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Quasi-oppositional differential evolution for optimal reactive power dispatch

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ABSTRACT

This paper presents quasi-oppositional differential evolution to solve reactive power dispatch problem of a power system. Differential evolution (DE) is a population-based stochastic parallel search evolutionary algorithm. Quasi-oppositional differential evolution has been used here to improve the effectiveness and quality of the solution. The proposed quasi-oppositional differential evolution (QODE) employs quasi-oppositional based learning (QOBL) for population initialization and also for generation jumping. Reactive power dispatch is an optimization problem that reduces grid congestion with more than one objective. The proposed method is used to find the settings of control variables such as generator terminal voltages, transformer tap settings and reactive power output of shunt VAR compensators in order to achieve minimum active power loss, improved voltage profile and enhanced voltage stability. In this study, QODE has been tested on IEEE 30-bus, 57-bus and 118-bus test systems. Test results of the proposed QODE approach have been compared with those obtained by other evolutionary methods reported in the literature. It is found that the proposed QODE based approach is able to provide better solution.

Introduction

The reactive power dispatch (RPD) plays an important role for improving economy and security of power system operation. Although the reactive power generation has no production cost, however it affects the overall generation cost by the way of the active power loss. The RPD is a nonlinear, non-convex and non-differentiable optimization problem. It minimizes active power loss and improves voltage profile and voltage stability by adjusting control variables such as generator voltages, transformer tap settings, and reactive power output of shunt VAR compensators in a power system while satisfying several equality and inequality constraints.

Several classical mathematical methods [1–8] such as linear programming, quadratic programming, gradient projection method, interior point method, reduced gradient method and Newton method have been applied to solve RPD problem of power system. These methods are computationally fast but these methods optimize the objective function by linearizing it. The RPD is a non-linear multimodal optimization problem with a mixture of discrete and continuous variables. It has multiple local optima. Hence, it is so hard to find the global optimum of reactive power dispatch problem by using classical mathematical methods. For these reasons, researchers have developed computational intelligence-based techniques to solve the RPD problem.

In recent years, computational intelligence-based techniques, such as evolutionary programming [9], adaptive genetic algorithm [10], particle swarm optimization [11], hybrid stochastic search technique [12], hybrid particle swarm optimization [13], multiagent-based particle swarm optimization [14], bacterial foraging based optimization [15], differential evolution [16,21], quantum-inspired evolutionary algorithm [17], self adaptive real coded genetic algorithm [18], seeker optimization algorithm [19], comprehensive learning particle swarm optimization (CLPSO) [20], biogeography-based optimization [22], hybrid shuffled frog leaping algorithm and Nelder–Mead simplex search [23], gravitational search algorithm [24], quasi-oppositional teaching learning based optimization [25], and opposition-based gravitational search algorithm [26] have been applied to solve RPD problem. These techniques have shown effectiveness in overcoming the disadvantages of classical methods.

Since the mid 1990s, many techniques originated from Darwin's natural evolution theory have emerged. These techniques are usually termed by "evolutionary computation methods" including evolutionary algorithms (EAs), swarm intelligence and artificial immune system. Differential evolution (DE) [27–29], a relatively new member in the family of evolutionary algorithms, first

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proposed over 1995–1997 by Storn and Price at Berkeley is a novel approach to numerical optimization. It is a population-based stochastic parallel search evolutionary algorithm which is very simple yet powerful. The main advantages of DE are its capability of solving optimization problems which require minimization process with nonlinear, non-differentiable and multi-modal objective functions.

The basic concept of opposition-based learning (OBL) [31–33] was originally introduced by Tizhoosh. The main idea behind OBL is for finding a better candidate solution and the simultaneous consideration of an estimate and its corresponding opposite estimate (i.e., guess and opposite guess) which is closer to the global optimum. OBL was first utilized to improve learning and back propagation in neural networks by Ventresca and Tizhoosh [34], and since then, it has been applied to many EAs, such as differential evolution [35], particle swarm optimization [36] and ant colony optimization [37].

Quasi-oppositional based learning (QOBL) is implemented on differential evolution (DE). The proposed quasi-oppositional differential evolution (QODE) along with basic differential evolution (DE) is applied to solve the RPD problem. The RPD is a combinatorial optimization problem involving nonlinear functions having multiple local optima and nonlinear and discontinuous constraints. In order to evaluate the proposed method, the proposed QODE is tested on IEEE 30-bus, 57-bus and 118-bus test systems with different objective functions that reflect active power loss minimization, voltage profile improvement and voltage stability enhancement. Test results obtained from QODE have been compared with those obtained by other evolutionary methods reported in the literature. From numerical results, it is found that the proposed QODE based approach provides better solution.

Problem formulation

The objective of the RPD is to minimize the active power loss and to improve voltage profile and voltage stability while satisfying equality and inequality constraints. Three objective functions and constraints are formulated as follows.

Objective functions

Minimization of active power loss

Minimization of active power loss in the transmission lines can be formulated as follows

$$\text{Minimize } F_1 = P_{loss} = \sum_{k=1}^{NTL} g_k \left[V_i^2 + V_j^2 - 2V_iV_j\cos(\delta_i - \delta_j) \right] \tag{1}$$

where P_{loss} denotes active power loss of the power system, NTL is the number of transmission lines, g_k is the conductance of branch k connected between ith bus and jth bus, V_i and V_j are the voltage magnitudes of the ith and jth buses, δ_i and δ_j are the voltage phase angles of the ith and jth buses.

The vector of dependent variables x may be represented as

$$x^T = [P_{G1}, V_{L1}, \ldots, V_{LNPQ}, Q_{G1}, \ldots, Q_{GNG}] \tag{2}$$

where P_{G1} denotes the slack bus power, V_L is the PQ bus voltage, Q_G is the reactive power output of the generator, NG is the number of generator bus, NPQ is the number of PQ bus.

The vector of control variables u may be represented as

$$u^T = [V_{G1}, \ldots, V_{GNG}, Q_{c1}, \ldots, Q_{cNC}, T_1, \ldots, T_{NT}] \tag{3}$$

where NC and NT are the number of shunt VAR compensators and the number of tap changing transformers, V_G is the terminal voltage at the voltage controlled bus, Q_c is the output of shunt VAR compensator and T is the tap setting of the tap changing transformer.

Voltage profile improvement

The objective is to minimize the voltage deviation of all load (PQ) buses from 1 p.u. As a result the power system operates more securely and service quality is also improved. The objective function can be formulated as follows

$$\text{Minimize } F_2 = \sum_{i=1}^{NPQ} |V_i - 1.0| \tag{4}$$

where NPQ is the number of load buses in the power system.

Voltage stability enhancement

Voltage stability problem is the ability of a power system to maintain acceptable voltages at all bus bars in the system under normal operating condition. A system experiences a state of voltage instability when the system is being subjected to a disturbance, increase in load demand or change in system configuration which causes a progressive and uncontrollable decrease in voltage. Weak system, system with long transmission lines and heavily loaded system are much prone to voltage instability problem.

Voltage instability is a major threat for secure and reliable operation of a large scale power system. The loss of voltage stability can manifest in the form of progressive drop of voltage magnitudes, triggering unintentional load shedding and even leading to cascading outages or system-wide blackouts. Recently, a number of major blackouts around the world [39] have taken place due to voltage instability.

Voltage stability can be classified into long-term and short-term concerns depending on the time frame of interest. Analysis techniques can generally fall into static method and dynamic method. The static method is necessary for analyzing long-term voltage stability problem where as dynamic method is necessary for analyzing short-term voltage stability problem. The former is based on steady state modeling of the network i.e. via algebraic equations and relies on power flow. The latter is based on the time domain simulation, which models the system via differential–algebraic equations to account for the dynamic nature of system components in particular loads. Here, long-term voltage stability problem has been considered.

Enhancement of voltage stability of a system is an important parameter of power system planning and operation. Voltage stability enhancement can be done by minimizing the voltage stability indicator i.e. L-index value at each bus of a power system. The L-index of a bus indicates the proximity of voltage collapse condition of that bus. L-index L_j of jth bus is defined as follows [40]

$$L_j = \left| 1 - \sum_{i=1}^{NPV} F_{ji} \frac{V_i}{V_j} \right| \quad \text{where } j = 1, 2, \ldots, NPQ \tag{5}$$

$$F_{ji} = -[\Upsilon_1]^{-1}[\Upsilon_2] \tag{6}$$

where NPV is the number of PV bus and NPQ is the number of PQ bus. Υ_1 and Υ_2 are the sub-matrices of the system YBUS obtained after segregating the PQ and PV bus bar parameters as described in (5).

$$\begin{bmatrix} I_{PQ} \\ I_{PV} \end{bmatrix} = \begin{bmatrix} \Upsilon_1 \Upsilon_2 \\ \Upsilon_3 \Upsilon_4 \end{bmatrix} \begin{bmatrix} V_{PQ} \\ V_{PV} \end{bmatrix} \tag{7}$$

L-index is calculated for all the PQ buses. L_j represents no load case and voltage collapse case of bus j in the range of 0 and 1 respectively. Hence, a global system indicator L describing the stability of a complete system is given as follows:

$$L = \max(L_j), \quad \text{where } j = 1, 2, \ldots, NPQ \tag{8}$$

Lower value of L represents a more stable system. In the RPD problem, inaccurate tuning of control variable settings may increase voltage stability margin of the system [21]. In order to improve voltage stability and to move the system far from the voltage collapse point, the objective function [24] can be defined as follows:

$$\text{Minimize } F_3 = L_{\max} \tag{9}$$

where $L_{\max}$ is the maximum value of L-index.

Constraints

The objective functions are subjected to the equality constraints imposed by the physical laws governing the transmission system as well as the inequality constraints imposed by the equipment ratings given below.

Equality constraints

These constraints are load flow equations as described below:

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] = 0,$$
$$i = 1, 2, \ldots, NB \tag{10}$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \sin(\delta_i - \delta_j) B_{ij} \cos(\delta_i - \delta_j)] = 0,$$
$$i = 1, 2, \ldots, NB \tag{11}$$

where NB is the number of buses, P_{Gi} and Q_{Gi} are active and reactive power generation at the ith bus, P_{Di} and Q_{Di} are active and reactive power demand at the ith bus, G_{ij} and B_{ij} are the transfer conductance and susceptance between ith bus and jth bus respectively.

Inequality constraints

Generator constraints. The generator voltage magnitudes and reactive power outputs are constrained by design specifications. The lower and upper limits of generator voltage magnitude and reactive power output are given below:

$$V_{Gi}^{\min} \leqslant V_{Gi} \leqslant V_{Gi}^{\max}, \quad i = 1, 2, \ldots, NG \tag{12}$$

$$Q_{Gi}^{\min} \leqslant Q_{Gi} \leqslant Q_{Gi}^{\max}, \quad i = 1, 2, \ldots, NG \tag{13}$$

Shunt VAR compensator constraints. Reactive power output of shunt VAR compensators must be restricted within their lower and upper limits as follows:

$$Q_{ci}^{\min} \leqslant Q_{ci} \leqslant Q_{ci}^{\max}, \quad i = 1, 2, \ldots, NC \tag{14}$$

Transformer constraints. The upper and lower values for the transformer tap settings are limited by physical considerations and these are given below:

$$T_i^{\min} \leqslant T_i \leqslant T_i^{\max}, \quad i = 1, 2, \ldots, NT \tag{15}$$

Security constraints. These include the constraints on voltage magnitudes at PQ buses and transmission line loadings. Voltage of each PQ bus must be within its lower and operating limits. Line flow through each transmission line must be within its capacity limits. These are described as follows:

$$V_{Li}^{\min} \leqslant V_{Li} \leqslant V_{Li}^{\max}, \quad i = 1, 2, \ldots, NPQ \tag{16}$$

$$S_{li} \leqslant S_{li}^{\max}, \quad i = 1, 2, \ldots, NTL \tag{17}$$

Description of quasi-oppositional differential evolution

A brief description of differential evolution

Differential evolution (DE) is a type of evolutionary algorithm originally proposed by Price and Storn [29] for optimization problems over a continuous domain. DE is exceptionally simple, significantly faster and robust. The basic idea of DE is to adapt the search during the evolutionary process. At the start of the evolution, the perturbations are large since parent populations are far away from each other. As the evolutionary process matures, the population converges to a small region and the perturbations adaptively become small. As a result, the evolutionary algorithm performs a global exploratory search during the early stages of the evolutionary process and local exploitation during the mature stage of the search. In DE the fittest of an offspring competes one-to-one with that of corresponding parent which is different from other evolutionary algorithms. This one-to-one competition gives rise to faster convergence rate. Price and Storn gave the working principle of DE with simple strategy in [29]. Later on, they suggested ten different strategies of DE [30]. Strategy-7 (DE/rad/1/bin) is the most successful and widely used strategy. The key parameters of control in DE are population size (N_P), scaling factor (S_F) and crossover rate (C_R). The optimization process in DE is carried out with three basic operations: mutation, crossover and selection. The DE algorithm is described as follows.

Initialization

The initial population of N_P vectors is randomly selected based on uniform probability distribution for all variables to cover the entire search uniformly. Each individual X_i is a vector that contains as many parameters as the problem decision variables D. Random values are assigned to each decision parameter in every vector according to:

$$X_{ij}^0 \sim U(X_j^{\min}, X_j^{\max}) \tag{18}$$

where $i = 1, \ldots, N_P$ and $j = 1, \ldots, D$; $X_j^{\min}$ and $X_j^{\max}$ are the lower and upper bounds of the jth decision variable; $U\left(X_j^{\min}, X_j^{\max}\right)$ denotes a uniform random variable ranging over $\left[X_j^{\min}, X_j^{\max}\right]$. X_{ij}^0 is the initial jth variable of ith population. All the vectors should satisfy the constraints. Evaluate the value of the cost function $f\left(X_i^0\right)$ of each vector.

Mutation

DE generates new parameter vectors by adding the weighted difference vector between two population members to a third member. For each target vector X_i^k at kth iteration the noisy vector $X_i'^k$ is obtained by

$$X_i'^k = X_a^k + S_F\left(X_b^k - X_c^k\right), \quad i \in N_P \tag{19}$$

where X_a^k, X_b^k and X_c^k are selected randomly from N_P vectors at kth iteration and $a \neq b \neq c \neq i$. The scaling factor (S_F), in the range $0 < S_F \leqslant 1.2$, controls the amount of perturbation added to the parent vector. The noisy vectors should satisfy the constraint.

Crossover

Perform crossover for each target vector X_i^k with its noisy vector $X_i'^k$ and create a trial vector $X_i''^k$ such that

$$X_i''^k = \begin{cases} X_i'^k, & \text{if } \rho \leqslant C_R \\ X_i^k, & \text{otherwise} \end{cases}, \quad i \in N_P \tag{20}$$

where ρ is an uniformly distributed random number within $[0, 1]$. The crossover constant (C_R), in the range $0 \leqslant C_R \leqslant 1$, controls the diversity of the population and aids the algorithm to escape from local optima.

Selection

Perform selection for each target vector, X_i^k by comparing its cost with that of the trial vector, $X_i^{//k}$. The vector that has lesser cost of the two would survive for the next iteration.

$$X_i^{k+1} = \begin{cases} X_i^{//k}, & \text{if } f\left(X_i^{//k}\right) \leqslant f\left(X_i^k\right) \\ X_i^k, & \text{otherwise} \end{cases}, \quad i \in N_P \tag{21}$$

The process is repeated until the maximum number of iterations or no improvement is seen in the best individual after many iterations.

Fig. 1 shows the flowchart of differential evolution.

Opposition-based learning

Opposition-based learning (OBL) was developed by Tizhoosh [31] to improve candidate solution by considering current population as well as its opposite population at the same time.

Evolutionary optimization methods start with some initial population and try to improve them toward some optimal solution. The process of searching terminates when some predefined criteria are satisfied. The process is started with random guesses in the absence of a priori information about the solution.

The process can be improved by starting with a closer i.e. fitter solution by simultaneously checking the opposite solution. By doing this, the fitter one (guess or opposite guess) may be chosen as an initial solution. According to the theory of probability, 50% of the time, a guess is further from the solution than its opposite guess. Therefore, process starts with the closer of the two guesses. The same approach can be applied not only to the initial solution but also continuously to each solution in the current population.

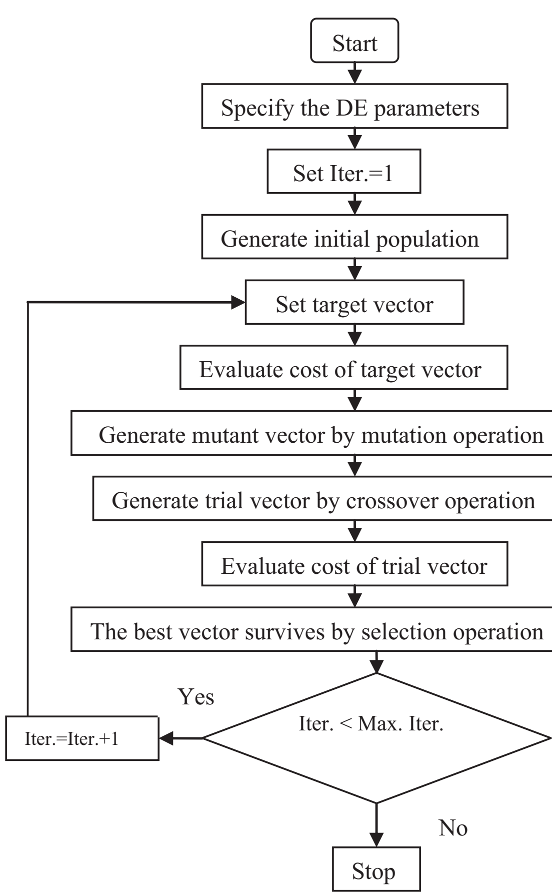

Fig. 1. Flowchart of differential evolution.

Quasi-opposition-based learning

Quasi-opposition-based learning (QOBL) was introduced by Rahnamayan et al. [38] to improve candidate solution by considering current population as well as its quasi-opposite population at the same time.

The process can be improved by starting with a closer i.e. fitter solution by simultaneously checking the quasi-opposite solution. By doing this, the fitter one (guess or quasi-opposite guess) may be chosen as an initial solution. The process starts with the closer of the two guesses. The same approach can be applied not only to the initial solution but also continuously to each solution in the current population. It is proved that a quasi-opposite number is usually closer than a random number to the solution. It is also proved that a quasi-opposite number is usually closer than an opposite number to the solution [38]. The idea of QOBL technique is used in population initialization and generation jumping.

Definition of opposite number and quasi-opposite number

If x be a real number between $[lb, ub]$, its opposite number (x_o) and its quasi-opposite number (x_{qo}) are defined as

$$x_o = lb + lu - x \tag{22}$$

and

$$x_{qo} = rand\left[\left(\frac{lb + lu}{2}\right), (lb + lu - x)\right]$$

Similarly, this definition can be extended to higher dimensions [31] as stated in the next sub-section.

Definition of opposite point and quasi-opposite point

Let $X = (x_1, x_2, \ldots, x_n)$ be a point in n-dimensional space where $x_i \in [lb_i, ub_i]$ and $i \in 1, 2, \ldots, n$. The opposite point $X_o = (x_{o1}, x_{o2}, \ldots, x_{on})$ is completely defined by its components as in (23).

$$x_{oi} = lb_i + ub_i - x_i \tag{23}$$

The quasi-opposite point $X_{qo} = (x_{qo1}, x_{qo2}, \ldots, x_{qon})$ is completely defined by its components as in (24).

$$x_{qoi} = rand\left[\left(\frac{lb_i + lu_i}{2}\right), (lb_i + lu_i - x_i)\right] \tag{24}$$

By employing the definition of quasi-opposite point, the quasi-opposition-based optimization is defined in the following sub-section.

Quasi-opposition based optimization

Let $X = (x_1, x_2, \ldots, x_n)$ be a point in n-dimensional space i.e. a candidate solution. Assume $f = (\bullet)$ is a fitness function which is used to measure the candidate's fitness. According to the definition of the quasi-opposite point, $X_{qo} = (x_{qo1}, x_{qo2}, \ldots, x_{qon})$ is the quasi-opposite of $X = (x_1, x_2, \ldots, x_n)$. Now, if $f(X_{qo}) < f(X)$ (for a minimization problem), then point X can be replaced with X_{qo}; otherwise, the process is continued with X. Hence, the point and its quasi-opposite point are evaluated simultaneously in order to continue with the fitter one.

Quasi-oppositional differential evolution

In the present work, the concept of the quasi-opposition-based learning [38] is incorporated in differential evolution. The original DE is chosen as a parent algorithm and the quasi-opposition-based ideas are embedded in DE. Fig. 2 shows the flowchart of QODE algorithm.

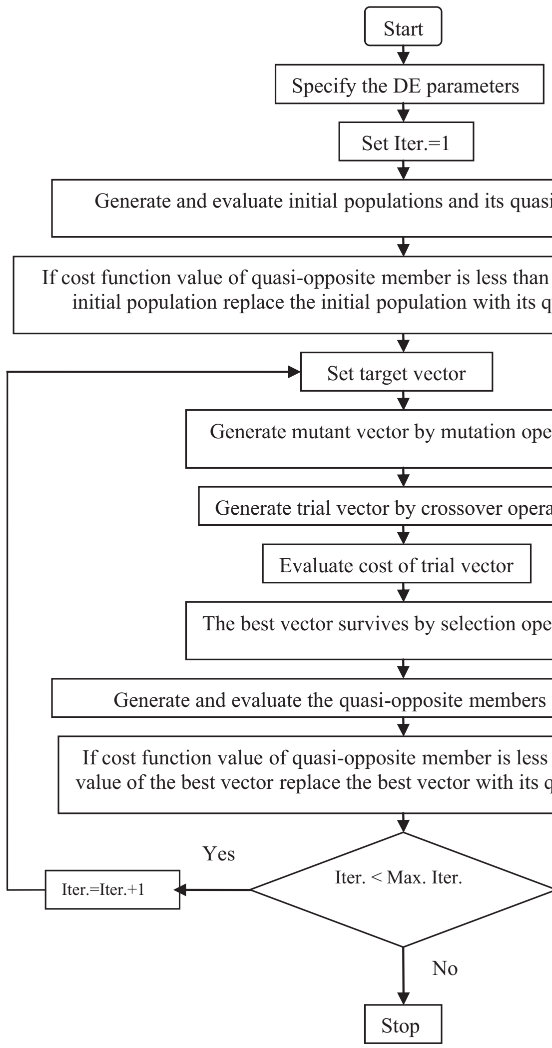

Fig. 2. Flowchart of QODE.

Table 1
Optimal value of control variables obtained from QODE for IEEE 30 bus system for different cases.

Control variable	Active power loss minimization	Voltage stability enhancement	Improvement of voltage profile	Minimization of active power loss with improvement of voltage profile
V_1	1.0500	1.0500	1.0500	1.0500
V_2	1.0338	1.0336	1.0340	1.0334
V_5	1.0058	1.0055	1.0059	1.0057
V_8	1.0230	1.0232	1.0235	1.0232
V_{11}	1.0913	1.0911	1.0917	1.0915
V_{13}	1.0400	1.0402	1.0396	1.0404
T_{6-9}	0.9994	0.9875	1.0157	1.0189
T_{6-10}	1.0012	1.0031	1.0274	1.0212
T_{4-12}	0.9983	1.0222	1.0087	1.0058
T_{28-27}	1.0141	0.9895	0.9817	0.9977
Q_{c10}	0.0030	0.0000	0.0095	0.0242
Q_{c12}	0.0199	0.0032	0.0068	0.0469
Q_{c15}	0.0475	0.0500	0.0301	0.0092
Q_{c17}	0.0334	0.0314	0.0079	0.0027
Q_{c20}	0.0182	0.0500	0.0247	0.0307
Q_{c21}	0.0250	0.0249	0.0171	0.0318
Q_{c23}	0.0342	0.0486	0.0301	0.0073
Q_{c24}	0.0345	0.0403	0.0500	0.0311
Q_{c29}	0.0000	0.0500	0.0178	0.0207
Power loss (MW)	2.6867	7.0812	9.2745	3.1010
Voltage deviation	0.4609	0.8886	0.0607	0.2590
L_{max}	0.0581	0.0238	0.0543	0.0647

Table 2
Comparison of performance for active power loss minimization of IEEE 30 bus system.

Techniques	Best loss (MW)	Average loss (MW)	Worst loss (MW)	CPU time (s)
QODE	2.6867	2.6879	2.6895	82.074
DE	3.0762	3.0782	3.0981	74.675
GSA [24]	4.5143	–	–	94.693
DE [21]	4.5550	–	–	–
CLPSO [20]	4.5615	–	–	138
PSO [20]	4.6282	–	–	130
SARGA [18]	4.5740	–	–	–
QOTLBO [25]	4.5594	–	–	–
OGSA [26]	4.4984	–	–	89.19

Application of the proposed method

The proposed QODE and DE have been applied to solve RPD problems. Three different test systems with three different objective functions have been studied to verify its applicability. Programs have been written in MATLAB-7 language and executed on a 3.0 GHz Pentium-IV personal computer. In order to demonstrate the effectiveness of the proposed QODE for solution of three different RPD problems, IEEE 30-bus, 57-bus and 118-bus test systems have been considered. The results obtained from proposed QODE method are compared with those obtained from other evolutionary methods reported in the literature.

IEEE 30-bus system

The line data, bus data, generator data and the minimum and maximum limits for the control variables have been adapted from [4]. The system has six generators at buses 1, 2, 5, 8, 11 and 13 and four transformers with off nominal tap ratio at lines 6–9, 6–10, 4–12, and 28–27. In addition, shunt VAR compensating devices are assumed to be connected at bus bars 10, 12, 15, 17, 20, 21, 23, 24 and 29. as in [41]. The total system active power demand is 2.834 p.u. at 100 MVA base. In this study, 50 test runs are performed to solve the RPD problem for different objective functions. Different types of RPD problem for this test system are solved by using QODE and DE.

Minimization of active power loss

The proposed QODE and DE have been applied for minimization of active power loss as the objective function. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 100 respectively for this test system.

The optimal values of control variables obtained from the proposed QODE are given in Table 1. The best, average and worst active power loss and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 2. The active power loss obtained from gravitational search algorithm (GSA) [24], differential evolution (DE) [21], comprehensive learning particle swarm optimization (CLPSO) [20], particle swarm optimization (PSO) [20], self adaptive real coded genetic algorithm (SARGA) [18], quasi-oppositional teaching learning based optimization (QOTLBO) [25] and opposition-based gravitational search algorithm (OGSA) [26] are also shown in Table 2. The convergence characteristic obtained from proposed QODE and DE for minimum active power loss solution is shown in Fig. 3. It is seen from Table 2, that active power loss obtained from QODE is the least among all other methods. It is also observed that QODE based approach performs best after that it follows DE, OGSA, GSA, QOTLBO, CLPSO, SARGA and PSO.

Enhancement of voltage stability

In this case, the proposed QODE and DE approach have been applied for enhancement of voltage stability i.e. minimization of L_{max}. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 50 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are shown in Table 1. The best, average and worst active power loss and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 3. The L_{max} obtained from gravitational search algorithm (GSA) [24], differential evolution (DE) [21], quasi-oppositional teaching learning based optimization (QOTLBO) [25] and opposition-based gravitational search algorithm (OGSA) [26] are also shown in Table 3. The convergence characteristic obtained from proposed QODE and DE for L_{max} minimization is shown in Fig. 4. It is seen from Table 3 that the value of L_{max} obtained from QODE is the lowest among all other methods. It is also observed that QODE based

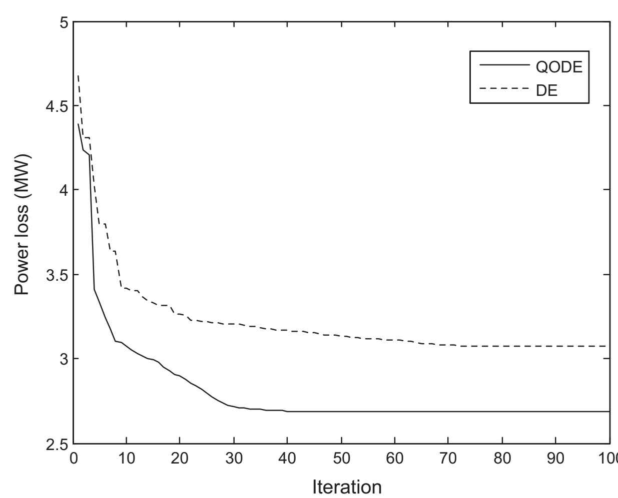

Fig. 3. Active power loss convergence characteristics for IEEE 30 bus system.

Table 3
Comparison of performance for L_{max} minimization of IEEE 30 bus system.

Techniques	Best L_{max}	Average L_{max}	Worst L_{max}	CPU time (s)
QODE	0.0238	0.0241	0.0246	50.473
DE	0.0256	0.0261	0.067	42.035
GSA [24]	0.1160	–	–	225.26
DE [21]	0.1246	–	–	–
QOTLBO [25]	0.1242	–	–	–
OGSA [26]	0.1230	–	–	185.16

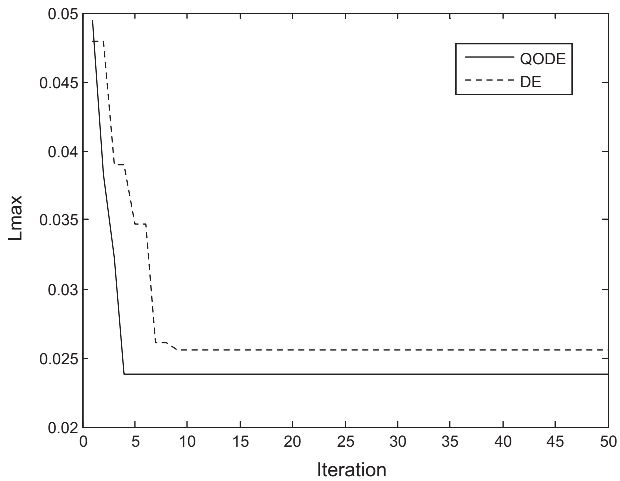

Fig. 4. L_{max} convergence characteristics for IEEE 30 bus system.

approach performs best after that it follows DE, GSA, OGSA and QOTLBO.

Improvement of voltage profile

In this case, the proposed QODE and DE have been applied for improvement of voltage profile. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 1. The best, average and worst voltage deviation and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 4. The voltage deviation obtained from gravitational search algorithm (GSA) [24], differential evolution (DE) [21], quasi-oppositional teaching learning based optimization (QOTLBO) [25] and opposition-based gravitational search algorithm (OGSA) [26] are also shown in Table 4. The convergence characteristic obtained from proposed QODE and DE for voltage deviation is shown in Fig. 5. It is seen from Table 4, that voltage deviation obtained from QODE is the lowest among all other methods. It is also observed that QODE based approach performs best after that it follows DE, GSA, OGSA and QOTLBO.

Minimization of active power loss with improvement of voltage profile

In this case, a twofold objective function is formed in order to minimize the active power loss and improve voltage profile by minimizing the voltage deviation of all load buses from 1.0 p.u. The objective function can be expressed as

$$\text{Minimize } F_4 = P_{loss} + \psi \sum_{i=1}^{NPQ} |V_i - 1.0| \tag{25}$$

where ψ is a weighting factor to be selected by the user. The value of ψ in this case is chosen as 0.01 [20].

The problem is solved by using proposed QODE and DE. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 100 respectively for this test system. The results obtained from proposed QODE for optimal values of control variables are shown in Table 1.

IEEE 57-bus system

The standard IEEE 57-bus system consists of 80 transmission lines, seven generators at buses 1, 2, 3, 6, 8, 9, 12 and 15 branches under load tap setting transformer branches. The reactive power sources are considered at buses 18, 25 and 53. The system line data, bus data, generator data and the minimum and maximum limits for the control variables have been adapted from [19] and [42]. The upper and lower limits of reactive power sources and transformer tap settings are taken from [24]. The total system active power demand is 12.508 p.u. and reactive power demand is 3.364 p.u. at 100 MVA base. In this study, 50 test runs are performed to solve the RPD problem for different objective functions. Different types of RPD problem for this test system are solved by using QODE and DE.

Table 4
Comparison of performance for Voltage deviation of IEEE 30 bus system.

Techniques	Best voltage deviation	Average voltage deviation	Worst voltage deviation	CPU time (s)
QODE	0.0607	0.0609	0.0612	81.310
DE	0.0623	0.0626	0.0631	74.506
GSA [24]	0.0676	–	–	198.65
DE [21]	0.0911	–	–	–
QOTLBO [25]	0.0856	–	–	–
OGSA [26]	0.0640	–	–	190.14

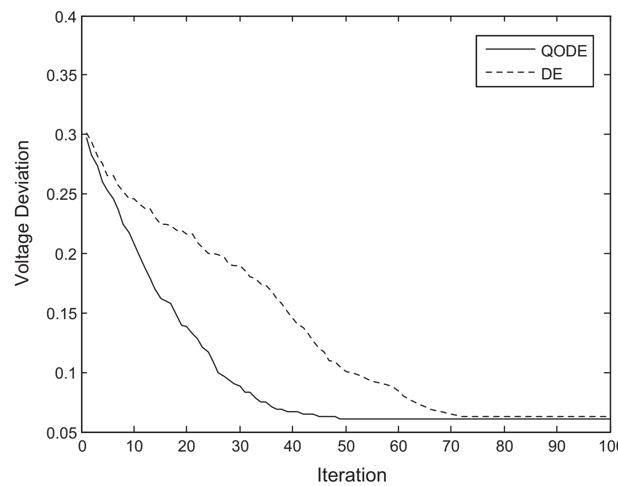

Fig. 5. Voltage deviation convergence characteristics for IEEE 30 bus system.

Minimization of active power loss

The proposed QODE and DE approach is applied for minimization of active power loss as the objective function. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 5. The best, average and worst active power loss and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 6. The active power loss obtained from gravitational search algorithm (GSA) [24], comprehensive learning particle swarm optimization (CLPSO) [19], seeker optimization algorithm (SOA) [19] and opposition-based gravitational search algorithm (OGSA) [26] are also shown in Table 6. The convergence characteristic obtained from proposed QODE and DE for minimum active power loss solution is shown in Fig. 6. It is seen from Table 6 that active power loss obtained from QODE is the lowest among all other methods. It is also observed that QODE based approach performs best after that it follows DE, OGSA, GSA, SOA and CLPSO.

Table 5
Optimal value of control variables obtained from QODE for IEEE 57 bus system for different cases.

Control loss variable	Active power minimization	Voltage stability enhancement	Improvement of voltage profile	Minimization of active power loss with improvement of voltage profile
V_1	1.0400	1.0400	1.0400	1.0400
V_2	1.0101	1.0103	1.0099	1.0107
V_3	0.9849	0.9847	0.9851	0.9850
V_6	0.9805	0.9800	0.9803	0.9806
V_8	1.0054	1.0050	1.0051	1.0047
V_9	0.9803	0.9805	0.9804	0.9800
V_{12}	1.0147	1.0150	1.0152	1.0149
T_{4-18}	1.0987	0.9801	0.9831	0.9805
T_{4-18}	1.0820	0.9526	0.9510	0.9529
T_{21-20}	0.9221	0.9501	0.9507	0.9505
T_{24-26}	1.0171	1.0045	1.0043	1.0047
T_{7-29}	0.9960	0.9777	0.9769	0.9775
T_{34-32}	1.0999	0.9138	0.9139	0.9136
T_{11-41}	1.0750	0.9465	0.9461	0.9463
T_{15-45}	0.9541	0.9269	0.9258	0.9265
T_{14-46}	0.9370	0.9962	0.9957	0.9960
T_{10-51}	1.0160	1.0385	1.0379	1.0388
T_{13-49}	1.0998	0.9052	0.9053	0.9048
T_{11-43}	1.0980	0.9240	0.9229	0.9245
T_{40-56}	0.9799	0.9875	0.9868	0.9877
T_{39-57}	1.0246	1.0098	1.0095	1.0092
T_{9-55}	1.0371	0.9373	0.9367	0.9375
Q_{c18}	0.0488	0.0401	0.0000	0.0116
Q_{c25}	0.0012	0.0590	0.0008	0.0428
Q_{c53}	0.0001	0.0166	0.0583	0.0464
Power loss (MW)	15.8473	34.9690	29.9137	16.3628
Voltage deviation	3.6588	1.0947	0.6634	1.0003
L_{max}	0.1625	0.0977	2.7554	0.1369

Table 6
Comparison of performance for active power loss minimization of IEEE 57 bus system.

Techniques	Best loss (MW)	Average loss (MW)	Worst loss (MW)	CPU time (s)
QODE	15.8473	15.8986	15.9249	101.29
DE	16.7857	16.8533	16.9276	93.84
GSA [24]	23.4611	–	–	321
CLPSO [19]	24.5152	–	–	423
SOA [19]	24.2654	–	–	382
OGSA [26]	23.43	–	–	309.12

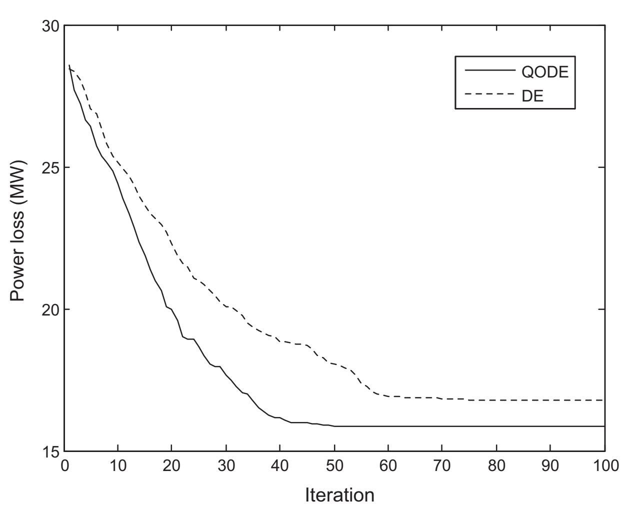

Fig. 6. Active power loss convergence characteristics for IEEE 57 bus system.

Table 7
Comparison of performance for L_{max} minimization of IEEE 57 bus system.

Techniques	Best L_{max}	Average L_{max}	Worst L_{max}	CPU time (s)
QODE	0.0977	0.0982	0.0990	114.017
DE	0.1034	0.1038	0.1045	105.954
OGSA [26]	0.1759	0.1841	0.1900	–

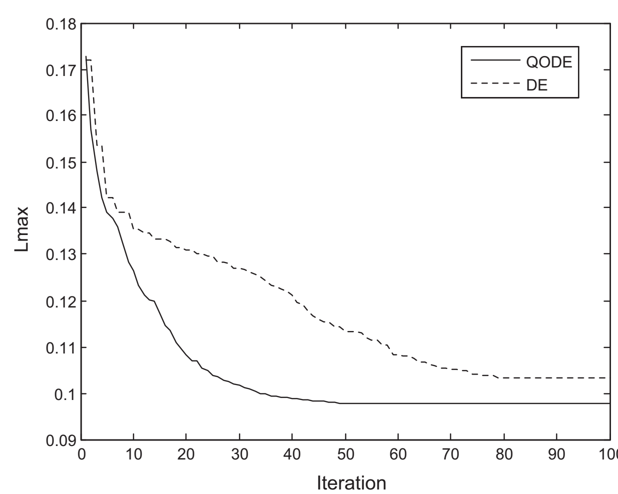

Fig. 7. L_{max} convergence characteristics for IEEE 57 bus system.

Enhancement of voltage stability

In this case, the proposed QODE and DE are applied for enhancement of voltage stability i.e. minimization of L_{max}. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0,

Table 8
Comparison of performance for voltage deviation of IEEE 57 bus system.

Technique	Best voltage deviation	Average voltage deviation	Worst voltage deviation	CPU time (s)
QODE	0.6634	0.6649	0.6655	103.75
DE	0.6792	0.6805	0.6812	94.89
OGSA [26]	0.6982	–	–	419.17

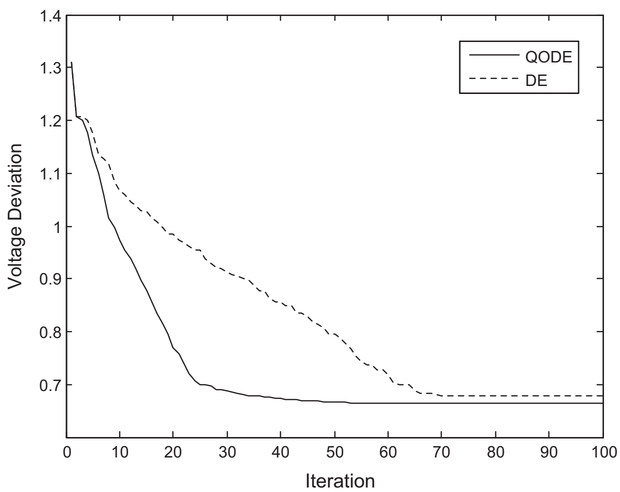

Fig. 8. Voltage deviation convergence characteristics for IEEE 57 bus system.

1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 5. The best, average and worst L_{max} and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 7. The L_{max} obtained from opposition-based gravitational search algorithm (OGSA) [26] is also shown in Table 7. The convergence characteristic obtained from proposed QODE and DE for L_{max} minimization is shown in Fig. 7. It is seen from Table 7 that the value of L_{max} obtained from QODE is the lowest among all other methods. It is also observed that QODE based approach performs best after that it follows DE, and OGSA.

Table 10
Comparison of performance for active power loss minimization of IEEE 118 bus system.

Techniques	Best loss (MW)	Average loss (MW)	Worst loss (MW)	CPU time (s)
QODE	80.9257	81.2145	81.5336	312.54
DE	82.2473	82.6514	83.0175	301.07
GSA [24]	127.76	–	–	1198.65
CLPSO [20]	130.96	–	–	1472
PSO [20]	131.99	–	–	1215
QOTLBO [25]	112.2789	113.7693	115.4516	–
OGSA [26]	126.99	–	–	1152.32

Improvement of voltage profile

In this case, the proposed QODE and DE have been applied for improvement of voltage profile. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 5. The best, average and worst voltage deviation and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 8. The voltage deviation obtained from opposition-based gravitational search algorithm (OGSA) [26] is also shown in Table 8. The convergence characteristic obtained from proposed QODE and DE for voltage deviation is shown in Fig. 8. It is seen from Table 8, that voltage deviation obtained from QODE is lower than DE and OGSA.

Minimization of active power loss with improvement of voltage profile

In this case, a twofold objective function is formed in order to minimize the active power loss and improve voltage profile by minimizing the voltage deviation of all load buses from 1.0 p.u. as given in Eq. (25). The weighting factor ψ is selected by the user. The value of ψ in this case is chosen as 0.1.

The problem is solved by using QODE and DE. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 100, 1.0, 1.0 and 100 respectively for this test system. The results obtained from proposed QODE for optimal values of control variables are shown in Table 5.

Table 9
Optimal value of control variables obtained from QODE for IEEE 118 bus system for active power loss minimization.

Variable	QODE	Variable	QODE	Variable	QODE	Variable	QODE
V_1	0.9552	V_{49}	1.0250	V_{90}	0.9800	T_{65-66}	0.9349
V_4	0.9984	V_{54}	0.9550	V_{91}	0.9835	T_{68-69}	0.9345
V_6	0.9907	V_{55}	0.9516	V_{92}	0.9724	T_{81-82}	0.9359
V_8	1.0151	V_{56}	0.9543	V_{99}	−0.2599	Q_{c5}	1.0103
V_{10}	1.0500	V_{59}	0.9850	V_{100}	0.9693	Q_{c34}	0.0218
V_{12}	0.9903	V_{61}	0.9950	V_{103}	−0.0145	Q_{c37}	0.9532
V_{15}	0.9701	V_{62}	0.9980	V_{104}	0.9370	Q_{c44}	0.0678
V_{18}	0.9730	V_{65}	1.0050	V_{105}	0.9396	Q_{c45}	0.0644
V_{19}	0.9654	V_{66}	1.0500	V_{107}	0.9520	Q_{c46}	0.0000
V_{24}	0.9920	V_{69}	1.0350	V_{110}	0.9567	Q_{c48}	0.0992
V_{25}	1.0500	V_{70}	0.9857	V_{111}	0.9800	Q_{c74}	0.0771
V_{26}	1.0154	V_{72}	0.9800	V_{112}	0.9750	Q_{c79}	0.0852
V_{27}	0.9680	V_{73}	0.9910	V_{113}	0.9930	Q_{c82}	0.1203
V_{31}	0.9671	V_{74}	0.9655	V_{116}	1.0050	Q_{c83}	0.0805
V_{32}	0.9682	V_{76}	0.9422	T_{8-5}	0.9811	Q_{c105}	0.0828
V_{34}	0.9853	V_{77}	1.0058	T_{26-25}	0.9603	Q_{c107}	0.1975
V_{36}	0.9793	V_{80}	1.0400	T_{30-17}	0.9611	Q_{c110}	0.0005
V_{40}	0.9700	V_{85}	0.9885	T_{38-37}	0.9360	Power loss (MW)	80.9257
V_{42}	0.9850	V_{87}	1.0150	T_{63-59}	0.9598	Voltage deviation	2.0904
V_{46}	1.0050	V_{89}	1.0050	T_{64-61}	0.9847	L_{max}	0.1100

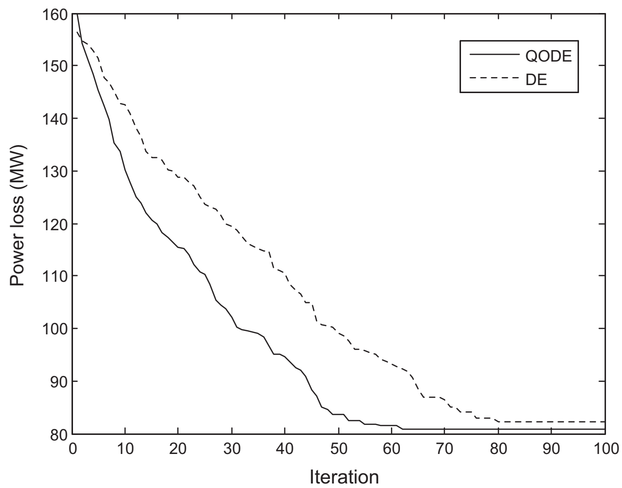

Fig. 9. Active power loss convergence characteristics for IEEE 118 bus system.

IEEE 118-bus system

The standard IEEE 118-bus system consists of 186 transmission lines, 54 generator buses, 64 load buses, 9 branches under load tap setting transformer and 14 reactive power sources. The system line data, bus data, generator data and the minimum and maximum limits for the control variables have been adapted from [20] and [43]. The upper and lower limits of reactive power sources and transformer tap settings are taken from [20]. The total system active power demand is 42.4200 p.u. and reactive power demand is 14.3800 p.u. at 100 MVA base. In this study, 50 test runs are performed to solve the RPD problem for different objective functions. Different types of RPD problem for this test system are solved by using QODE and DE.

Minimization of active power loss

The proposed QODE and DE have been applied for minimization of active power loss as the objective function. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 200, 1.0, 1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 9. The best, average and worst active power loss and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 10. The active power loss obtained from gravitational search algorithm (GSA) [24], comprehensive learning particle swarm optimization (CLPSO) [20], particle swarm optimization (PSO) [20], quasi-oppositional teaching learning based optimization (QOTLBO) [25] and opposition-based gravitational search algorithm (OGSA) [26] are also shown in Table 10. The convergence characteristic obtained from proposed QODE and DE for minimum active power loss solution is shown in Fig. 9. It is seen from Table 10 that active power loss obtained from QODE is the lowest among all other methods. It is also observed that QODE based approach performs best after that it follows DE, QOTLBO, OGSA, GSA, CLPSO and PSO.

Table 12
Comparison of performance for L_{max} minimization of IEEE 118 bus system.

Techniques	Best L_{max}	Average L_{max}	Worst L_{max}	CPU time (s)
QODE	0.0500	0.0506	0.0514	312.657
DE	0.0589	0.0592	0.0597	301.789
CLPSO [20]	0.0965	–	–	–
PSO [20]	0.1388	–	–	–
QOTLBO [25]	0.0608	0.0616	0.0631	–
OGSA [26]	0.06000	–	–	1112.1

Enhancement of voltage stability

In this case, the proposed QODE and DE have been applied for enhancement of voltage stability i.e. minimization of L_{max}. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 200, 1.0, 1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 11. The best, average and worst L_{max} and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 12. The L_{max} obtained from comprehensive learning particle swarm optimization (CLPSO) [20], particle swarm optimization (PSO) [20], quasi-oppositional teaching learning based optimization (QOTLBO) [25] and opposition-based gravitational search algorithm (OGSA) [26] are also shown in Table 12. The convergence characteristic obtained from proposed QODE and DE for L_{max} minimization is shown in Fig. 10. It is seen from Table 12 that the value of L_{max} obtained from QODE

Table 11
Optimal value of control variables obtained from QODE for IEEE 118 bus system for voltage stability enhancement.

Variable	QODE	Variable	QODE	Variable	QODE	Variable	QODE
V_1	0.9548	V_{49}	1.0253	V_{90}	0.9853	T_{65-66}	0.9345
V_4	0.9975	V_{54}	0.9550	V_{91}	0.9800	T_{68-69}	0.9346
V_6	0.9901	V_{55}	0.9529	V_{92}	1.0039	T_{81-82}	0.9353
V_8	1.0153	V_{56}	0.9556	V_{99}	1.0101	Q_{c5}	−0.2931
V_{10}	1.0500	V_{59}	0.9851	V_{100}	1.0172	Q_{c34}	0.0000
V_{12}	0.9901	V_{61}	0.9953	V_{103}	1.0039	Q_{c37}	−0.1504
V_{15}	0.9703	V_{62}	0.9984	V_{104}	0.9756	Q_{c44}	0.0929
V_{18}	0.9729	V_{65}	1.0052	V_{105}	0.9702	Q_{c45}	0.0976
V_{19}	0.9652	V_{66}	1.0504	V_{107}	0.9520	Q_{c46}	0.0000
V_{24}	0.9924	V_{69}	1.0350	V_{110}	0.9730	Q_{c48}	0.1382
V_{25}	1.0497	V_{70}	0.9868	V_{111}	0.9803	Q_{c74}	0.0000
V_{26}	1.0153	V_{72}	0.9807	V_{112}	0.9751	Q_{c79}	0.0222
V_{27}	0.9685	V_{73}	0.9911	V_{113}	0.9933	Q_{c82}	0.1529
V_{31}	0.9671	V_{74}	0.9623	V_{116}	1.0051	Q_{c83}	0.1973
V_{32}	0.9695	V_{76}	0.9430	T_{8-5}	0.9805	Q_{c105}	0.0000
V_{34}	0.9838	V_{77}	1.0061	T_{26-25}	0.9602	Q_{c107}	0.0000
V_{36}	0.9800	V_{80}	1.0404	T_{30-17}	0.9607	Q_{c110}	0.0003
V_{40}	0.9705	V_{85}	0.9888	T_{38-37}	0.9351	Power loss (MW)	94.0059
V_{42}	0.9851	V_{87}	1.0153	T_{63-59}	0.9597	Voltage deviation	1.6066
V_{46}	1.0053	V_{89}	1.0055	T_{64-61}	0.9848	L_{max}	0.0500

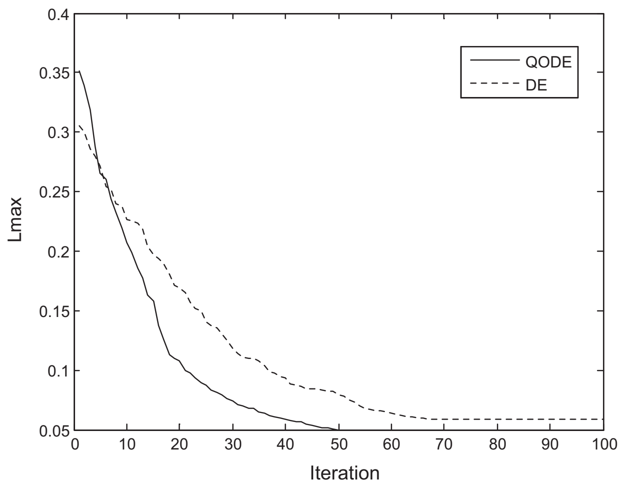

Fig. 10. L_{max} convergence characteristics for IEEE 118 bus system.

is the lowest among all other methods. It is also observed that QODE based approach performs best after that it follows DE, OGSA, QOTLBO, OGSA, CLPSO and PSO.

Improvement of voltage profile

In this case, the proposed QODE and DE have been applied for improvement of voltage profile. Here, the population size (N_P), scaling factor (S_F), crossover rate (C_R) and the maximum iteration number (N_{max}) have been selected 200, 1.0, 1.0 and 100 respectively for this test system. The optimal values of control variables obtained from the proposed QODE are given in Table 13. The best, average and worst voltage deviation and average CPU time among 50 runs of solutions obtained from proposed QODE and DE are summarized in Table 14. The voltage deviation obtained from comprehensive learning particle swarm optimization (CLPSO) [20] and particle swarm optimization (PSO) [20] is also shown in Table 14. The convergence characteristic obtained from proposed QODE and DE for voltage deviation is shown in Fig. 11. It is seen from Table 14, that the voltage deviation obtained from QODE is the lowest among all other methods. It is also observed that QODE based approach performs best after that it follows DE, CLPSO and PSO.

Table 14
Comparison of performance for voltage deviation of IEEE 118 bus system.

Techniques	Best voltage deviation	Average voltage deviation	Worst voltage deviation	CPU time (s)
QODE	1.6008	1.6012	1.6019	301.4058
DE	1.6167	1.6172	1.6178	290.0573
CLPSO [20]	1.6177	–	–	–
PSO [20]	2.2359	–	–	–

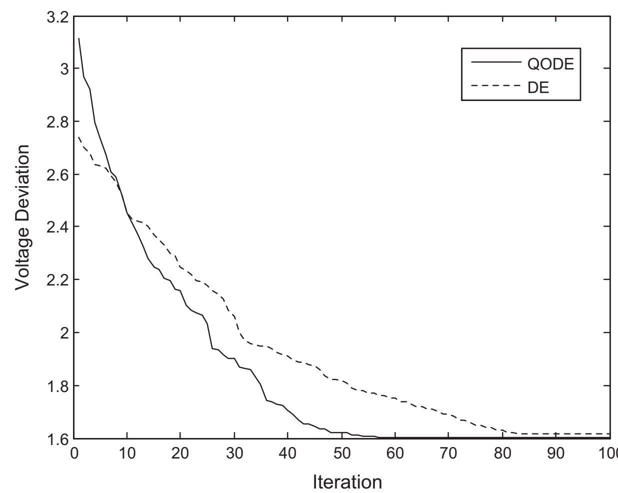

Fig. 11. Voltage deviation convergence characteristics for IEEE 118 bus system.

Conclusion

In this paper, QODE is demonstrated and successfully applied to solve RPD problem. The RPD problem is formulated as a nonlinear optimization problem with equality and inequality constraints of power system. In this study, different objective functions such as minimization of active power loss and enhancement of voltage profile and voltage stability are considered. The proposed QODE approach is tested on IEEE 30-bus, 57-bus and 118-bus test systems to demonstrate its effectiveness. Due to incorporation of QOBL technique in DE, proposed QODE approach is able to provide

Table 13
Optimal value of control variables obtained from QODE for IEEE 118 bus system for improvement of voltage profile.

Variable	QODE	Variable	QODE	Variable	QODE	Variable	QODE
V_1	0.9553	V_{49}	1.0253	V_{90}	0.9853	T_{65-66}	0.9347
V_4	0.9981	V_{54}	0.9551	V_{91}	0.9804	T_{68-69}	0.9351
V_6	0.9905	V_{55}	0.9557	V_{92}	0.9994	T_{81-82}	0.9354
V_8	1.0152	V_{56}	0.9565	V_{99}	1.0100	Q_{c5}	−0.3398
V_{10}	1.0498	V_{59}	0.9849	V_{100}	1.0171	Q_{c34}	−0.1214
V_{12}	0.9901	V_{61}	0.9953	V_{103}	1.0044	Q_{c37}	−0.1308
V_{15}	0.9706	V_{62}	0.9981	V_{104}	0.9768	Q_{c44}	0.0835
V_{18}	0.9731	V_{65}	1.0054	V_{105}	0.9737	Q_{c45}	0.0973
V_{19}	0.9655	V_{66}	1.0500	V_{107}	0.9520	Q_{c46}	0.0000
V_{24}	0.9923	V_{69}	1.0350	V_{110}	0.9733	Q_{c48}	0.0649
V_{25}	1.0500	V_{70}	0.9871	V_{111}	0.9801	Q_{c74}	0.0000
V_{26}	1.0151	V_{72}	0.9803	V_{112}	0.9754	Q_{c79}	0.0824
V_{27}	0.9683	V_{73}	0.9908	V_{113}	0.9929	Q_{c82}	0.1579
V_{31}	0.9672	V_{74}	0.9634	V_{116}	1.0051	Q_{c83}	0.0123
V_{32}	0.9711	V_{76}	0.9431	T_{8-5}	0.9848	Q_{c105}	0.0000
V_{34}	0.9846	V_{77}	1.0063	T_{26-25}	0.9601	Q_{c107}	0.0000
V_{36}	0.9820	V_{80}	1.0401	T_{30-17}	0.9603	Q_{c110}	0.0316
V_{40}	0.9703	V_{85}	0.9891	T_{38-37}	0.9351	Power loss (MW)	83.9356
V_{42}	0.9851	V_{87}	1.0153	T_{63-59}	0.9605	Voltage deviation	1.6008
V_{46}	1.0054	V_{89}	1.0051	T_{64-61}	0.9849	L_{max}	0.0574

better quality solution and less iteration cycles than other evolutionary methods reported in the literature.

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