used as well.

It should be pointed out that the possibilities for using DOProC are far from being exhausted.The area for further investigations is the use of statistically dependent input quantities where input data are entered directly into the algorithm. The software can be also used for the reliability assessment of structural systems and development of such numerical procedures which should increase efficiency of DOProC in matrix calculations, for instance.

Appendix

A lite version of the computational modules in ProbCalc and other software applications which are based onDOProC can be downloaded at http://www.fast.vsb.cz/popv.

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