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Structural Reliability Analysis Using DOProC Method

M. Krejsa[a,]*, P. Janas[a], V. Krejsa[a]

[a]VSB - Technical University Ostrava, Faculty of Civil Engineering, Ludvika Podeste 1875/17, Ostrava – Poruba 708 33, Czech Republic

Abstract

Various calculation methods based on the theory of probability and statistics are used for structural analysis and reliability assessment. Those methods so-called probabilistic have been becoming very popular recently. Using the probabilistic method, it is possible to analyse a safety margin defined in a computational model where at least some input characters are random. New method which are being developed now - Direct Optimized Probabilistic Calculation ("DOProC") is a purely numerical method which uses no simulation or approximation techniques. Results of the probabilistic tasks are more accurate and, often, more fast to reach.

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Keywords: DOProC; direct optimized probabilistic calculation; probCalc; probabilistic methods; reliability assessment; random variable; probability of failure

1. Introduction

The paper is focused on the probabilistic methods may be used advantageously in engineering, e.g. in civil engineering for solving structural analysis problems where a computational model contains random variables. Primary probability approaches are presented and developed for the modelling and analysis of uncertainty [41], and for evaluating the associated effects on safety and reliability [1]. An important part in structural failure analysis is modelling and quantification of various sources of uncertainty. In structural theory of reliability there are analysed aleatory uncertainty and epistemic uncertainty [19]. Aleatory uncertainty is related to the randomness of physical quantities (strength of material such as variability in yield strength of steel, and others various material parameters, such as [33]) and can be modelled as random variables in probabilistic form. Epistemic uncertainty is knowledge-based and is coming out from imperfection of the calculation model (discrepancy between behaviour of real

* Corresponding author. Tel.: +420 597 321 303; fax: +420 597 321 358.
E-mail address: martin.krejsa@vsb.cz .

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structure and its computational simplified representation in numerical model [7, 43], e.g. FEM) or limited availability of random input data.

2. Formulation of structural reliability

Probabilistic calculation model can be defined in probabilistic tasks in general as the function of n random variables X_1, X_2,…, X_n expressed by statistical moments, parametric probability distributions or empirical distributions of probability in form of non-parametrically defined bounded histograms. Resulting random variable Z, expressed generally as:

$$Z = g(X_1, X_2, \ldots, X_n), \tag{1}$$

is also random variable.

The structure must satisfy the condition of reliability, based e.g. on the assumption:

$$R \geq E \rightarrow Z = R - E \geq 0, \tag{2}$$

where R is resistance of structure, E load effect and Z safety margin. Taking into account all randomness in loads [4, 32], manufacturing and assembly imperfections and the environment properties in which designed structure performs its function, resistance and load effect are to be considered as statistically dependent or independent random variables.

Common notation of the theoretical time-invariant structural reliability problem - estimated failure probability p_f, can be defined relative to the criterion of reliability (2) as:

$$p_f = P(R < E) = \int_{D_f} f(x_1, x_2, \ldots, x_n)\, \mathrm{d}x_1, \mathrm{d}x_2, \ldots, \mathrm{d}x_n \cdot \tag{3}$$

where D_f is failure area of the safety margin $Z(\mathbf{X}) < 0$ as a function $f(\mathbf{X})$ of joint probability density of random variables $\mathbf{X} = X_1, X_2, \ldots, X_n$.

Determination of failure probability p_f based on the explicit calculation of the integral (3) is very complicated and generally unmanageable. For solution of this integral have been developed series of probabilistic methods [31] – see below.

3. Overview of probabilistic computational methods

The probabilistic computational methods for reliability calculations can be split e.g. into the following groups:

- Simulation based techniques. The simulation techniques have their origin in crude Monte Carlo simulation method (MC) [37], which generates a large sample set of limit state evaluations and approximates the true value of the probability of failure by identifying the number of samples falling into the failure domain. In order to further improve the computational efficiency many variance reduction techniques have been proposed:
- Stratified sampling techniques. Stratified Sampling and Latin Hypercube Sampling (LHS) [35] represent the special type of MC numerical simulation which uses the stratification of the theoretical probability distribution function of input random variables. The whole space of each random variable is divided into subsets of equal probability from which is generated outcome [39].
- Advanced simulation techniques. The sampling process of this kind of simulation techniques focuses in the failure region and helps faster convergence to the true failure probability. On this principle was developed several methods, e.g.: Importance Sampling [36], Adaptive Sampling [5], Directional Simulation [3, 8], Line Sampling, Design Point Sampling, Axis Orthogonal Importance Sampling method, Subset simulation [2], Descriptive sampling [40] and Slice Sampling [38].

- Approximation techniques. The principal methods FORM/SORM use analytical approximations in which the reliability index β is interpreted as the minimum distance from the origin to the limit state surface in standardized normal space and the most likely failure point – „design point", is searched [17]. In the First-Order Reliability Method - FORM, an approximation to the probability of failure is obtained by linearizing the limit-state surface (the boundary of the failure domain) [18]. The Second-Order Reliability Method - SORM, improves on this approximation by using a parabolic, quadratic or higher order surface fitted at the design point as the integration boundary [16].
- Response Surface methodologies. The Response Surface Method (RSM) is efficient and widely applicable method in structural reliability analysis [20], which is based on the moving least square method and design experiments. In this method, typically first-order or second-order polynomials are chosen to approximate the real limit state function.
- Perturbation techniques. Algorithms based on perturbation estimates form one of stochastic approximation algorithms [42] and used e.g. in Stochastic Finite Element Method (SFEM).
- Artificial Neural Network. The Artificial Neural Network (ANN) algorithms introduced as universal function approximations are also increasingly used for structural reliability assessment.

The following mentioned newly developed probabilistic method can't classify into any mentioned category.

4. Direct determined probabilistic calculation – DOProC method

The proposed method: Direct Optimized Probabilistic Calculation – DOProC, solves the integral (3) pure numerical way that is based on basis of probability theory and does not require any simulation or approximation technique. This is highly effective way of probabilistic calculation in terms of computation time and accuracy of the solution for many probabilistic tasks. The novelty of the proposed method lies in an optimized numerical integration. In summary was published e.g. in [10, 11].

4.1. Theoretical Background

Similar to many other probabilistic methods the random input quantities such as the load, geometry, material properties or imperfections can be in DOProC defined using non-parametric (empirical) distribution of probability expressed by means of bounded histograms. It is also possible to use parametric distributions [6], typically based on observations, often of long-term data [9, 44].

In probabilistic tasks are input random variables often statistically dependent - for example cross-section properties, or strength and stiffness characteristics of the materials. In the calculations carried out by DOProC method can be statistically dependent input random variable expressed by the so-called multidimensional histograms (double, triple) [14], which can be used also for the calculation of so/called *numerical correlation index* for the characterization of the dependence not only for the linear relationship between two variables, but also for nonlinear dependence, or even for more than two random variables [15].

The basic computation algorithm of DOProC is based on general terms and procedures used in probabilistic theories. Let the histogram B be an arbitrary function f of histograms A_j where j ranges from 1 to n. Then:

$$B = f\left(A_1, A_2, A_3,..., A_j,...A_n\right) . \tag{4}$$

Each histogram A_j consists of i_j interval, where i_j ranges from 1 to N_j. Each interval is limited with value $a_{j(i\text{-}1)}$ from below and with value $a_{j(i)}$ from above. This means, that for the interval i_j = 1, the values will be as follows:

$$a_{j(0)} \le a_{j,1} < a_{j(1)} , \tag{5}$$

where

$$a_{j(1)} = a_{j(0)} + \Delta a_j \ , \tag{6}$$

and

$$\Delta a_j = \frac{a_{j,\max} - a_{j,\min}}{N_j} \cdot \tag{7}$$

In i_j interval of the histogram of the independence random variable A_j, the following formula is valid:

$$a_{j(i-1)} \le a_{j,i_j} < a_{j(i)} \ . \tag{8}$$

Values a_{j,i_j} in that interval are defined usually as mean value:

$$a_{j,i_j} = \frac{a_{j(i)} - a_{j(i-1)}}{2} \ . \tag{9}$$

Similar relations are valid for the histogram of the random variable B. If there are N_B intervals, the values of the histogram in the interval i_B are in range from $b_{(i_B\text{-}1)}$ to $b_{(i_B)}$ with the representative mean value b_{i_B}. They can be expressed as follows:

$$b_k = f\left(a_{1,i_1}, a_{2,i_2}, a_{3,i_3}, ..., a_{j,i_j}, ..., a_{n,i_n}\right)_k , \tag{10}$$

where k is the serial number of the combination of intervals of independent random variables A_j, which ranges from 1 to $k_{\max}$:

$$k_{\max} = N_1 \cdot N_2 \cdot N_3 \cdot \ldots \cdot N_j \cdot \ldots \cdot N_n \ . \tag{11}$$

If the values a_{1,i_1}, a_{2,i_2}, a_{3,i_3}, ..., a_{j,i_j}, ..., a_{n,i_n} are statistically independent, the probability of occurrence of the value b_k for the combination of k intervals of the histograms of independent random variables A_j is the product of probabilities of all potential occurrence of values a_{j,i_j}, which are included in the expression (10):

$$p(b_k) = \left\lfloor p\left(a_{1,i_1}\right) \cdot p\left(a_{2,i_2}\right) \cdot p\left(a_{3,i_3}\right) \cdot \ldots \cdot p\left(a_{j,i_j}\right) \cdot \ldots \cdot p\left(a_{n,i_n}\right) \right\rfloor_k \cdot \tag{12}$$

Each value b_k will have the probability $p(b_k)$ and will be included into the corresponding interval b_{i_B}. All of values b_k which have to been included into the interval b_{i_B} will correspond the probability:

$$p\left(b_{i_B}\right) = \sum_k p(b_k) \ \cdot \tag{13}$$

Also have to be valid:

$$\sum_{i_B=1}^{N_B} p\left(b_{i_B}\right) = 1 \ \cdot \tag{14}$$

Performing these numerical operations of the probabilistic calculation with two random variables A_1 and A_j expressed by histograms is shown in scheme on the Fig. 1.

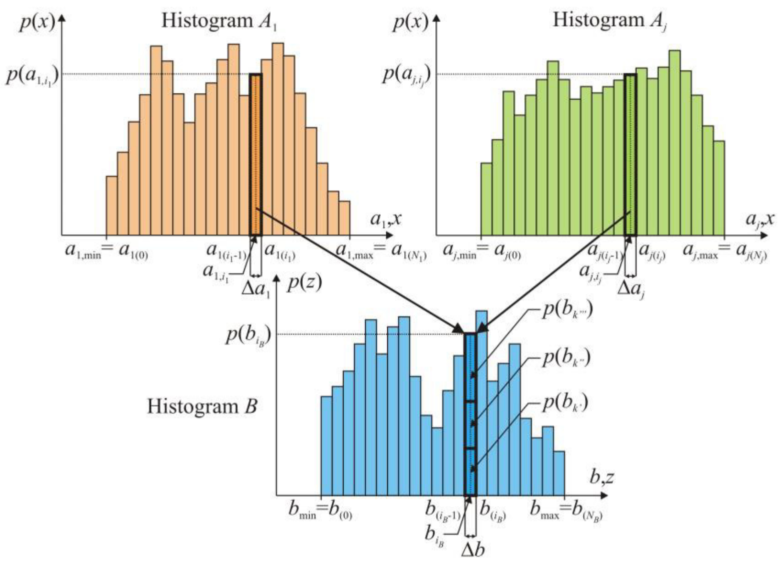

Fig. 1. Principle of the performing the numerical operations with the histograms of two statistically independent random variables.

The number of intervals i_j in each histogram of the random variables A_j can be similar as the number of i_B intervals in the histogram of the resulting random variable *B*. The number of intervals is very important for the total number of needed numerical operations and required computing time. On top of this, the accuracy of the calculation depends considerably on the number of intervals. If there are too many random quantities, the tasks require too much time even if advanced computational facilities are available. Therefore, efforts have been made to optimize calculations in order to reduce the number of operations, while retaining reliable calculation results:

- The grouping of input and output variables. This procedure can be used e.g. in situations where the random variable input or output variables can be expressed using one joint histogram. This leads to a large reduction of computational operations.
- Parallelization. The calculation algorithm of DOProC method is advantageous for use on computers with two or more CPUs or their cores. The basic computational algorithm of DOProC - Eq. (10) and (12), can be divided the number of computational operations up to as many parts as there are available execution units, and after partial calculations can be put together from partial results into the histogram of resulting variable, e.g. histogram of safety margin *Z*.
- Interval optimizing. The purpose of this computational procedure is to reduce the intervals of each variable involved in the calculation. Input random variables don't affect the outcome of the probabilistic calculation as well - are differently sensitive. For input variables that affect the outcome probability less, therefore the number of classes can be reduced. Custom probabilistic calculation is then carried out with the minimum number of intervals for each input random variables.
- Zone optimizing. The intervals of each individual histogram are clearly defined during the calculation using one to three types of zones, depending on influence on resulting probability of failure (contribute always, may or may not contribute, contribute never). The calculation then will be limited only on intervals of input random variables which clearly don't contribute the resulting value of failure probability.
- Trend optimizing. This optimization of probabilistic calculation follows the zonal optimizing. This optimization of probabilistic calculation determines the trends of changes in the histograms of input variables when defining individual zones.

Such procedures can be combined, thereby achieving an even stronger acceleration of the calculation. Mentioned computational procedures have been described comprehensively in [10, 11].

4.2. Application

The algorithm of DOProC method has been implemented in several software codes [28], and has been used in many cases in probabilistic tasks and reliability assessments [23]. For the application of the DOProC method [30] can be used software titled ProbCalc [12, 27]. In ProbCalc is relatively easy to implement analytical and numerical transformation probabilistic model of solved tasks. The ProbCalc is extensively useful in solving of probabilistic tasks of engineering practice, especially on probabilistic reliability assessment according to the current standards [34], see Fig. 2.

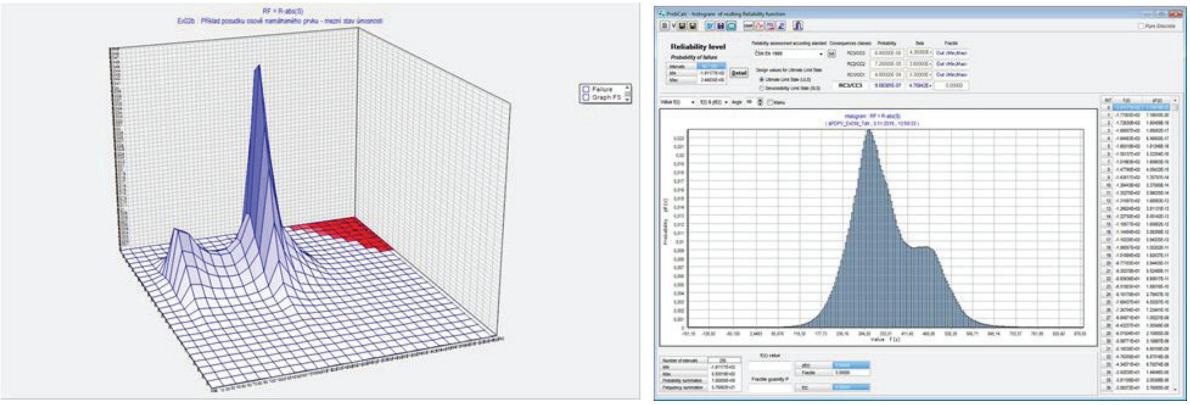

Fig. 2. Desktop of ProbCalc software: Resulting histogram of safety margin and resulting reliability assessment according Eurocodes.

Special software applications HistAn2D and HistAn3D were developed for creation of the double (Fig. 3) and triple histograms which describe the statistical dependence between two or three random variables (for instance, for strength properties or construction parameters of construction materials, see [14, 15]).

In [22, 25, 26] was published in detail the methodology for probabilistic assessment of structures exposed to fatigue, focusing on the determination of acceptable size of fatigue crack and definition of the regular inspection system. This relatively advanced probabilistic task was solved using ProbCalc, but also using new application under development titled FCProbCalc [21, 29], which allows in a user friendly environment to calculate the probability of fatigue crack progression.

The comprehensively methodology for probabilistic design and reliability assessment of anchor reinforcement in long mining and underground works was also utilized [13, 24]. It was also established a program Anchor, with which is possible to realize the probabilistic calculation very flexibly.

5. Summary

This paper discussed probabilistic methods for the estimation of structural failure analysis with focus on new method under development – the Direct Optimized Probabilistic Calculation ("DOProC"). The highlight of the DOProC lies in an efficient and accurate optimized numerical integration useful to many probabilistic tasks and failure analysis. The algorithm has been implemented in software codes, which seem to be very effective probabilistic computational tools.

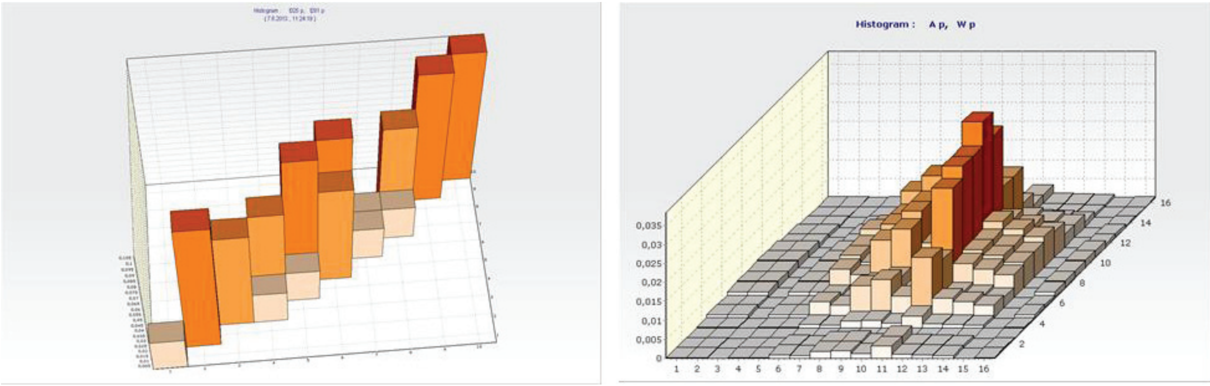

Fig. 3. A double histogram created in HistAn2D shows statistics dependence of two random variables (left) and behavior of two statistically independent random quantities

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Appendix A. An example appendix

A lite version of the ProbCalc and the other software applications based on the DOProC method can be downloaded at http://www.fast.vsb.cz/popv.

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