Knowledge-Based Systems 000 (2017) 1–9

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Fuzzy best-worst multi-criteria decision-making method and its applications

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ARTICLE INFO

Article history:
Received 16 August 2016
Revised 4 January 2017
Accepted 5 January 2017
Available online xxx

Keywords:
Fuzzy best-worst method
Fuzzy sets
Fuzzy reference comparison
Consistency ratio
Multi-criteria decision-making

ABSTRACT

Considering the vagueness frequently representing in decision data due to the lack of complete information and the ambiguity arising from the qualitative judgment of decision-makers, the crisp values of criteria may be inadequate to model the real-life multi-criteria decision-making (MCDM) issues. In this paper, the latest MCDM method, namely best-worst method (BWM) was extended to the fuzzy environment. The reference comparisons for the best criterion and for the worst criterion were described by linguistic terms of decision-makers, which can be expressed in triangular fuzzy numbers. Then, the graded mean integration representation (GMIR) method was employed to calculate the weights of criteria and alternatives with respect to different criteria under fuzzy environment. According to the concept of BWM, the nonlinearly constrained optimization problem was built for determining the fuzzy weights of criteria and alternatives with respect to different criteria. The fuzzy ranking scores of alternatives can be derived from the fuzzy weights of alternatives with respect to different criteria multiplied by fuzzy weights of the corresponding criteria, and then the crisp ranking score of alternatives can be calculated by employing GMIR method for optimal alternative selection. Meanwhile, the consistency ratio was proposed for fuzzy BWM to check the reliability of fuzzy preference comparisons. Three case studies were performed to illustrate the effectiveness and feasibility of the proposed fuzzy BWM. The results indicate the proposed fuzzy BWM can not only obtain reasonable preference ranking for alternatives but also has higher comparison consistency than the BWM.

1. Introduction

Decision-making refers to the selection of optimal or satisfactory alternative from a set of alternatives [1,2]. When multiple criteria are considered, the decision-making can be called as Multi-criteria decision-making (MCDM) [3,4]. The essence of MCDM is the ranking of all the alternatives and then the selection of optimal one by employing certain approach and existing decision information with consideration of different criteria. MCDM is an important part of modern decision science, systems engineering, and management science, which has obtained a wide range of applications in many fields, such as engineering, economics, and management [5–8]. Based on the solution space of studied issue, MCDM can be divided into two classes, namely multi-attribute decision-making (MADM) and multi-objective decision-making (MODM). For MADM, the decision variables are discrete, and the number of alternatives is limited, which can also be called as discrete MCDM. For MODM, it contains continuous decision variables and an unlimited number of alternatives, which can also be called as continuous MCDM. The MADM firstly evaluates the multiple alternatives and lists the superior and inferior alternatives in order, and then selects the optimal one. However, the MODM employs the vector-based optimization technique, which is a kind of mathematical programming method.

In the existing studies, MADM (the discrete MCDM) is commonly labeled as MCDM [3,9]. Therefore, we will also use MCDM to represent MADM in this paper. Since the Operations Research (OR) was proposed by three OR pioneers Churchman, Ackoff, and Arnoff in 1957, the MCDM has made great achievement in the aspects of theory and method [10]. In the past years, several MCDM methods have been proposed by researchers, such as TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [11,12], VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) [13,14], GDM (grey decision-making) [15], ELECTRE (Elimination and Choice Expressing REality) [16], MEEM (matter-element extension model) [17], SWARA (step-wise weight assessment ratio analysis) [18], AHP (Analytic Hierarchy Process) [19,20], and ANP (Analytic Network Process) [21,22]. When dealing with practical issues, MCDM consists of two parts: one is the acquisition of decision information including criteria weights and criteria values;

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http://dx.doi.org/10.1016/j.knosys.2017.01.010
0950-7051/

the other is the aggregation of information by a certain approach and then ranks the alternatives. However, due to the complexity as well as uncertainty of objective things and the fuzziness of human thinking, the employment of fuzzy information to reflect the decision information may be a better way in many practical MCDM issues [23–25]. Meanwhile, many fuzzy-based MCDM methods have been proposed and widely used in recent years, such as fuzzy TOPSIS [26–29] and fuzzy ELECTRE [30–32], which have been employed in many practical issues, such as emergency management evaluation [33], new product development evaluation [34], power distribution system planning evaluation [35], and situation assessments [36].

As the latest MCDM method, the best-worst multi-criteria decision-making method (BWM) was proposed by Rezaei in 2015, which can obtain the weights of criteria and alternatives with respect to different criteria based on pairwise comparisons with the need of less compared data [9]. Meanwhile, the BWM can effectively remedy the inconsistency derived from pairwise comparisons. Different from AHP, the BWM employs a 1–9 scale to perform the pairwise comparisons. Moreover, quite different from AHP, BWM only executes reference comparisons, which means it only needs to determine the preference of the best criterion over all the other criteria and the preference of all the criteria over the worst criterion by using a number between 1 and 9. This procedure is much easier, more accurate and at less redundant because it does not execute secondary comparisons [9]. However, the human qualitative judgments (such as the 1–9 scale-based pairwise comparisons by decision-makers in BWM) usually hold the characteristics of ambiguity and intangibility, and the information of criteria in real world have the drawbacks of vague and uncertain [24,37–38]. Therefore, the reference comparisons of BWM can be executed by employing fuzzy number other than crisp value in some practical issues, which may be more in line with the actual situations and can obtain more convincing ranking results. In this paper, BWM was extended to the fuzzy environment, and a fuzzy-based BWM was proposed which the reference comparisons were executed by using the fuzzy comparing judgments.

The rest of this paper is organized as follows: Section 2 introduces the basic concept of fuzzy BWM; the fuzzy BWM is applied in three cases, and the results are compared with the BWM in Section 3; Section 4 gives the conclusions and future research.

2. Fuzzy best-worst MCDM method

2.1. Triangular fuzzy numbers

In 1965, Prof. L. A. Zadeh proposed the fuzzy set theory [23]. As an extension of classical set theory, the fuzzy set theory can solve the practical problems under uncertainty environment. A fuzzy set $\tilde{a}$ is a pair (U, m), where U is a set and m: $U \to [0, 1]$ is the membership function, donated by $\mu_{\tilde{a}}(x)$. By referring to $\mu_{\tilde{a}}(x)$, each element x in a universe of discourse X can be mapped to a real number in the interval [0, 1].

Definition 1. Let $\tilde{a} \in F(R)$ be a fuzzy number if:

(1) there exists $x_0 \in R$ such that $\mu_{\tilde{a}}(x_0) = 1$;
(2) for any $\alpha \in [0, 1]$, $\tilde{a}_\alpha = [x, \mu_{\tilde{a}_\alpha}(x) \geq \alpha]$ is a closed interval.

Here, R is the set of real numbers, and $F(R)$ represents the fuzzy set.

Definition 2. A fuzzy number $\tilde{a}$ on R is defined as a triangular fuzzy number (TFN) if its membership function $\mu_{\tilde{a}}(x) : R \to [0, 1]$ is equal to

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < l \\ \dfrac{x-l}{m-l}, & l \leq x < m \\ \dfrac{u-x}{u-m}, & m \leq x \leq u \\ 0, & x > u \end{cases} \tag{1}$$

where l, m, and u respectively represent the lower, modal, and upper value of the support of $\tilde{a}$, all of which are crisp numbers$(-\infty < l \leq m \leq u < \infty)$. A TFN can be represented as a triplet (l, m, u).

Table 1
Transformation rules of linguistic variables of decision-makers.

Linguistic terms	Membership function
Equally importance (EI)	(1,1,1)
Weakly important(WI)	(2/3,1,3/2)
Fairly Important (FI)	(3/2,2,5/2)
Very important(VI)	(5/2,3,7/2)
Absolutely important(AI)	(7/2,4,9/2)

For the basic operational laws of two TFNs, the readers can refer to [39].

Definition 3. Let the graded mean integration representation (GMIR) $R(\tilde{a})$ of a TFN $\tilde{a}$ represent the ranking of triangular fuzzy number [38,40,41].

Let $\tilde{a}_i = (l_i, m_i, u_i)$, and the GMIR $R(\tilde{a}_i)$ of TFN $\tilde{a}_i$ can be calculated by

$$R(\tilde{a}_i) = \frac{l_i + 4m_i + u_i}{6} \tag{2}$$

2.2. Fuzzy best-worst method

Suppose there are n criteria for a research object, and the fuzzy pairwise comparisons on these n criteria can be performed based on the linguistic variables (terms) of decision-makers, such as 'Equally importance (EI)', 'Weakly important (WI)', 'Fairly Important (FI)', 'Very important (VI)', and 'Absolutely important (AI)'. Then, the linguistic evaluations of decision-makers need to be transformed to fuzzy ratings (represented by TFNs), and the rules of transformation are listed in Table 1 [42,43].

Then, the fuzzy comparison matrix can be obtained as follows,

$$\tilde{A} = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix} \end{array} \tag{3}$$

where $\tilde{a}_{ij}$ represents the relative fuzzy preference of criterion i to criterion j, which is a triangular fuzzy number; $\tilde{a}_{ij} = (1, 1, 1)$ when $i=j$.

From the basic principle of BWM (the readers can refer to [9]), we can learn that it is not necessary to execute n fuzzy pairwise comparisons in order to obtain a completed matrix $\tilde{A}$.

Definition 4. A pairwise comparison $\tilde{a}_{ij}$ is defined as a fuzzy reference comparison if i is the best element and/or j is the worst element.

Please cite this article as: S. Guo, H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowledge-Based Systems (2017), http://dx.doi.org/10.1016/j.knosys.2017.01.010

For $\tilde{A}$, there are totally $2n$-3 (n-2 Best-to-Others fuzzy comparisons + n-2 Others-to-Worst fuzzy comparisons +1 Best-to-Worst fuzzy comparison) fuzzy reference comparisons, which need to be executed for fuzzy BWM.

Both the fuzzy weights of criteria and the fuzzy weights of alternatives with respect to different criteria can be determined by using fuzzy BWM. For determining the fuzzy weights of criteria, the fuzzy comparisons on relative criteria should be executed. For determining the fuzzy weights of alternatives with respect to different criteria, the related alternatives should be fuzzily compared against each criterion. Finally, the fuzzy ranking scores of alternatives can be derived from the fuzzy weights of alternatives with respect to different criteria multiplied by the fuzzy weights of the corresponding criteria, and then the crisp ranking scores of alternatives (if need) can be calculated by employing GMIR method for optimal alternative selection. Therefore, the logic and procedure of fuzzy comparison for determining the weights of criteria and alternatives are similar.

In this paper, we will elaborate the detailed steps of fuzzy BWM for determining the fuzzy weights of criteria. It should be noted that this detailed steps can also be used for the determination of fuzzy weights of alternatives.

Step 1. Build the decision criteria system.

The decision criteria system consists of a set of decision criteria, which is very important for reasonably performing the evaluation on alternatives. The values of decision criteria can reflect the performances of different alternatives. Suppose there are n decision criteria $\{c_1, c_2, \cdots, c_n\}$.

Step 2. Determine the best (most important) criterion and the worst (least important) criterion.

Based on the built decision criteria system, the best criterion and the worst criterion should be identified by decision-makers in this step. The best criterion is represented as c_B, and the worst criterion is labeled as c_W.

Step 3. Execute the fuzzy reference comparisons for the best criterion.

The fuzzy reference comparison is very important for fuzzy BWM. According to Definition 4, the fuzzy reference comparison includes two parts: one part is the pairwise comparison $\tilde{a}_{ij}$ in the case that i is the best element, and here c_i is the best criterion c_B; the other is the pairwise comparison $\tilde{a}_{ij}$ in the case that j is the worst element, and here c_j is the worst criterion c_W. In this step, the first part will be performed.

By using the linguistic terms of decision-makers listed in Table 1, the fuzzy preferences of the best criterion over all the criteria can be determined. Then, the obtained fuzzy preferences are transformed to TFNs according to the transformation rules shown in Table 1. The obtained fuzzy Best-to-Others vector is:

$$\tilde{A}_B = (\tilde{a}_{B1}, \tilde{a}_{B2}, \cdots, \tilde{a}_{Bn}) \tag{4}$$

where $\tilde{A}_B$ represents the fuzzy Best-to-Others vector; $\tilde{a}_{Bj}$ represents the fuzzy preference of the best criterion c_B over criterion j, $j = 1, 2, \cdots, n$. It can be known that $\tilde{a}_{BB} = (1, 1, 1)$.

Step 4. Execute the fuzzy reference comparisons for the worst criterion.

In this step, the other part of fuzzy reference comparison will be done. By using the linguistic evaluations of decision-makers listed in Table 1, the fuzzy preferences of all the criteria over the worst criterion can be determined, and then they are transformed to TFNs according to the transformation rules listed in Table 1. The fuzzy Others-to-Worst vector can be obtained as:

$$\tilde{A}_W = (\tilde{a}_{1W}, \tilde{a}_{2W}, \cdots, \tilde{a}_{nW}) \tag{5}$$

where $\tilde{A}_W$ represents the fuzzy Others-to-Worst vector; $\tilde{a}_{iW}$ represents the fuzzy preference of criterion i over the worst criterion c_W, $i = 1, 2, \cdots, n$. It can be known that $\tilde{a}_{WW} = (1, 1, 1)$.

Step 5. Determine the optimal fuzzy weights $(\tilde{w}_1^*, \tilde{w}_2^*, \cdots, \tilde{w}_n^*)$.

The optimal fuzzy weight for each criterion is the one where, for each fuzzy pair $\tilde{w}_B/\tilde{w}_j$ and $\tilde{w}_j/\tilde{w}_W$, it should have $\tilde{w}_B/\tilde{w}_j = \tilde{a}_{Bj}$ and $\tilde{w}_j/\tilde{w}_W = \tilde{a}_{jW}$. To satisfy these conditions for all j, it should determine a solution where the maximum absolute gaps $|\frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{a}_{Bj}|$ and $|\frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{a}_{jW}|$ for all j are minimized. It should be noted that $\tilde{w}_B$, $\tilde{w}_j$ and $\tilde{w}_W$ in fuzzy BWM are triangular fuzzy numbers, which are very different from that in BWM. In some cases, we prefer to use $\tilde{w}_j = (l_j^w, m_j^w, u_j^w)$ for optimal alternative selection. For example, when we select the optimal electric vehicle charging station using fuzzy TOPSIS method, we need to use (l_j^w, m_j^w, u_j^w) to reflect the fuzzy weight of criterion j, not a crisp value. However, in some cases, we need a crisp value after obtaining fuzzy weight of criterion based on the linguistic variables of decision-makers. That is to say, the fuzzy weight of criterion represented by TFN $\tilde{w}_j = (l_j^w, m_j^w, u_j^w)$ needs to be transformed to a crisp value. In this paper, it also needs the transformed crisp value of fuzzy weight $\tilde{w}$ of criterion because we need to build the constraint conditions for solving just like that in [9]. We use the graded mean integration representation (GMIR) (see Eq. (2)) to transform the fuzzy weight of criterion to crisp weight.

Therefore, we can obtain the constrained optimization problem for determining the optimal fuzzy weights $(\tilde{w}_1^*, \tilde{w}_2^*, \cdots, \tilde{w}_n^*)$ as follows.

$$\begin{aligned} &\min \quad \max_j \left\{ \left| \frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{a}_{Bj} \right|, \left| \frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{a}_{jW} \right| \right\} \\ &s.t. \quad \begin{cases} \sum_{j=1}^{n} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0 \\ j = 1, 2, \cdots, n \end{cases} \end{aligned} \tag{6}$$

where $\tilde{w}_B = (l_B^w, m_B^w, u_B^w)$, $\tilde{w}_j = (l_j^w, m_j^w, u_j^w)$, $\tilde{w}_W = (l_W^w, m_W^w, u_W^w)$, $\tilde{a}_{Bj} = (l_{Bj}, m_{Bj}, u_{Bj})$, $\tilde{a}_{jW} = (l_{jW}, m_{jW}, u_{jW})$.

Eq. (6) can be transferred to the following nonlinearly constrained optimization problem.

$$\begin{aligned} &\min \quad \tilde{\xi} \\ &s.t. \quad \begin{cases} \left| \frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{a}_{Bj} \right| \le \tilde{\xi} \\ \left| \frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{a}_{jW} \right| \le \tilde{\xi} \\ \sum_{j=1}^{n} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0 \\ j = 1, 2, \cdots, n \end{cases} \end{aligned} \tag{7}$$

where $\tilde{\xi} = (l^{\xi}, m^{\xi}, u^{\xi})$.

Please cite this article as: S. Guo, H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowledge-Based Systems (2017), http://dx.doi.org/10.1016/j.knosys.2017.01.010

JID: KNOSYS [m5G;January 9, 2017;13:39]

Table 2
Consistency index (CI) for fuzzy BWM.

Linguistic terms	Equally importance (EI)	Weakly important (WI)	Fairly Important (FI)	Very important (VI)	Absolutely important (AI)
$\tilde{a}_{BW}$	(1, 1, 1)	(2/3, 1, 3/2)	(3/2, 2, 5/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)
CI	3.00	3.80	5.29	6.69	8.04

Considering $l^{\xi} \le m^{\xi} \le u^{\xi}$, we suppose $\tilde{\xi}^* = (k^*, k^*, k^*)$,$k^* \le l^{\xi}$, then Eq. (7) can be transferred as

$$\min \quad \tilde{\xi}^*$$
$$s.t. \quad \begin{cases} \left| \dfrac{(l_B^w, m_B^w, u_B^w)}{(l_j^w, m_j^w, u_j^w)} - (l_{Bj}, m_{Bj}, u_{Bj}) \right| \le (k^*, k^*, k^*) \\ \left| \dfrac{(l_j^w, m_j^w, u_j^w)}{(l_W^w, m_W^w, u_W^w)} - (l_{jW}, m_{jW}, u_{jW}) \right| \le (k^*, k^*, k^*) \\ \sum_{j=1}^{n} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0 \\ j = 1, 2, \cdots, n \end{cases} \tag{8}$$

By solving Eq. (8), the optimal fuzzy weights $(\tilde{w}_1^*, \tilde{w}_2^*, \cdots, \tilde{w}_n^*)$ can be obtained.

2.3. Consistency ratio for fuzzy BWM

Consistency ratio (CR) is an important indicator to check the consistency degree of pairwise comparison. In this section, the CR is proposed for the fuzzy BWM.

Definition 5. A fuzzy comparison is fully consistent when $\tilde{a}_{Bj} \times \tilde{a}_{jW} = \tilde{a}_{BW}$, where $\tilde{a}_{BW}$,$\tilde{a}_{Bj}$, and $\tilde{a}_{jW}$ are the fuzzy preference of the best criterion over the worst criterion, the fuzzy preference of the best criterion over the criterion *j*, and the fuzzy preference of the criterion *j* over the worst criterion, respectively.

In practice, there may exist inconsistent for criterion *j* related to pairwise comparison. The consistency ratio is employed to check how consistent a fuzzy pairwise comparison is. The CR for fuzzy BWM can be calculated as follows.

According to Table 1, the maximum possible fuzzy value of $\tilde{a}_{BW}$ is (7/2, 4, 9/2), which corresponds to the linguistic terms 'Absolutely important (AI)' given by decision-maker. When $\tilde{a}_{Bj} \times \tilde{a}_{jW} \ne \tilde{a}_{BW}$, which means $\tilde{a}_{Bj} \times \tilde{a}_{jW}$ may be higher or lower than $\tilde{a}_{BW}$, the inconsistency of fuzzy pairwise comparison will occur. When both $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$ are equal to $\tilde{a}_{BW}$, the inequality will reach the greatest, which results in $\tilde{\xi}$. Considering the occurrence of the greatest inequality, according to the equality relation $(\tilde{w}_B/\tilde{w}_j) \times (\tilde{w}_j/\tilde{w}_W) = \tilde{w}_B/\tilde{w}_W$, the following equation can be obtained as

$$\left(\tilde{a}_{Bj} - \tilde{\xi}\right) \times \left(\tilde{a}_{jW} - \tilde{\xi}\right) = \left(\tilde{a}_{BW} + \tilde{\xi}\right) \tag{9}$$

As for the maximum fuzzy inconsistency $\tilde{a}_{Bj} = \tilde{a}_{jW} = \tilde{a}_{BW}$, Eq. (9) can be written as

$$\left(\tilde{a}_{BW} - \tilde{\xi}\right) \times \left(\tilde{a}_{BW} - \tilde{\xi}\right) = \left(\tilde{a}_{BW} + \tilde{\xi}\right) \tag{10}$$

Then, Eq. (10) can be derived as

$$\tilde{\xi}^2 - (1 + 2\tilde{a}_{BW})\tilde{\xi} + \left(\tilde{a}_{BW}^2 - \tilde{a}_{BW}\right) = 0 \tag{11}$$

where $\tilde{\xi} = (l^{\xi}, m^{\xi}, u^{\xi})$, $\tilde{a}_{BW} = (l_{BW}, m_{BW}, u_{BW})$.

For $\tilde{a}_{BW} = (l_{BW}, m_{BW}, u_{BW})$, the maximum possible fuzzy value is (7/2, 4, 9/2), which indicates $l_{BW} = 7/2$, $m_{BW} = 4$, and $u_{BW} = 9/2$. It shows the maximum value of l_{BW}, m_{BW}, and u_{BW} cannot exceed 9/2. In this case, if we use the upper boundary u_{BW} to calculate the consistency index, all the object (data) affiliated to TFN $\tilde{a}_{BW}$ can use this consistency index to remain the fuzzy consistency ratio effective and workable because the consistency index corresponding to u_{BW} is the largest in the interval $[l_{BW}, u_{BW}]$. Meanwhile, $\tilde{\xi}$ can also be represented by a crisp value ξ. For other cases such as $\tilde{a}_{BW} = (5/2, 3, 7/2)$, $\tilde{a}_{BW} = (3/2, 2, 5/2)$, $\tilde{a}_{BW} = (2/3, 1, 3/2)$, and $\tilde{a}_{BW} = (1, 1, 1)$, we can perform the same process. Therefore, Eq. (11) can be transferred to

$$\xi^2 - (1 + 2u_{BW})\xi + \left(u_{BW}^2 - u_{BW}\right) = 0 \tag{12}$$

where u_{BW} = 1, 3/2, 5/2, 7/2, and 9/2, respectively.

By solving Eq. (12) for different u_{BW}, the maximum possible ξ can be found, which is employed as the consistency index for fuzzy BWM. The obtained consistency index (CI) with regards to different linguistic terms of decision-makers for fuzzy BWM are listed in Table 2.

Table 3
The linguistic terms for fuzzy preferences of the best criterion over all the criteria in Case study 1.

Criteria	C1	C2	C3
Best criterion C3	AI	WI	EI

3. Case studies

In this section, three practical cases are selected for the application and verification of the proposed fuzzy BWM.

3.1. Case study 1

A company needs to select an optimal transportation mode to deliver the products to a market. Rezaei J employed BWM method to tackle this issue [9]. For comparison, we adopt the transportation mode selection example mentioned in [9] as the case study 1 in this paper. Due to the ambiguity and intangibility of decision-maker when he/she performs the evaluation, the fuzzy BWM is employed to select the optimal transportation mode.

Three criteria, namely 'load flexibility' (C1), 'accessibility' (C2), and 'cost' (C3) are selected for optimal transportation mode selection issue [9] (Step 1). The 'cost' (C3) and 'load flexibility' (C1) are respectively the best and the worst criterion based on the opinions from the company (Step 2). The fuzzy reference comparisons are performed, and the linguistic terms of decision-maker for fuzzy preferences of the best criterion over all the criteria are listed in Table 3. Then, the fuzzy Best-to-Others vector can be obtained according to Table 1 and Eq. (4) as follows (Step 3)

$$\tilde{A}_B = [(7/2, 4, 9/2), (2/3, 1, 3/2), (1, 1, 1)]$$

The fuzzy reference comparisons for the worst criterion are executed, and the linguistic evaluations of decision-makers for the fuzzy preferences of all the criteria over the worst criterion are listed in Table 4. Then, the fuzzy Others-to-Worst vector can be obtained according to Table 1 and Eq. (5) as follows (Step 4)

$$\tilde{A}_W = [(1, 1, 1), (3/2, 2, 5/2), (7/2, 4, 9/2)]$$

Based on the above analysis, for getting the optimal fuzzy weights of all the criteria, the following nonlinearly constrained

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Table 4
The linguistic terms for fuzzy preferences of all the criteria over the worst criterion in Case study 1.

Criteria	Worst criterion C1
C1	EI
C2	FI
C3	AI

optimization problem can be built according to Eq. (8) (Step 5).

$$\min \quad \tilde{\xi}^*$$
$$s.t. \begin{cases} \left|\frac{(l_3^w,m_3^w,u_3^w)}{(l_1^w,m_1^w,u_1^w)} - (l_{31},m_{31},u_{31})\right| \le (k^*,k^*,k^*) \\ \left|\frac{(l_3^w,m_3^w,u_3^w)}{(l_2^w,m_2^w,u_2^w)} - (l_{32},m_{32},u_{32})\right| \le (k^*,k^*,k^*) \\ \left|\frac{(l_3^w,m_3^w,u_3^w)}{(l_3^w,m_3^w,u_3^w)} - (l_{33},m_{33},u_{33})\right| \le (k^*,k^*,k^*) \\ \left|\frac{(l_1^w,m_1^w,u_1^w)}{(l_1^w,m_1^w,u_1^w)} - (l_{11},m_{11},u_{11})\right| \le (k^*,k^*,k^*) \\ \left|\frac{(l_2^w,m_2^w,u_2^w)}{(l_1^w,m_1^w,u_1^w)} - (l_{21},m_{21},u_{21})\right| \le (k^*,k^*,k^*) \\ \left|\frac{(l_3^w,m_3^w,u_3^w)}{(l_1^w,m_1^w,u_1^w)} - (l_{31},m_{31},u_{31})\right| \le (k^*,k^*,k^*) \\ \sum_{j=1}^{3} R(\tilde{w}_j) = 1 \\ l_j^w \le m_j^w \le u_j^w \\ l_j^w \ge 0 \\ j = 1,2,3 \end{cases} \tag{13}$$

Then, we can obtain the following nonlinearly constrained optimization problem represented by concrete numbers.

$$\min \quad k^*$$
$$s.t. \begin{cases} l_3 - 3.5*u_1 \le k*u_1;\ l_3 - 3.5*u_1 \ge -k*u_1; \\ m_3 - 4*m_1 \le k*m_1;\ m_3 - 4*m_1 \ge -k*m_1; \\ u_3 - 4.5*l_1 \le k*l_1;\ u_3 - 4.5*l_1 \ge -k*l_1; \\ l_3 - 0.67*u_2 \le k*u_2;\ l_3 - 0.67*u_2 \ge -k*u_2; \\ m_3 - 1*m_2 \le k*m_2;\ m_3 - 1*m_2 \ge -k*m_2; \\ u_3 - 1.5*l_2 \le k*l_2;\ u_3 - 1.5*l_2 \ge -k*l_2; \\ l_2 - 1.5*u_1 \le k*u_1;\ l_2 - 1.5*u_1 \ge -k*u_1; \\ m_2 - 2*m_1 \le k*m_1;\ m_2 - 2*m_1 \ge -k*m_1; \\ u_2 - 2.5*l_2 \le k*l_1;\ u_2 - 2.5*l_2 \ge -k*l_1; \\ \frac{1}{6}*l_1 + \frac{1}{6}*4*m_1 + \frac{1}{6}*u_1 + \frac{1}{6}*l_2 + \\ \frac{1}{6}*4*m_2 + \frac{1}{6}*u_2 + \frac{1}{6}*l_3 + \frac{1}{6}*4*m_3 + \frac{1}{6}*u_3 = 1 \\ l_1 \le m_1 \le u_1; \\ l_2 \le m_2 \le u_2; \\ l_3 \le m_3 \le u_3; \\ l_1 > 0;\ l_2 > 0;\ l_3 > 0 \\ k \ge 0 \end{cases} \tag{14}$$

By solving Eq. (14), the optimal fuzzy weights of three criteria ('load flexibility', 'accessibility', and 'cost') can be calculated, which are (see also Fig. 1)

$\tilde{w}_1^* = (0.1341, 0.1449, 0.1449)$; $\tilde{w}_2^* = (0.2823, 0.3550, 0.3952)$;
$\tilde{w}_3^* = (0.4423, 0.5146, 0.5431)$;
$\tilde{\xi}^* = (0.4495, 0.4495, 0.4495)$;

Then, we can obtain the crisp weights (namely the GMIRs of fuzzy weights) of three criteria 'load flexibility', 'accessibility' and 'cost', which are

$w_1^* = 0.1431$; $w_2^* = 0.3496$; $w_3^* = 0.5073$

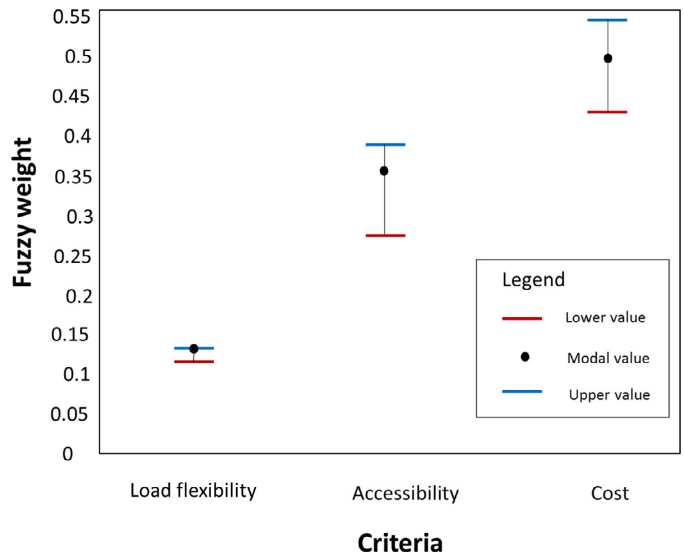

Fig. 1. Optimal fuzzy weights of three criteria in Case study 1.

Table 5
The linguistic terms for fuzzy preferences of the best criterion over all the criteria in Case study 2.

Criteria	C1	C2	C3	C4	C5
Best criterion C2	WI	EI	FI	WI	AI

The weights of three criteria 'load flexibility', 'accessibility', and 'cost' are respectively 0.07414, 0.3387, and 0.5899 by employing BWM method [9]. It can be seen that the preference orders of three criteria are the same between BWM and fuzzy BWM although there are slight gaps among each of criteria weights.

Because $a_{BW} = a_{31} = (7/2, 4, 9/2)$, the consistency index for this case is 8.04. The consistency ratio is 0.4495/8.04 = 0.0559, which indicates a very high consistency because the consistency ratio 0.0559 is very close to zero. From [9], we can learn the consistency ratio for this same case by using BWM is 0.058, which is larger than that of fuzzy BWM. Therefore, it can be concluded that the fuzzy BWM shows higher comparison consistency than the BWM because the fuzzy BWM can take the ambiguity and intangibility of decision makers into consideration.

3.2. Case study 2

How to select a high cost-performance car is an important issue faced by many people. A buyer uses five criteria, namely quality (C1), price (C2), comfort (C3), safety (C4), and style (C5) to comprehensively evaluate the car alternatives, and then makes the purchase decision [44]. The fuzzy BWM is used to select the optimal car which can consider the ambiguity and intangibility of buyer when he/she makes the purchase decision.

Five criteria, namely 'quality' (C1), 'price' (C2), 'comfort' (C3), 'safety' (C4), and 'style' (C5) are selected for optimal car selection (Step 1). 'Price' (C2) and 'style' (C5) are respectively the best criterion and the worst criterion (Step 2). The linguistic terms of buyer for fuzzy preferences of the best criterion over all the criteria are listed in Table 5. Then, the fuzzy Best-to-Others vector can be obtained as

$$\tilde{A}_B = [(2/3, 1, 3/2), (1, 1, 1), (3/2, 2, 5/2), (2/3, 1, 3/2), (7/2, 4, 9/2)]$$

The linguistic terms of buyer for the fuzzy preferences of all the criteria over the worst criterion are listed in Table 6. Then, the

Please cite this article as: S. Guo, H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowledge-Based Systems (2017), http://dx.doi.org/10.1016/j.knosys.2017.01.010

Table 6
The linguistic terms for fuzzy preferences of all the criteria over the worst criterion in Case study 2.

Criteria	Worst criterion C5
C1	FI
C2	AI
C3	FI
C4	WI
C5	EI

fuzzy Others-to-Worst vector can be obtained as follows (Step 4),

$$\tilde{A}_W = [(3/2, 2, 5/2), (7/2, 4, 9/2), (3/2, 2, 5/2), (2/3, 1, 3/2), (1, 1, 1)]$$

Then, the nonlinearly constrained optimization problem for optimal car selection can be built as follows

$$\min \quad k^*$$
$$s.t.\begin{cases} l_2 - 0.67 * u_1 \le k * u_1;\ l_2 - 0.67 * u_1 \ge -k * u_1; \\ m_2 - 1 * m_1 \le k * m_1;\ m_2 - 1 * m_1 \ge -k * m_1; \\ u_2 - 1.5 * l_1 \le k * l_1;\ u_2 - 1.5 * l_1 \ge -k * l_1; \\ l_2 - 1.5 * u_3 \le k * u_3;\ l_2 - 1.5 * u_3 \ge -k * u_3; \\ m_2 - 2 * m_3 \le k * m_3;\ m_2 - 2 * m_3 \ge -k * m_3; \\ u_2 - 2.5 * l_3 \le k * l_3;\ u_2 - 2.5 * l_3 \ge -k * l_3; \\ l_2 - 0.67 * u_4 \le k * u_4;\ l_2 - 0.67 * u_4 \ge -k * u_4; \\ m_2 - 1 * m_4 \le k * m_4;\ m_2 - 1 * m_4 \ge -k * m_4; \\ u_2 - 1.5 * l_4 \le k * l_4;\ u_2 - 1.5 * l_4 \ge -k * l_4; \\ l_2 - 3.5 * u_5 \le k * u_5;\ l_2 - 3.5 * u_5 \ge -k * u_5; \\ m_2 - 4 * m_5 \le k * m_5;\ m_2 - 4 * m_5 \ge -k * m_5; \\ u_2 - 4.5 * l_5 \le k * l_5;\ u_2 - 4.5 * l_5 \ge -k * l_5; \\ l_1 - 1.5 * u_5 \le k * u_5;\ l_1 - 1.5 * u_5 \ge -k * u_5; \\ m_1 - 2 * m_5 \le k * m_5;\ m_1 - 2 * m_5 \ge -k * m_5; \\ u_1 - 2.5 * l_5 \le k * l_5;\ u_1 - 2.5 * l_5 \ge -k * l_5; \\ l_3 - 1.5 * u_5 \le k * u_5;\ l_3 - 1.5 * u_5 \ge -k * u_5; \\ m_3 - 2 * m_5 \le k * m_5;\ m_3 - 2 * m_5 \ge -k * m_5; \\ u_3 - 2.5 * l_5 \le k * l_5;\ u_3 - 2.5 * l_5 \ge -k * l_5; \\ l_4 - 0.67 * u_5 \le k * u_5;\ l_4 - 0.67 * u_5 \ge -k * u_5; \\ m_4 - 1 * m_5 \le k * m_5;\ m_4 - 1 * m_5 \ge -k * m_5; \\ u_4 - 1.5 * l_5 \le k * l_5;\ u_4 - 1.5 * l_5 \ge -k * l_5; \\ \frac{1}{6} * l_1 + \frac{1}{6} * 4 * m_1 + \frac{1}{6} * u_1 + \frac{1}{6} * l_2 + \frac{1}{6} * 4 * m_2 + \frac{1}{6} * u_2 \\ \quad + \frac{1}{6} * l_3 + \frac{1}{6} * 4 * m_3 + \frac{1}{6} * u_3 + \frac{1}{6} * l_4 + \frac{1}{6} * 4 * m_4 \\ \quad + \frac{1}{6} * u_4 + \frac{1}{6} * l_5 + \frac{1}{6} * 4 * m_5 + \frac{1}{6} * u_5 = 1; \\ l_1 \le m_1 \le u_1;\ l_2 \le m_2 \le u_2;\ l_3 \le m_3 \le u_3;\ l_4 \le m_4 \le u_4; \\ \quad l_5 \le m_5 \le u_5; \\ l_1 > 0;\ l_2 > 0;\ l_3 > 0;\ l_4 > 0;\ l_5 > 0;\ k \ge 0 \end{cases} \tag{15}$$

By solving Eq. (15), the optimal fuzzy weights of five criteria (namely quality, price, comfort, safety, and style) can be calculated, which are (see also Fig. 2)

$$\tilde{w}_1^* = (0.2361, 0.2428, 0.2745);\ \tilde{w}_2^* = (0.2792, 0.2792, 0.3094)$$
$$\tilde{w}_3^* = (0.1801, 0.2146, 0.2745);\ \tilde{w}_4^* = (0.1506, 0.1558, 0.1911)$$
$$\tilde{w}_5^* = (0.0834, 0.0870, 0.1031)$$
$$\tilde{\xi}^* = (0.7913, 0.7913, 0.7913)$$

The crisp weights (GMIRs) for those five criteria can be calculated as

$$w_1^* = 0.2470;\ w_2^* = 0.2842;\ w_3^* = 0.2189;\ w_4^* = 0.1608;\ w_5^* = 0.0891$$

Therefore, it can be seen that price≻quality≻comfort ≻ safety ≻style, which is in accordance with the preference order obtained by employing BWM (The BWM-based criteria weight determination can be found in [44]).

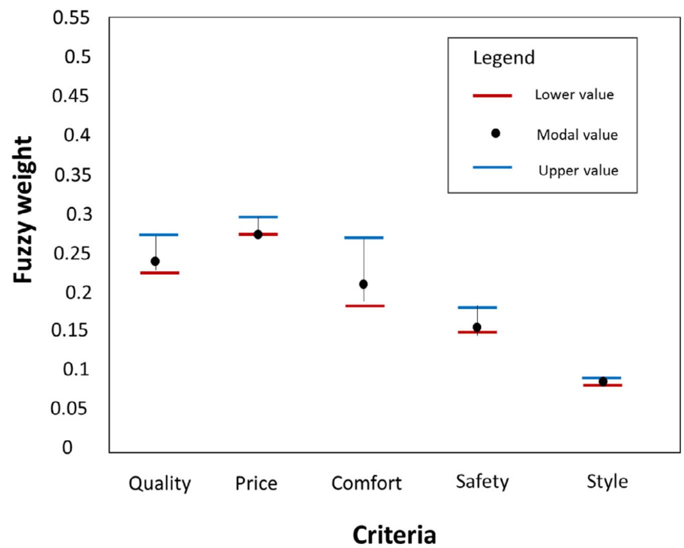

Fig. 2. Optimal fuzzy weights of five criteria in Case study 2.

Table 7
The linguistic terms for fuzzy preferences of the best criterion over all the criteria in Case study 3.

Criteria	C1	C2	C3	C4
Best criterion C1	EI	VI	WI	WI

Because $a_{BW} = a_{25} = (7/2, 4, 9/2)$, the consistency index for this case is 8.04. The consistency ratio is 0.7913/8.04 = 0.0984, which is much lower than that obtained by using BWM approach (i.e. 0.2237), Therefore, it can be concluded again that the fuzzy BWM shows higher comparison consistency than the BWM. Meanwhile, the single optimal fuzzy weights of those five criteria can be obtained using fuzzy BWM, while the multi-optimality occurs by using BWM [44].

3.3. Case study 3

Supplier development is an important part of supplier relationship management, which is a main business process of supply chain management. Besides supplier capabilities, the supplier's willingness to collaborate is also vital to the buying company, which should be considered for supplier development issue [45]. The buying company can employ four representative criteria, namely 'willingness to improve performance' (C1), 'willingness to share information' (C2), 'willingness to rely on each other' (C3), and 'willingness to become involved in a long-term relationship' (C4) to evaluate the supplier performance [41]. In [41], the weights of these four criteria were calculated by BWM. In this paper, their weights will be determined by the proposed fuzzy BWM, which takes the intangibility and ambiguity of decision-maker into consideration.

Four criteria C1, C2, C3, and C4 are used for evaluating supplier performance (Step 1). 'Willingness to improve performance' (C1) is selected as the best criterion, and 'willingness to share information' (C2) is regards as the worst criterion (Step 2). The linguistic terms of decision-maker for fuzzy preferences of the best criterion over all the criteria are listed in Table 7. Then, the fuzzy Best-to-Others vector can be obtained as

$$\tilde{A}_B = [(1, 1, 1), (5/2, 3, 7/2), (2/3, 1, 3/2), (2/3, 1, 3/2)]$$

The linguistic terms of decision-maker for the fuzzy preferences of all the criteria over the worst criterion are listed in Table 8.

Table 8
The linguistic terms for fuzzy preferences of all the criteria over the worst criterion in Case study 3.

Criteria	Worst criterion C2
C1	VI
C2	EI
C3	FI
C4	FI

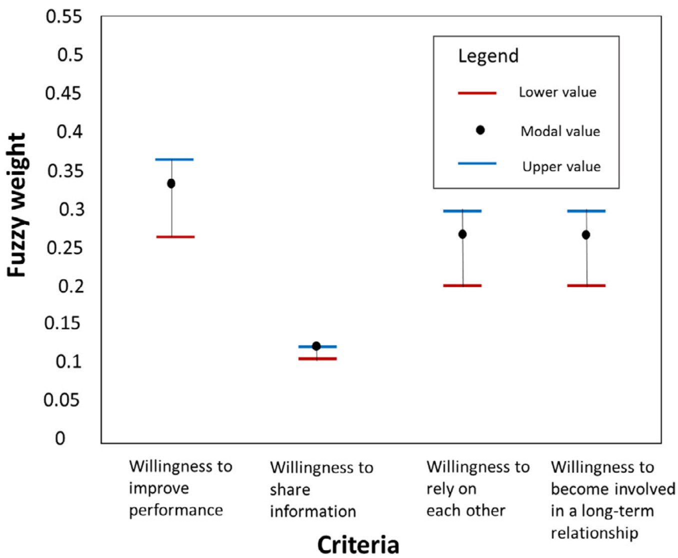

Fig. 3. Optimal fuzzy weights of five criteria in Case study 3.

Then, the fuzzy Others-to-Worst vector can be obtained as follows (Step 4),

$$\tilde{A}_W = [(5/2, 3, 7/2), (1, 1, 1), (3/2, 2, 5/2), (3/2, 2, 5/2)]$$

Then, the nonlinearly constrained optimization problem for supplier performance evaluation can be built as follows

$$\min \quad k^*$$
$$s.t. \begin{cases} l_1 - 2.5 * u_2 \le k * u_2; \, l_1 - 2.5 * u_2 \ge -k * u_2; \\ m_1 - 3 * m_2 \le k * m_2; \, m_1 - 3 * m_2 \ge -k * m_2; \\ u_1 - 3.5 * l_2 \le k * l_2; \, u_1 - 3.5 * l_2 \ge -k * l_2; \\ l_1 - 0.67 * u_3 \le k * u_3; \, l_1 - 0.67 * u_3 \ge -k * u_3; \\ m_1 - 1 * m_3 \le k * m_3; \, m_1 - 1 * m_3 \ge -k * m_3; \\ u_1 - 1.5 * l_3 \le k * l_3; \, u_1 - 1.5 * l_3 \ge -k * l_3; \\ l_1 - 0.67 * u_4 \le k * u_4; \, l_1 - 0.67 * u_4 \ge -k * u_4; \\ m_1 - 1 * m_4 \le k * m_4; \, m_1 - 1 * m_4 \ge -k * m_4; \\ u_1 - 1.5 * l_4 \le k * l_4; \, u_1 - 1.5 * l_4 \ge -k * l_4; \\ l_3 - 1.5 * u_2 \le k * u_2; \, l_3 - 1.5 * u_2 \ge -k * u_2; \\ m_3 - 2 * m_2 \le k * m_2; \, m_3 - 2 * m_2 \ge -k * m_2; \\ u_3 - 2.5 * l_2 \le k * l_2; \, u_3 - 2.5 * l_2 \ge -k * l_2; \\ l_4 - 1.5 * u_2 \le k * u_2; \, l_4 - 1.5 * u_2 \ge -k * u_2; \\ m_4 - 2 * m_2 \le k * m_2; \, m_4 - 2 * m_2 \ge -k * m_2; \\ u_4 - 2.5 * l_2 \le k * l_2; \, u_4 - 2.5 * l_2 \ge -k * l_2; \\ \frac{1}{6} * l_1 + \frac{1}{6} * 4 * m_1 + \frac{1}{6} * u_1 + \frac{1}{6} * l_2 + \frac{1}{6} * 4 * m_2 \\ \quad + \frac{1}{6} * u_2 + \frac{1}{6} * l_3 + \frac{1}{6} * 4 * m_3 + \frac{1}{6} * u_3 + \frac{1}{6} * l_4 \\ \quad + \frac{1}{6} * 4 * m_4 + \frac{1}{6} * u_4 = 1; \\ l_1 \le m_1 \le u_1; \, l_2 \le m_2 \le u_2; \, l_3 \le m_3 \le u_3; \, l_4 \le m_4 \le u_4; \\ l_1 > 0; \, l_2 > 0; \, l_3 > 0; \, l_4 > 0; \, k \ge 0 \end{cases} \tag{16}$$

By solving Eq. (16), the optimal fuzzy weights of four criteria can be calculated, which are (see also Fig. 3)

$$\tilde{w}_1^* = (0.2799, 0.3417, 0.3685); \, \tilde{w}_2^* = (0.1129, 0.1236, 0.1236); \, \tilde{w}_3^* = (0.2123, 0.2764, 0.3089); \, \tilde{w}_4^* = (0.2123, 0.2764, 0.3089); \, \tilde{\xi}^* = (0.2361, 0.2361, 0.2361)$$

The crisp weights (GMIRs) for four criteria C1, C2, C3 and C4 can be calculated as

$$w_1^* = 0.3359; \, w_2^* = 0.1218; \, w_3^* = 0.2712; \, w_4^* = 0.2712$$

Therefore, it can be seen that 'willingness to improve performance' (C1) is the most important criterion in terms of supplier's willingness for supplier performance evaluation, the next important criteria are 'willingness to rely on each other' (C3) and 'willingness to become involved in a long-term relationship' (C4), and 'willingness to share information' (C2) is ranked as the least important criterion. The weight ranking result is closely consistent with that obtained from BWM technique (The criteria weight determination detail by BWM method can be found in [45]).

Because $a_{BW} = a_{12} = (5/2, 3, 7/2)$, the consistency index for this case is 6.69. The consistency ratio is 0.2361/6.69 = 0.0353, which is largely lower than that of BWM technique (i.e. 0.382), Therefore, it can be concluded again that the fuzzy BWM shows higher comparison consistency than the BWM.

4. Conclusions and future research

The BWM proposed in 2015 is a promising vector-based MCDM method. In general, the fuzzy set theory can be employed to tackle the issues with the characteristics of vagueness and ambiguity. In this paper, the BWM is extended to the fuzzy environment, and a fuzzy BWM technique is proposed which combines the latest MCDM approach and fuzzy set theory. In the decision-making process, the reference comparisons for criteria and alternatives are more suitable to employ the linguistic variables instead of crisp values in some cases. Meanwhile, the GMIR method is used for the constrained optimization problem construction to derive the weights of criteria and alternatives, which achieves the extension of BWM under fuzzy environment. Three cases, namely optimal transportation mode selection, car purchase decision, and supplier performance evaluation are employed to verify the applicability of the proposed fuzzy BWM. The results show the fuzzy BWM outperforms the BWM because the fuzzy BWM can obtain a higher comparison consistency.

Compared with the BWM, the proposed fuzzy BWM has several advantages as follows.

(1) Due to the vagueness of decision data and the ambiguity of decision-maker, the involvement of fuzzy concept into MCDM can obtain much more reliable decision result. The fuzzy BWM technique which combines the fuzzy set theory and BWM can obtain more highly reliable weights than the BWM because it can provide more consistent comparisons (namely lower consistency ratio).
(2) Usually, the decision-maker feels very confused when he/she compares different criteria and alternatives by using too detailed scales, such as 1–9 scales used in AHP. The decision-maker cannot accurately distinguish the different between adjacent scales, such as 7 and 8 scale. Compared with the BWM using 1–9 scales, the proposed fuzzy BWM only uses five granularities of linguistic terms, namely 'Equally importance (EI)', 'Weakly important(WI)', 'Fairly Important (FI)', 'Very important(VI)', and 'Absolutely important(AI)' to perform the reference comparisons for criteria and alternatives, which can help the decision-maker make reference comparisons more accurately and easily.

(3) When the number of criteria is larger than three, the multi-optimality will likely occur by employing BWM [44], which cannot obtain the effective preference rank and then fails to make the decision. However, the proposed fuzzy BWM can escape from the multi-optimality issue and derive single optimal weights.

The proposed fuzzy BWM can also be combined with other MCDM methods, such as TOPSIS and VIKOR. Meanwhile, the fuzzy BWM can be extended to include group decision-making which can take more than one decision-maker into consideration. It should be mentioned that the decision-makers may have different priori knowledge for linguistic terms choice and possess different concepts to establish the parameters of the triangular membership functions in group decision-making. For solving this issue, the multi-granular linguistic approach is effective and promising [46,47], which is our following research work. This is also a shortcoming in this paper. Meanwhile, we can also attempt bionic intelligence algorithms, such as monarch butterfly optimization (MBO) [48], earthworm optimization algorithm (EWA) [49], elephant herding optimization (EHO) [50], and moth search (MS) algorithm [51] to solve the nonlinearly constrained optimization problem (namely Eq. (8)) in the future study.

In recent years, some flexible and useful linguistic approaches, such as 2-tuple linguistic approach [52,53], ordinal linguistic approach [54], and unbalanced linguistic approach [55,56] have been developed to deal with linguistic term sets, which have captured extensive attentions. Providing the linkage of these linguistic approaches to BWM technique and exploring the 2-tuple (and/or ordinal) linguistic modeling of fuzzy BWM linguistic (numerical) scale problems are interesting and promising topics in future research.

In the future study, we will also apply this fuzzy BWM in some other real-world problems, such as the optimal electric vehicle charging station selection and the comprehensive benefit evaluation of eco-industrial parks.

Author contributions

Sen Guo conceived, designed and performed the research; Haoran Zhao contributed structuring the format of the paper and analyzing the data; Sen Guo wrote the paper.

Acknowledgments

The authors would like to thank the Editor in Chief Prof. Jie Lu, Associate Editor Prof. Enrique Herrera-Viedma and anonymous referees, whose comments and suggestions are very helpful in improving this paper and enlightening us on fuzzy BWM in the future research. This study is supported by the National Natural Science Foundation of China [grant number 71373076].

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Please cite this article as: S. Guo, H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowledge-Based Systems (2017), http://dx.doi.org/10.1016/j.knosys.2017.01.010