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A new algorithm of image segmentation using curve fitting based higher order polynomial smoothing

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ABSTRACT

Image segmentation plays an efficient role in image analysis which discriminates the objects from its background in pixel level. In accordance with the application, image segmentation is widely spread over various fields. The motivation of this paper is to focus on the application of statistical analysis in image segmentation. In this paper, we have incorporated curve fitting technique on an image to acquire the segmented image thereby extracting information from the images. By using higher order polynomial smoothing curve, appropriate result is obtained from detection of the object. Furthermore, we have calculated the image quality metrics which is a method of statistical analysis to get the quality measures and performance analysis of images. Extensive experiments show that the proposed approach outperforms the existing approaches namely histogram based segmentation, edge detection based segmentation, Ostu's segmentation and Watershed segmentation. The outcome is derived by applying the proposed algorithm and results obtained are appreciable.

1. Introduction

Image Segmentation is the process of partitioning a digital image into multiple regions or sets of pixel [1–3] and is basically used to locate objects and their boundaries. The concept behind segmentation is to simplify the representation of an image into something that is more meaningful and easier to analyse. The segmentation algorithms can be divided into two broad categories based on the two important properties, namely, (1) Discontinuity and (2) Similarity. These two important properties are derived by gray level. Over the years segmentation is applied for various computer vision applications namely feature extraction, identification, image registration etc.

In this paper, we have introduced image segmentation technique using smoothing polynomial curve fitting technique considering the higher order polynomial distribution. By using the curve fitting process, we can construct an exact fitting curve where the data point can be constructed using mathematical function which also known as smooth function. In the context of data fitting, these curves are used for visualization of data. Here the data is nothing but considering the pixels. There are different orders of polynomial equation. Considering the higher order polynomial equation applied on an image provides more appropriate and promising results. Smoothing technique is much more prominent for denoising the outcome segmented images. Furthermore, several image quality assessment indices are applied for the performance analysis of the outcome derived by applying the proposed algorithm. There are two different approaches for image quality measurement

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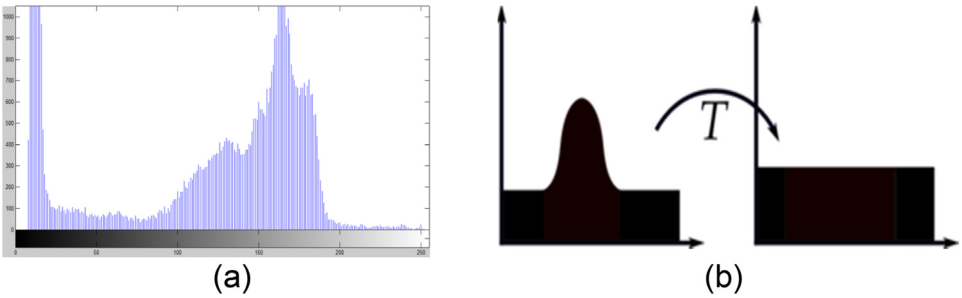

Fig. 1. (a) Image histogram (b) histogram equalization.

or metrics analysis. They are (1) Subjective approaches and (2) Objective approaches. When a human expert provides their decision about the quality of an image it is known as subjective measurements and when the measurement can be done using mathematical calculations it is known as objective measurements. Calculating the parameters using objective approaches on outcome segmented images by the proposed segmentation method, furnishes appreciable results when compared to other existing segmentation methods.

The rest of the paper is organized as follows. Section 2 presents the description related to work. In Section 3, the proposed method is presented. Experimentation and Results are explained in Section 4 while, Section 5 concludes the work.

2. Preliminaries

2.1. Image histogram and histogram equalization

An "image histogram" is a type of histogram that acts as a graphical representation of the tonal distribution in a digital image. It plots the number of pixels for each tonal value. By looking at the histogram for a specific image, a viewer will be able to judge the entire tonal distribution at a glance, as shown in Fig. 1. HE (Histogram Equalization) is a method of contrast adjustment which uses the image's histogram in image processing [4,15].

2.2. Polynomial curve fitting

The first degree polynomial equation is represented as:

$$n = mc + a \tag{1}$$

In Eq. (1), slope is m. With distinct 'c' coordinates, the 1st degree polynomial equation provides an exact fit over any two different points. If we increase the order of the above equation, the 2nd degree polynomial equation can be represented as:

$$n = mc^2 + ac + b \tag{2}$$

In Eq. (2), m,a and b are three different points to fit a curve. If we increase the order of the above equation, the 3rd degree polynomial equation can be denoted as:

$$n = mc^3 + ac^2 + bc + d \tag{3}$$

In the above equation, m, a, b and d are the four different points which exact fit. When we fit a curve, the important constraints to be taken care are point, angle and curvature. For the end condition, curvature and angle are considered as two important constraints. Other condition is known as identical end condition. For this condition an identical condition is taken for the smoothness of the curve. Three different degree equations are used for different conditions wherein Eq. (1) is to be fit for an angle and single point. In the same way Eq. (3) which represents 3rd order equation, derives the fit for angle, curvature and two different points.

2.3. Smoothing

Image smoothing is the most important step in digital image processing. Smoothing technique provides the different data set which helps to evaluate the quality of an image [5,6]. Histogram is one of the processes among many, which is used in smoothing an image.

Smoothing technique can be explained by relating it with a curve fitting technique using the following different methods:

Table 1
Image characteristics parameters.

Parameters [8,9]	Formulas
Peak Signal to Noise Ratio (PSNR)	$PSNR = 10\log\frac{(2^n-1)^2}{MSE} = 10\log\frac{255^2}{MSE}$
Signal to noise ratio (SNR)	$SNR = 10\log_{10}\left(\frac{\lambda^2}{\sigma^2}\right) = 10\log_{10}P^{-1}(N, 1-P_f)$
Pearson Correlation Coefficient (PCC)	$\rho_{x,y} = \frac{cov(x,y)}{\sigma_x\sigma_y} = \frac{E((X-\mu_x)(Y-\mu_y))}{\sigma_x\sigma_y}$
Mean Square Root (X_{rms})	$X_{rms} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\lvert x_n\rvert^2}$
Root Mean Square Error (RMSE)	$RMSE = \sqrt{\frac{1}{n}\sum_{i=0}^{n}(y_i - y'_i)^2}$
Mean Absolute Error (MAE)	$MAE = \frac{1}{n}\sum_{i=0}^{n}\lvert f_i - y_i\rvert = \frac{1}{n}\sum_{i=0}^{n}\lvert e_i\rvert$
SSIM	$SSIM(x, y) = \frac{(2\mu x\mu y+C1)(2\sigma xy+C2)}{(\mu_x^2+\mu_y^2+C_1)(\sigma_x^2+\sigma_y^2+C_2}$

MSE = mean square error; λ = mean; σ = Standard Deviation; σ_x = standard deviation of x dimension of image; σ_y = standard deviation of y dimension of image; μ_x = mean value of x dimension of image; μ_y = mean value of y dimension of image; y_i = predicted values for time i of regression dependent variabley'_i; σ_{xy} = cross-covariance for image; C_1, C_2 and C_3 are the constant values of luminance, contrast and structure respectively.

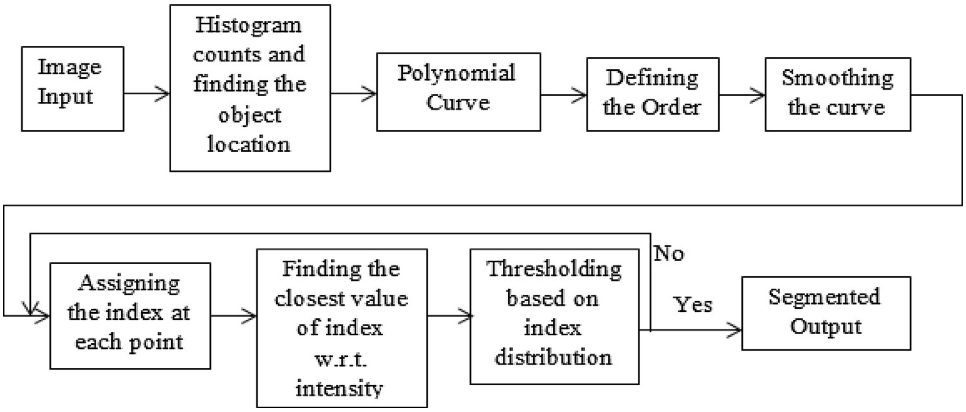

Fig. 2. Overview of the process.

(i) By using the mathematical function of curve fitting which also known as smooth function also. This function provides the details of different data in the images which can be correlated with fitting technique. These data are to be needed for the best fit.
(ii) For higher order polynomial equations, the data sets are needed to fit the curve more appropriately.

2.4. Image quality metrics

We have defined the different approaches of image quality measures as mentioned in Section 1. When we compared the distorted images with the original image (distortion-free) it is known as Objective image quality. Human Visualization system (HVS) is also a characteristic of measurement of image quality assessment [7,8]. Table 1 is listed about all the different parameters which helps to find the quality of images [8,9].

Universal image quality index (Q) [10] and the Structural Similarity Index (SSIM) [7,11] are two methods of Human visual systems (HVS) based image quality assessment.

3. Curve fitting based higher order polynomial smoothing image segmentation

3.1. Analysis of proposed method

Higher order smoothing polynomial curve fitting technique for image segmentation illustrated the better quality of images when compared to other existing image segmentation method. Fig. 2 describes the process of proposed segmentation method.

Before applying the concept of polynomial curve fitting, image is preprocessed for the input data type by defining the input image histogram's data for finding the location of object which is further processed for object detection and feature extraction, known as segmentation. The polynomial is defined as $p(x)$ of degree n that fits the data, $p(x(i))$ to $y(i)$, is a least square sense. The result p is a row vector of length $n + 1$ containing the polynomial in descending powers:

$$p(x) = p_1x^n + p_2x^{n-1} + \ldots + p_nx + p_{n+1} \tag{1}$$

where x is a substituting matrix.

The polynomial coefficient in $\hat{x}$ is illustrated by the equation given below.

$$\hat{x} = \frac{x - \mu_1}{\mu_2} \tag{2}$$

where $\mu_1 = Mean(x)$ and $\mu_2 = std(x)$.

To smoothing the fitting curve, we applied smoothing technique. A smoothing average filter smoothes data by replacing each data point with the neighboring data points defined within the span. This process is equivalent to low pass filtering with the response of the smoothing given by a difference equation given below.

$$y_8(i) = \frac{1}{2N+1}(y(i+N) + y(i+N-1) + \ldots + y(i-N)) \tag{3}$$

where $y_s(i)$ represents the smoothed value of i^{th} data point, N the number of neighbor data point on either side of $y_s(i)$, and *2N + 1* the span.

We have tested the different order of polynomial for an input image, as given below.

For quadratic polynomial: $y = p_1x^2 + p_2x + p_3$

where, coefficients are:

$p_1 = -0.020982$

$p_2 = 4.0561$

$p_3 = 226.81$

Norms of Residual = 9762.4

For cubic polynomial: $y = p_1x^3 + p_2x^2 + p_3x + p_4$

where, coefficients are:

$p_1 = 0.00047621$

$p_2 = -0.20313$

$p_3 = 22.599$

$p_4 = -163.37$

Norms of Residual = 9458.8

For 4th Degree polynomial: $y = p_1x^4 + p_2x^3 + p_3x^2 + p_4x + p_5$

where, coefficients are:

$p_1 = -8.0298e\text{-}06$

$p_2 = 0.0045714$

$p_3 = -0.87358$

$p_4 = 60.418$

$p_5 = -637.06$

Norms of Residual = 9086.7

For 4th degree polynomial, y = f(x) [15,200] for x values shows in Fig. 3. For cubic, y = f(x) [15,200], x = 15 then f(x) = 132 and if x = 200 then f(x) = 40.8,as shown in Fig. 4 and for quadratic, y = f(x) [15,200], x = 15 then f(x) = 132 and if x = 200, then f(x) = 40.8, as shown in Fig. 5.

The curve can be fitted by applying the functions and residue over an image. Curve fitting can be adjusted after considering the higher order polynomial values for the best fit. The data set of the function with respect to the residue values correlation and covariance coefficient values changes, which helps to smoothen the images better and extract the information. Close matching of the data set observes the best suited fitting curve. These series of data provide clearer view of the input image. The average of data set value will adjust the component of the images and cause overlapping of the higher order polynomial function values. The data are segmented in different index with respect to the intensity values. Thresholding technique is used for the segmentation method by applying threshold value (*Thr*) from the data set using the Eq. (4). After the segmentation of data sets, close matching is again checked for the nearest average data set values. In order to receive the better segmented output, higher order polynomial function datasets provides better segmentation of any input images. Block diagram of the proposed method is clearly shown in Fig. 2.

$$Thr = \frac{\sum_{k=1}^{m}\sum_{l=1}^{n} X(k, l)}{m \times n} \tag{4}$$

where $X(k, l)$ is defined as data sets, m and n is the height and width of data set image respectively.

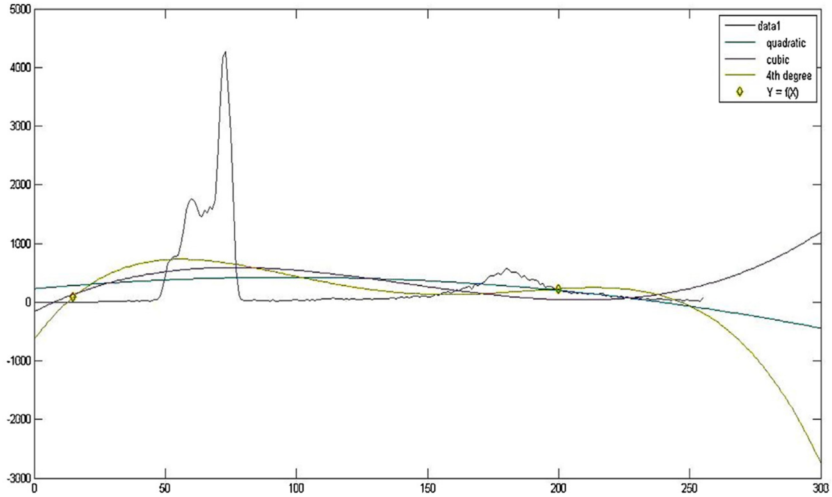

Fig. 3. 4th Degree polynomial curve fitting analysis for the function y = f(x) for [15, 200] and norms of residue is 9086.7.

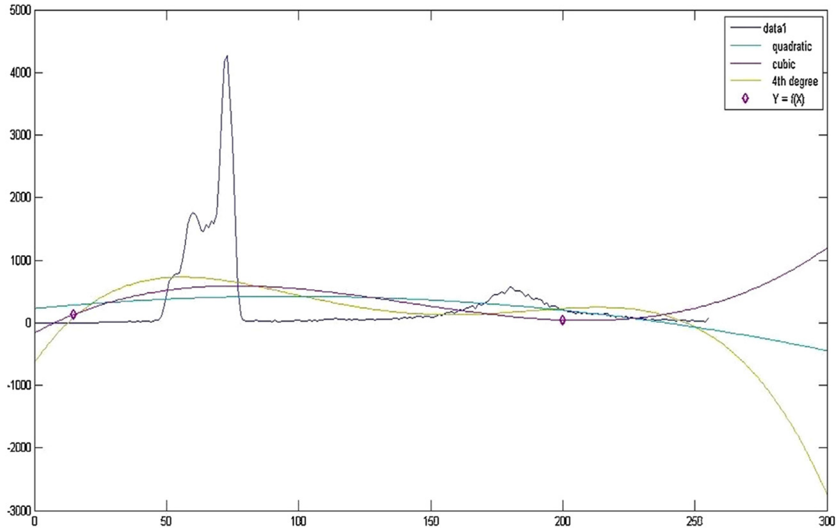

Fig. 4. Cubic polynomial curve fitting analysis for the function y = f(x). For [15, 200] and norms of residue is 9458.8.

The analysis of the figure given above with different order of polynomial concludes that higher order is more suitable to retain all the data with respect to the residue and curve fitting for that span. We have applied, all the data information according to the fitted curve, over an image for further processing and extraction of information from an image.

3.2. Correlations and covariance for data analysis

Correlation quantifies the strength of linear relationship between two variables. When there is no correlation between two variables, then there is no tendency for the values of the variables to increase or decrease in tandem. Two variables that are uncorrelated are not necessary independent, however they might have a non-linear relationship and calculating linear correlation before fitting a model, is a useful model to identify variables that have a simple relationship.

However in case of covariance, the strength of linear relationship between two variables in units is relative to their variances [12]. Correlations are standardized covariance giving a dimensionless quantity that measures the degree of linear relationship.

The plot for correlation vs covariance is derived in Figs. 6 and 7. It is noted from the above Fig. 7 that by taking the average variance of each data point of defined index for input image provides more smoothness and relationship with the index data point variables are linearly distributed. Hence, the detection of object from an image is much more promising.

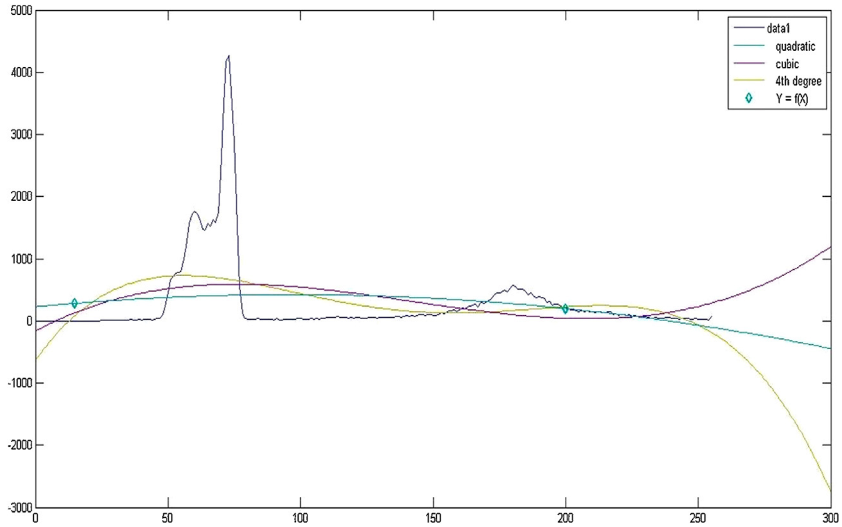

Fig. 5. Quadrant polynomial curve fitting analysis for the function y = f(x). For [15, 200] and norms of residue is 9762.4.

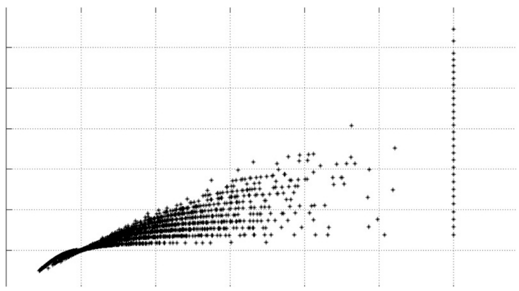

Fig. 6. Correlation-coefficient with covariance and average variance for input test image for 4th degree polynomial to fit and smooth the curve.

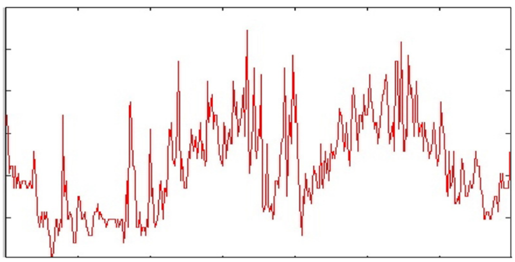

Fig. 7. Correlation-coefficient with covariance and variance for input test image for 4th degree polynomial to fit and before smooth the curve.

3.3. Analysis of test images

The process of our proposed method has already been defined in Fig. 2. The analysis of several test images using the proposed method mentioned in this paper, illustrates the segmentation output.

Figs. 8(b), 9(b) and 10(b) refer to the output segmented images using the proposed method, as described in the paper. The proposed method is also much more suitable for medical imaging as mentioned in Fig. 10(a). Output results using the proposed method of the input images are compared with other state-of-art segmentation methods i.e., Otsu's segmentation method and histogram based segmentation method, and the comparison outcome results are shown in Figs. 9 and 10.

The sagittal medical imaging also provides a fine segmented result using our proposed method, which is shown in Fig. 11.

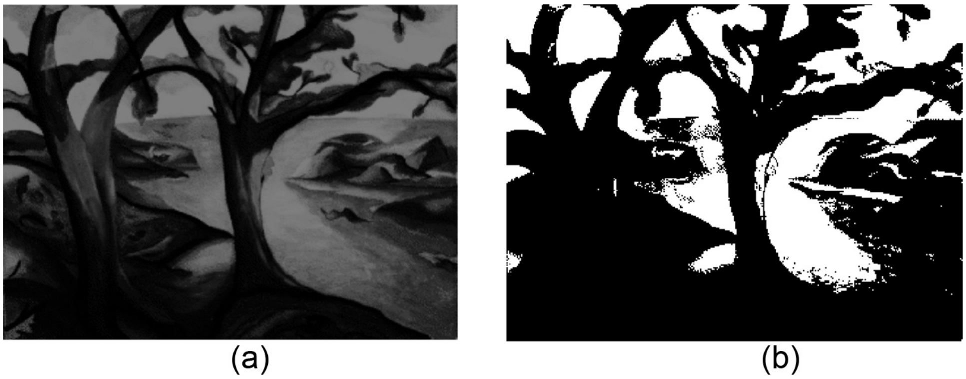

Fig. 8. (a) Original Input_test_1 (b) Image Segmentation using proposed method.

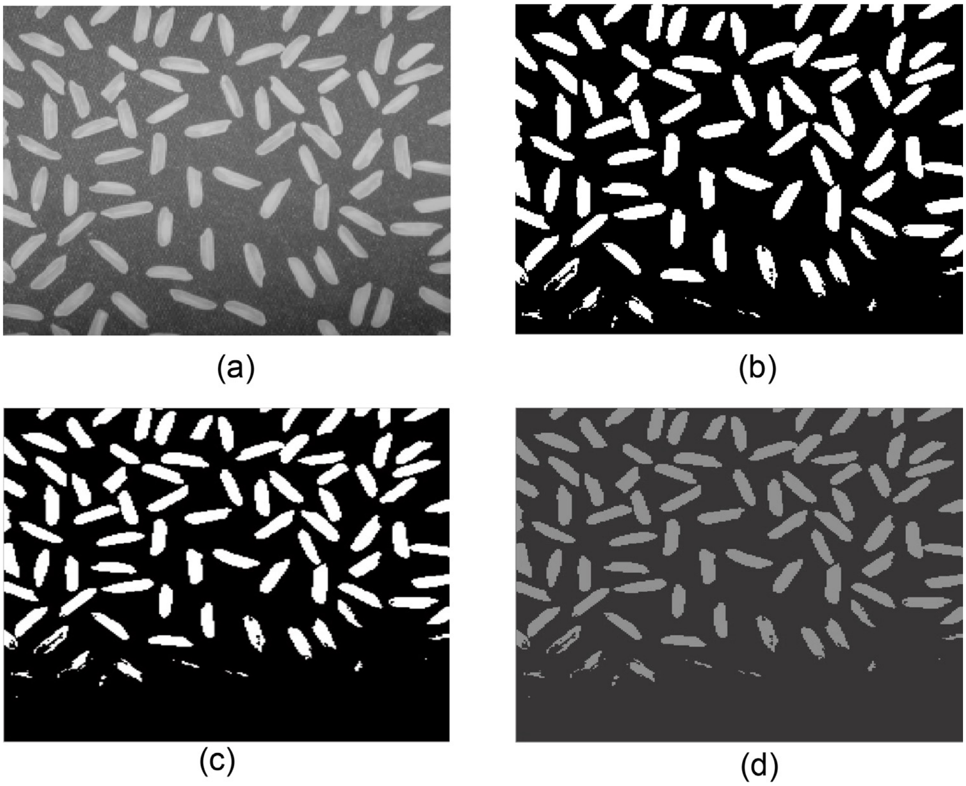

Fig. 9. (a) Original Input_test_2 (b) Image Segmentation using proposed method (c) Ostu's Segmentation output (d) Histogram based segmentation output.

4. Experimental results and discussion

Fig. 8–11 depict the observed segmented output images using our proposed segmentation method, named curve fitting based polynomial smoothing image segmentation. Furthermore, for the analysis of image quality metrics of outcome image by proposed method helps to illustrate the conclusion.

Quality assessment of an image is crucial for the system of image processing. We have already explained about the different image quality measurement technique. HVS is an important method which evaluates the image quality. To illustrate the quality measurement of proposed methods outcome results, we have compared it with existing image segmentation methods such as Histogram based segmentation, edge detection based segmentation, Ostu's segmentation and watershed segmentation methods. All the test images have been chosen for application noise. With respect to the different noise models, the parameters are calculated for image quality metrics analysis. The image quality metrics have already been defined in Table 1. We have calculated SSIM which (Structural Similarity Index Metrics) provides the information about the similarity of outcome images with the original images. Table 2 provides the calculation of outcome images by taking different test images.

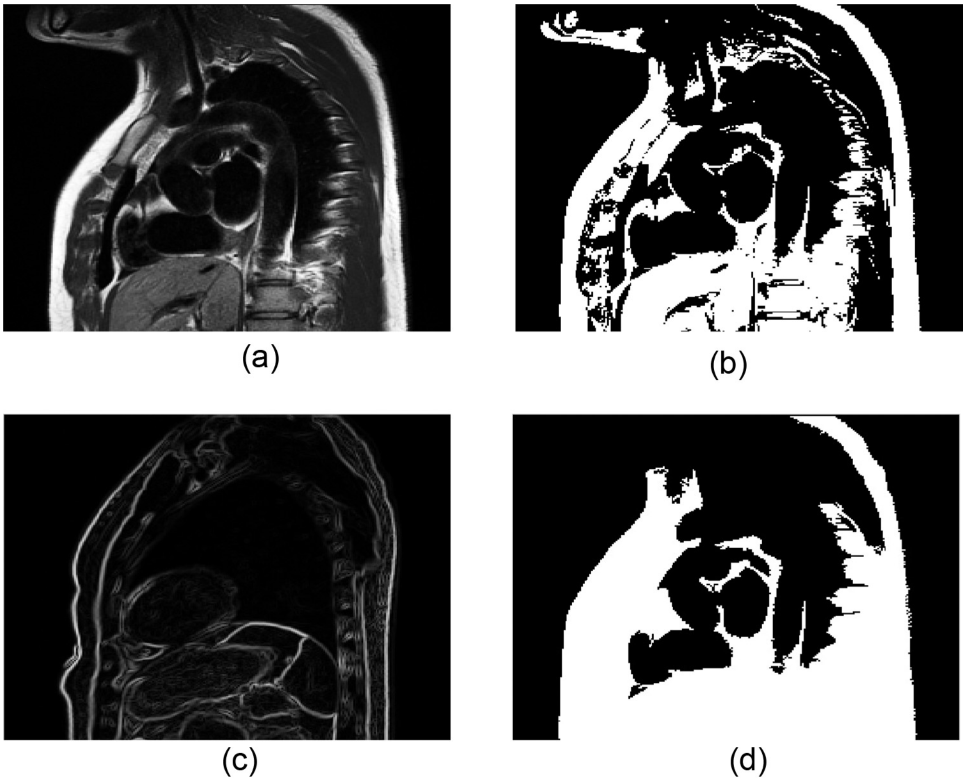

Fig. 10. (a) Original Input_test_3 (b) Image Segmentation using proposed method(c) Ostu's Segmentation output (d) Histogram based segmentation output.

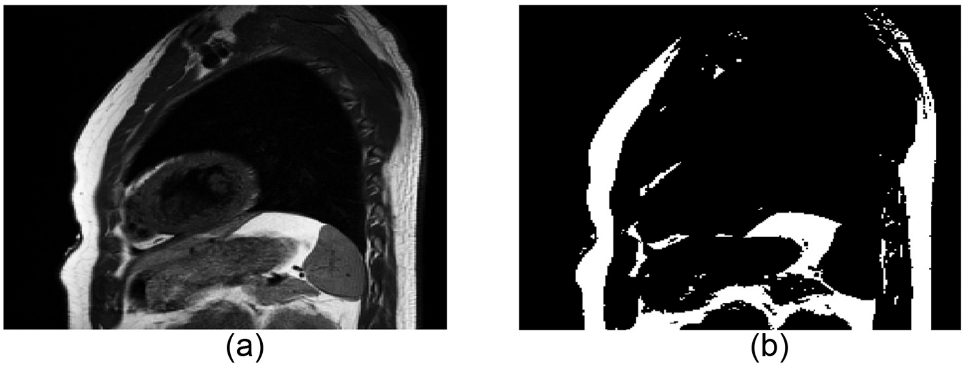

Fig. 11. (a) Original Input_test_4 (b) Image Segmentation using proposed method.

Table 2
Calculation of image quality measurements.

Test Inputs	PSNR	MSE	RMSE	PCC	SNR	MAE
Test Input_1	+15.33335 dB	1919.3084	43.80991	576.19843	−38.35746 dB	7.54360
Test Input_2	+15.29441 dB	1936.5934	44.00674	784.66470	−39.83405 dB	7.60651
peppers	+15.24952 dB	1956.7167	44.23479	1336.0898	−43.94377 dB	7.67808
coins	+15.36291 dB	1906.2891	43.66107	207.00404	−48.32485 dB	7.47725
mri	+15.58855 dB	1809.7778	42.54148	242.54148	−42.54148 dB	7.09717
cameraman	+15.11115 dB	2020.0628	44.94511	108.55623	−55.87819 dB	7.92207

Table 3 describes the comparison of Universal Quality Index with other existing segmentation methods, such as Otsu's segmentation and watershed segmentation. Universal quality index values of our proposed method are higher than other segmentation methods, which describes the quality of outcome for segmented images by proposed segmentation method.The proposed segmentation method provides fine and accurate results for better segmentation output images.

Table 3
Comparison of universal quality index.

Test Images	Histogram based Segmentation	Edge detection based Segmentation	Otsu's Segmentation	Watershed Segmentation	Proposed Method
Test Input_1	0.05265	0.07357	0.06155	0.08076	**0.13430**
Test Input_2	0.07981	0.06976	0.07557	0.06777	**0.14114**
peppers	0.09846	0.09871	0.05631	0.07585	**0.13226**
coins	0.04592	0.03672	0.05916	0.06802	**0.14532**
mri	0.02894	0.04982	0.06722	0.05771	**0.16652**
cameraman	0.07926	0.02796	0.03443	0.07916	**0.13499**

Table 4
Comparison of SSIM.

Test Images	Histogram based Segmentation [15]	Edge detection based Segmentation [15]	Otsu's Segmentation [13]	Watershed Segmentation [14]	Proposed Method
Test Input_1	0.8629	0.7261	0.5617	0.8942	0.9621
Test Input_2	0.8935	0.8490	0.8367	0.9234	0.9711
peppers	0.9128	0.9128	0.8160	0.8949	0.9134
cameraman	0.7276	0.9174	0.9137	0.8381	0.9893

Table 5
Comparison of PSNR (dB).

Test Images	Histogram based segmentation [15]	Edge Detection based Segmentation [15]	Otsu's Segmentation [13]	Watershed Segmentation [14]	Proposed Method
Test Input_1	+13.79846	+15.21225	+14.12834	+14.09275	**+15.33335**
Test Input_2	+13.98295	+14.91680	+13.98655	+13.88677	**+15.29441**
Peppers	+14.69014	+13.98319	+13.99090	+14.01231	**+15.24952**
Coins	+15.21989	+13.89721	+14.01483	+13.98190	**+15.36291**
Mri	+14.92596	+15.21850	+14.09313	+13.78341	**+15.58855**
Cameraman	+15.00903	+14.89949	+14.04588	+14.10470	**+15.11115**

Table 4 describes the Structural Similarity Index Metrics (SSIM) calculation. SSIM assessment is based on luminance, contrast and structure. SSIM attains higher values which indicate greater consistency result.

As observed from Table 5, using proposed method indicates fine quality of images with respect to noise for higher values of PSNR.

5. Conclusion

In this work, comparative analysis of various segmentation methods with proposed segmentation method has been presented. The other types of segmentation method have been compared with curve fitting based polynomial smoothing image segmentation i.e., proposed segmentation method by considering the image quality metrics. The results of proposed segmentation method and calculation of image quality measurement provides the best and accurate results with respect to analysis of calculation from parameters. The output of different segmentation methods and the proposed segmentation method have also been compared and discussed in Section 4. It can be inferred from the outcome of the results that using proposed method, detects the object clearly and gives more accuracy while calculating the quality index. Also the SSIM calculation gives higher values, indicating the fine output images. The higher values of universal index metrics conclude high accuracy in outcome of segmented images. Thus, the proposed method named curve fitting based polynomial smoothing image segmentation leads to a very promising future for assessment researches.

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