Solving Distributed Unit Commitment Problem With Walrasian Auction

Toshiyuki Miyamoto, *Member, IEEE*, Kazuyuki Mori, *Member, IEEE*, Shoichi Kitamura, and Yoshio Izui, *Member, IEEE*

***Abstract*—In the process of scheduling multiple generating units, an up/down decision for every generating unit has to be made for every hour on the planning horizon. Once the unit commitment is decided for every generating unit, a generation level (economic dispatch) from the committed unit is calculated to minimize the total operation cost. Such a problem is known as a unit commitment problem (UCP). Because an up/down pattern is expressed with a vector of binary variables, one has to solve a combinatorial optimization problem. Due to electricity liberalization or the introduction of distributed power sources in recent years, the need to solve the UCP by autonomous distributed agents, that is, solving distributed UCP (DUCP), comes about. This paper proposes a method of solving the DUCP with a Walrasian auction. The main feature of the proposed method is that not only a generation-level pattern but also an up/down pattern can be determined. The results of computational experiments show that the DUCP is solved through the cooperation of autonomous distributed agents by using the proposed method.**

***Index Terms*—Distributed optimization, energy management systems (EMS), energy saving, unit commitment problem (UCP), Walrasian auction.**

I. Introduction

In the process of scheduling multiple generating units, an up/down decision for every generating unit has to be made for every hour on the planning horizon. Once the unit commitment is decided for every generating unit, a generation level (economic dispatch) from the committed unit is calculated to minimize the total operation cost. Such a problem is known as a unit commitment problem (UCP). The UCP is formulated as a cost minimization problem under the energy balance and input–output characteristics of generating units, where the cost is the sum of the start-up cost of the generating units and energy cost. The UCP is an NP-hard problem; finding an exact solution seems difficult. Much research has been conducted on the UCP; methods with exact algorithms (e.g., branch and bound/cut) and heuristic algorithms (e.g., genetic algorithm) have been proposed [1].

Most existing research considered the UCP a centralized optimization problem. In recent years, the mechanism of energy supply has been greatly changed due to electricity liberalization or the introduction of distributed power sources. Considering the case where energy is supplied by plural business entities, the need to solve the UCP without disclosing secret information, such as device characteristics, arises. To do this, it is necessary to solve the UCP through the cooperation of autonomous distributed agents; the problem is referred to as distributed UCP (DUCP).

This paper targets a special district that is composed of independent business entities (agents); hereafter, such a district is referred to as a group. In the group, day-ahead markets for trading thermal and electrical energy are present. Each agent has both a thermal and an electrical demand and decides on an operational plan for the units by considering an energy trade in the group. We have proposed a distributed energy management system (DEMS) that aims to reduce the energy cost incurred by the entire group (group cost) through energy trading, with the cooperation of the multiagents [2]–[5]. In the DEMS, short-term planning of energy trading and generating units are determined through the cooperation of agents. We have proposed a method based on a multiattribute and multi-item auction in [2] and a method based on market-oriented programming [6] in [3]–[5] for one period problem, that is, the decision on the up/down pattern is not considered. In this paper, we propose a distributed method of solving the DUCP through a Walrasian auction. The main feature of the proposed method is that not only a generation-level pattern but also an up/down pattern is determined in a distributed manner.

This paper is organized as follows. Section II surveys related works of the DUCP. Section III briefly introduces the DUCP and the DEMS. Section IV proposes a distributed method for the DUCP. After evaluating the proposed method through computational experiments in Section V, this paper is concluded in Section VI.

II. Related Works

As for the DUCP, a number of distributed or agent-based approaches have been investigated. Nagata *et al.* [7] have studied a method of unit commitment scheduling in a decentralized power network based on a multiagent system. Although they deal with the unit commitment, the decision is made by a mobile agent and the previously fixed order of the generators. Li and Shahidehpour [8] and Mashhour and Moghaddas-Tafreshi [9] have conducted research on the price-based unit commitment (PBUC).

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T. Miyamoto is with the Graduate School of Engineering, Osaka University, Suita 565-0871, Japan (e-mail: miyamoto@eei.eng.osaka-u.ac.jp).

K. Mori, S. Kitamura, and Y. Izui are with the Advanced Technology Research and Development Center, Mitsubishi Electric Corporation, Amagasaki 661-8661, Japan.

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In the PBUC, unit commitment and economic dispatch are decided from given forecasted market prices, and satisfying hourly loads is no longer an obligation. Chung *et al.* [10] introduced the alternating direction method to the multiarea generation scheduling algorithm in interconnected electric power systems. This paper deals with unit commitment, but it is decided in a centralized manner. Ramachandran *et al.* [11] presented the application of a hybrid optimization algorithm for the distributed energy resource management of a smart grid in the energy market. Pantoja and Quijano [12] proposed a replicator dynamics strategy for dynamic resource allocation. Kumar Nunna and Doolla [13] discussed distributed energy resource management for multiple microgrids using multiagent systems. In these three studies, unit commitment is not considered. Sharma *et al.* [14] proposed a multiagent modeling for solving profit-based unit commitment and communication and negotiation stages for agents. A coordinating agent conducts the negotiation; thus, decision is not made in a fully distributed manner. In the above-mentioned studies on the DUCP, unit commitment is not considered, is decided in a centralized manner, or is decided in a distributed manner by abandoning constraints on hourly loads, whereas in this paper, we propose a distributed method of solving the DUCP, in which unit commitment and constraints on hourly loads are considered, through a Walrasian auction.

A Walrasian auction is a kind of simultaneous auction. Suppliers (respectively customers) state the amount of their supply (respectively demand) for the goods whose prices are decided by the market. Customers bid to maximize their own utility function; suppliers make offers to maximize their profits under the constraints of the production possibility set. The market updates the price to balance the demand and the supply. The price update procedure of a Walrasian auction is called tatônnement. According to the general equilibrium theory of theoretical economics, a vector of prices, in which supply and demand are balanced, exists under some conditions, such as the convexity of the utility function of customers and the production possibility set of supplies. Such a vector of prices is referred to as competitive market equilibrium. Therefore, an equilibrium point of demand and supply can be reached through a Walrasian auction. A tatônnement procedure does not always converge; however, we assume that it reaches equilibrium.

Some researchers use a Walrasian auction in their research on the UCP. Motto and Galiana [15] discussed issues and methods for attaining equilibrium in electric power auction markets with unit commitment. They use disincentive functions to make markets converge in the case where a straightforward Walrasian auction does not converge; they consider unit commitment but in a single-period UCP. Correia [16] formulated the DUCP as a probabilistic-dynamic-programming model and proposed a method where distributed agents obtain probabilities for an up/down decision through Monte Carlo simulations. Correia [16] used a Walrasian auction to decide the economic dispatch, and a centralized agent decided on a unit commitment when an unfeasible situation occurs. Gatterbauer and Ilić [17] proved that under certain circumstances, centralized unit commitment can lead to overall higher social welfare than decentralized unit commitment.

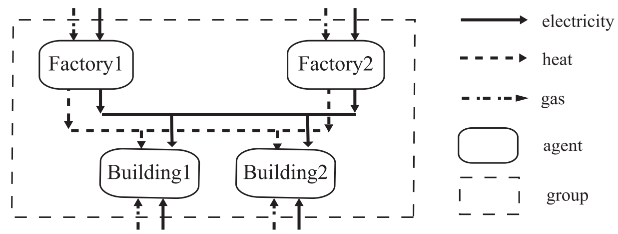

Fig. 1. Example group.

Sioshansi *et al.* [18] examined the issue of dispatch efficiency raised by the design of markets based on central versus self-commitment. Solving the UCP is not the objective of these two research studies.

III. Distributed Energy Management System

A. Overview

This paper targets a special district (group) that is composed of independent business entities (agents); the day-ahead market for trading thermal and electrical energy is present in the group. Each agent has thermal and electrical demand for 24 h on the next day, and some agents have generating units for thermal and/or electrical energy. An electrical energy shortage can be covered by purchasing electrical energy from outside of the group; however, whole thermal energy must be produced in the group. Selling thermal or electrical energy to outside of the group is not allowed. Fig. 1 depicts an example group that is composed of two factories (Factory1 and Factory2) and two buildings (Building1 and Building2). The arrows in the figure represent the flow of energy; a factory is a supplier that can sell thermal and electrical energy to customers, such as buildings. A supplier purchases electrical energy and input energy of generating units (in this paper, we use gas as the input energy), generates energy to satisfy its demand, and sells excess energy to customers in the group. A customer purchases energy from inside and outside of the group and satisfies its demand. The total cost of a group is referred to as a group cost.

The DEMS uses energy trading in the group to improve efficiency in energy use. For example, an agent with a cogeneration system, which can generate thermal and electrical energy, may have excess energy due to an imbalance of demands. By trading excess energy in the group, costs can be reduced in both the agent and the group. Moreover, assume that agents that possess efficient units produce extra energy for other agents; the group cost is also decreased in this case.

We consider an optimization problem of deciding on an operation plan of generating units to satisfy the energy demand of each agent. The objective of the problem could be minimizing the difference in profits by reducing the group costs among agents, minimizing a certain agent's profit, or minimizing the group cost. Minimizing the cost related to the entire group is the objective of all of the above-mentioned studies on the DUCP, and it is the most earth-conscious. Therefore, we adopt it as the objective of the DEMS. Moreover, it is known that an obtained solution by using a Walrasian auction satisfies the first-order necessary condition of the group cost minimization problem (see the Appendix or [19]). This means that if agents

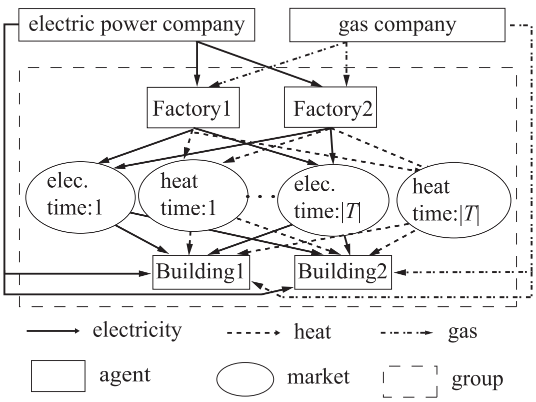

Fig. 2. Markets in the group as shown in Fig. 1.

seek their own profit and bid honestly, the entire group cost is minimized. In this paper, we assume that all agents are rational; they honestly decide and bid. A research topic on making agents honest is known as the market design [20]; it is beyond this paper's scope.

Generally, a generating unit cannot operate below the lower limit (minimum output) or above the upper limit (maximum output). When a unit begins operating, it must continuously be in operation for a certain period of time (minimum uptime). Similarly, when a unit stops operating, it must continuously be suspended for a certain period of time (minimum downtime). An operation plan of a unit is composed of up/down and generation-level patterns. An up/down pattern is represented by a vector of binary variables; a generation-level pattern is represented by a vector of continuous variables. Therefore, the DEMS must solve a 0–1 mixed integer, nonlinear program through the cooperation of agents.

B. Distributed Unit Commitment Problem

Let us formulate the UCP for the DEMS as a centralized problem first. We assume that every variable is non-negative. A set of agents is denoted by $\mathcal{P}$. A scheduling horizon is denoted by $T = \{1, 2, \ldots, |T|\}$. Agent $i \in \mathcal{P}$ has electrical demand $\mathrm{DE}_i(t)$ and thermal demand $\mathrm{DH}_i(t)$. The set of generating units of agent i is denoted by $\mathcal{U}_i$; subscript k is used to represent a unit.

An agent is either a supplier or a customer; it can trade energy through markets in the group. Fig. 2 depicts the markets in the group as shown in Fig. 1. As shown in the figure, a market is set up for each time and energy. Let $\alpha_x(t)$ be the price of energy x, $\mathrm{BE}_i^e(t)$ (respectively $\mathrm{BE}_i(t)$) be the amount of purchased electrical energy from outside (respectively inside) of the group, $\mathrm{BG}_i(t)$ be the amount of purchased gas, $\mathrm{BH}_i(t)$ be the amount of purchased thermal energy, and $\mathrm{SE}_i(t)$ (respectively $\mathrm{SH}_i(t)$) be the amount of sold electrical (respectively thermal) energy. Then the cost $\mathrm{EC}_i(t)$ with respect to purchased/sold energy for agent i at time t is expressed as follows:

$$\begin{aligned}\mathrm{EC}_i(t) = {} & \alpha_{\mathrm{BE}^e}(t)\mathrm{BE}_i^e(t) + \alpha_{\mathrm{BG}}(t)\mathrm{BG}_i(t) \\ & + \alpha_{\mathrm{BE}}(t)\mathrm{BE}_i(t) + \alpha_{\mathrm{BH}}(t)\mathrm{BH}_i(t) \\ & - \alpha_{\mathrm{SE}}(t)\mathrm{SE}_i(t) - \alpha_{\mathrm{SH}}(t)\mathrm{SH}_i(t). \qquad (1)\end{aligned}$$

Let $A_{ik}(t)$ be a binary variable that represents the up/down status of unit $u_{ik} \in \mathcal{U}_i$ (1 for up and 0 for down) and Cos_{ik} be the start-up cost of u_{ik}. Then the start-up cost $\mathrm{SC}_{ik}(t)$ of unit u_{ik} at time t is expressed as follows:

$$\mathrm{SC}_{ik}(t) = \begin{cases}\mathrm{Cos}_{ik} & \text{if } A_{ik}(t) - A_{ik}(t-1) = 1\\ 0 & \text{otherwise}\end{cases} \qquad (2)$$

where $A_{ik}(0)$ represents the initial state of unit u_{ik} and is given, A_{ik} is referred to as an up/down pattern of unit u_{ik}, and an up/down pattern of all units in agent i is denoted by $\mathcal{A}_i = (A_{i1}, \ldots, A_{i|\mathcal{U}_i|})$.

The cost $\mathrm{TC}_i(t)$ of agent i at time t is denoted as follows:

$$\mathrm{TC}_i(t) = \mathrm{EC}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{SC}_{ik}(t). \qquad (3)$$

A vector of input (respectively output) energies for unit u_{ik} of agent i at time t is denoted by $I_{ik}(t) = (\mathrm{IE}_{ik}(t), \mathrm{IH}_{ik}(t), \mathrm{IG}_{ik}(t))$ (respectively $O_{ik}(t) = (\mathrm{OE}_{ik}(t), \mathrm{OH}_{ik}(t))$), where IE_{ik} (respectively OE_{ik}) is the amount of input (respectively output) electrical energy, IH_{ik} (respectively OH_{ik}) is the amount of input (respectively output) thermal energy, and IG_{ik} is the amount of gas. The input–output characteristic of unit u_{ik} is given as function Γ_{ik}, and the following equation must hold:

$$O_{ik}(t) = \Gamma_{ik}(I_{ik}(t)). \qquad (4)$$

Each unit has an upper limit and a lower limit on outputs; $\overline{\mathrm{OE}}_{ik}$ (respectively $\underline{\mathrm{OE}}_{ik} > 0$) represents the maximum output (respectively minimum output) of electrical energy, and $\overline{\mathrm{OH}}_{ik}$ (respectively $\underline{\mathrm{OH}}_{ik} > 0$) represents the maximum output (respectively minimum output) of thermal energy.

The minimum uptime (respectively minimum downtime) of unit u_{ik} is denoted by T_{ik}^{up} (respectively T_{ik}^{down}). X_{ik} is a variable that counts up/downtime, and is updated as follows:

$$X_{ik}(t) = \begin{cases}X_{ik}(t-1) + 1 & \text{if } A_{ik}(t) = A_{ik}(t-1)\\ 1 & \text{otherwise}\end{cases} \qquad (5)$$

where $X_{ik}(0)$ is given.

The UCP is a cost minimization problem with decision variables $A_{ik}(t)$, $\mathrm{IE}_{ik}(t)$, $\mathrm{IH}_{ik}(t)$, $\mathrm{IG}_{ik}(t)$, $\mathrm{SE}_i(t)$, $\mathrm{SH}_i(t)$, $\mathrm{BE}_i(t)$, and $\mathrm{BH}_i(t)$ and is formulated as follows:

$$\min \sum_{i \in \mathcal{P}} \sum_{t \in T} \mathrm{TC}_i(t) \qquad (6)$$

$$\begin{aligned}\text{subject to } & \mathrm{BE}_i^e(t) + \mathrm{BE}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{OE}_{ik}(t) \\ & = \mathrm{DE}_i(t) + \mathrm{SE}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{IE}_{ik}(t) \\ & \qquad (t \in T, i \in \mathcal{P}) \qquad (7)\end{aligned}$$

$$\begin{aligned}& \mathrm{BH}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{OH}_{ik}(t) \\ & = \mathrm{DH}_i(t) + \mathrm{SH}_i(t) + \mathrm{WH}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{IH}_{ik}(t) \\ & \qquad (t \in T, i \in \mathcal{P}) \qquad (8)\end{aligned}$$

$$\mathrm{BG}_i(t) = \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{IG}_{ik}(t) \quad (t \in T, i \in \mathcal{P}) \qquad (9)$$

$$\underline{\mathrm{OE}}_{ik} \cdot A_{ik}(t) \le \mathrm{OE}_{ik}(t) \le \overline{\mathrm{OE}}_{ik} \cdot A_{ik}(t) \quad (t \in T, i \in \mathcal{P}, u_{ik} \in \mathcal{U}_i) \tag{10}$$

$$\underline{\mathrm{OH}}_{ik} \cdot A_{ik}(t) \le \mathrm{OH}_{ik}(t) \le \overline{\mathrm{OH}}_{ik} \cdot A_{ik}(t) \quad (t \in T, i \in \mathcal{P}, u_{ik} \in \mathcal{U}_i) \tag{11}$$

$$A_{ik}(t) - A_{ik}(t-1) = -1 \Rightarrow X_{ik}(t-1) \ge T_{ik}^{\mathrm{up}} \quad (t \in T, i \in \mathcal{P}, u_{ik} \in \mathcal{U}_i) \tag{12}$$

$$A_{ik}(t) - A_{ik}(t-1) = 1 \Rightarrow X_{ik}(t-1) \ge T_{ik}^{\mathrm{down}} \quad (t \in T, i \in \mathcal{P}, u_{ik} \in \mathcal{U}_i) \tag{13}$$

$$A_{ik}(t)(O_{ik}(t) - \Gamma_{ik}(I_{ik}(t))) = 0 \quad (t \in T, i \in \mathcal{P}, u_{ik} \in \mathcal{U}_i) \tag{14}$$

$$\sum_{i \in \mathcal{P}} \mathrm{BE}_i(t) = \sum_{i \in \mathcal{P}} \mathrm{SE}_i(t) \quad (t \in T) \tag{15}$$

$$\sum_{i \in \mathcal{P}} \mathrm{BH}_i(t) = \sum_{i \in \mathcal{P}} \mathrm{SH}_i(t) \quad (t \in T) \tag{16}$$

where $\mathrm{WH}_i(t)$ represents waste thermal energy.

Equations (7)–(9) represent the energy balance in an agent for electrical energy, thermal energy, and gas. Equations (10) and (11) represent output constraints of units; (12) and (13) represent minimum uptime and downtime constraints. Equation (14) indicates the input–output characteristics of units, and (15) and (16) represent the energy balance in markets. Note that an agent can become a supplier or a customer. Therefore, when an agent is a supplier, $\mathrm{BE}_i(t) = 0$ and $\mathrm{BH}_i(t) = 0$; when an agent is a customer, $\mathrm{SE}_i(t) = 0$ and $\mathrm{SH}_i(t) = 0$. In this paper, transmission and ramping constraints are ignored just to simplify the problem. The input–output characteristics of units are commonly represented by quadric cost functions from outputs instead of (14). Because we want to use cogeneration systems that produce thermal and electrical energy, we give the characteristics of nonlinear functions from inputs to outputs.

In the DUCP, agent i autonomously decides $A_{ik}(t)$, $\mathrm{IE}_{ik}(t)$, $\mathrm{IH}_{ik}(t)$, $\mathrm{IG}_{ik}(t)$, $\mathrm{SE}_i(t)$, $\mathrm{SH}_i(t)$, $\mathrm{BE}_i(t)$, and $\mathrm{BH}_i(t)$. In this case, two issues arise: 1) how to minimize the objective given by (6) and 2) how to balance the energy among agents constrained by (15) and (16). To minimize the objective, one may have to modify the model, depending on the tool/solver used. In the case of mixed integer problem solvers, the input–output characteristic of units and conditional statements, (5), (12)–(14), must be linearized. To linearize input–output characteristics, a piece-wise linear function is usually used. Conditional statements can be linearized by using a large number called "big-M." Please refer to a textbook, such as [21], for more detail. Because linearization of input–output characteristics badly affects the convergence of the proposed method, we use a nonlinear problem solver. In this case, one may have to divide the problem into two levels: 1) a higher-level problem for $A_{ik}(t)$ and 2) a lower-level problem for other variables. We have proposed a method to achieve energy balance by using the markets in [3]–[5]. In these studies, we have not considered the problem of finding an up/down pattern $A_{ik}(t)$. Thus, the method can solve the lower-level problem distributively. In this paper, we propose a method to solve the higher-level problem distributively.

C. Walrasian Auction Method for DUCP

To use a Walrasian auction, one needs to identify: 1) the goods traded; 2) the agents trading; and 3) the agents' bidding strategies. In the DEMS, the goods are electrical or thermal energy, and the agents are independent business entities in the group.

The procedure of Walrasian auctions is as follows.

Step 1 *Set Up Markets:* A market is set up for each item; the initial price is given.

Step 2 *Show the Price:* Markets show the price of the goods to agents.

Step 3 *Bid:* Suppliers and customers decide (and bid) on their supply and demand, based on the bidding strategy.

Step 4 *Update the Price:* Each market raises the price or lowers it down in excessive demand and supply, respectively. Let Dem be the sum of demands, Sup be the sum of supplies, and γ be a small number. Then the price updating formula is expressed as follows:

$$\alpha = \alpha + \gamma \cdot (\mathrm{Dem} - \mathrm{Sup}). \tag{17}$$

When the demand equals the supply in every market, this process terminates; otherwise, it goes back to step 2.

The price updated by (17) represents the dual value in the optimization theory of the equality constraint. The general equilibrium theory of theoretical economics assures agents that when the utility function of customers and the production possibility set of suppliers are strictly convex, all markets reach an equilibrium point of demand and supply when the dual value is used as the price in each market from any initial price using sufficiently small γ.

The bidding strategy of agent i is as follows, where $A_{ik}(t)$ is assumed to be given:

$$\min \sum_{t \in T} \mathrm{TC}_i(t) \tag{18}$$

$$\text{subject to } \mathrm{BE}_i^e(t) + \mathrm{BE}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{OE}_{ik}(t) = \mathrm{DE}_i(t) + \mathrm{SE}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{IE}_{ik}(t) \quad (t \in T) \tag{19}$$

$$\mathrm{BH}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{OH}_{ik}(t) = \mathrm{DH}_i(t) + \mathrm{SH}_i(t) + \mathrm{WH}_i(t) + \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{IH}_{ik}(t) \quad (t \in T) \tag{20}$$

$$\mathrm{BG}_i(t) = \sum_{u_{ik} \in \mathcal{U}_i} \mathrm{IG}_{ik}(t) \quad (t \in T) \tag{21}$$

$$\underline{\mathrm{OE}}_{ik} \cdot A_{ik}(t) \le \mathrm{OE}_{ik}(t) \le \overline{\mathrm{OE}}_{ik} \cdot A_{ik}(t) \quad (t \in T, u_{ik} \in \mathcal{U}_i) \tag{22}$$

$$\underline{\mathrm{OH}}_{ik} \cdot A_{ik}(t) \le \mathrm{OH}_{ik}(t) \le \overline{\mathrm{OH}}_{ik} \cdot A_{ik}(t) \quad (t \in T, u_{ik} \in \mathcal{U}_i) \tag{23}$$

$$A_{ik}(t) - A_{ik}(t-1) = -1 \Rightarrow X_{ik}(t-1) \ge T_{ik}^{\mathrm{up}} \quad (t \in T, u_{ik} \in \mathcal{U}_i) \tag{24}$$

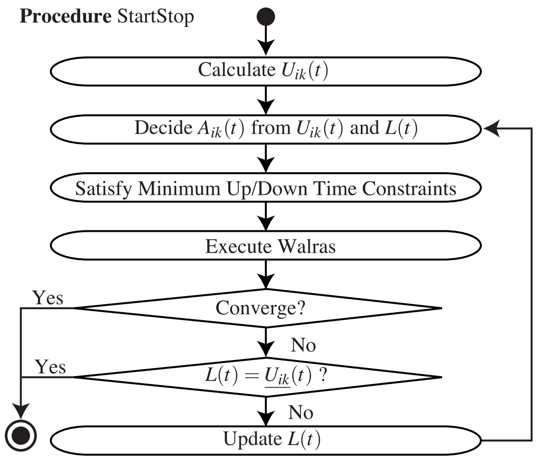

Fig. 3. Flowchart of procedure StartStop.

$$A_{ik}(t) - A_{ik}(t-1) = 1 \Rightarrow X_{ik}(t-1) \geq T_{ik}^{\text{down}} \quad (t \in T, u_{ik} \in \mathcal{U}_i) \tag{25}$$

$$A_{ik}(t)(O_{ik}(t) - \Gamma_{ik}(I_{ik}(t))) = 0 \quad (t \in T, u_{ik} \in \mathcal{U}_i). \tag{26}$$

IV. Proposed Method

A. Outline

In the DUCP, agents autonomously decide up/down and generation-level patterns of generating units. If agents selfishly decide their up/down patterns, the procedure does not converge, due to the vibration of the patterns. However, information about an agent's demand and the input–output characteristics of its units are secret from other agents; one cannot decide up/down patterns with a centralized agent. Therefore, deciding optimal up/down patterns in a distributed manner is an important issue in the DUCP.

Fig. 3 depicts the flowchart of the proposed procedure `StartStop` for the DUCP. `StartStop` has a double loop structure. The inner loop, which corresponds to the execute Walras step, is a Walrasian auction and decides the generation level. The outer loop decides the up/down patterns. We introduce a threshold $U_{ik}(t)$ for unit u_{ik} of agent i and a common variable $L(t)$ among agents to do so. Agent i operates unit u_{ik} at time t if $U_{ik}(t) \geq L(t)$. Thus, efficient operation of units is expected if we can find an appropriate value of $U_{ik}(t)$. Furthermore, agents do not need to disclose their secrets.

If $L(t)$ decreases monotonically, the number of operating units increases monotonically, thus, the vibration of the patterns never occurs. To update the value of $L(t)$, one does not need to find the maximum $U_{ik}(t)$ among suspending units; this can be achieved through the distributed election algorithm [22].

The following sections describe each step in detail.

B. Calculate $U_{ik}(t)$

In this step, a relaxed DUCP that is obtained by changing the minimum output $\underline{Y}_{ik}$, which is $\underline{\mathrm{OE}}_{ik}$ or $\underline{\mathrm{OH}}_{ik}$, of each unit to 0 is solved with a Walrasian auction. In this case, a solution can be found because it is unnecessary to decide on the up/down patterns. Let us denote the output level of unit u_{ik} in the relaxed problem by $\underline{Y}^*_{ik}(t)$. Then $U_{ik}(t)$ is calculated as follows:

$$U_{ik}(t) = \frac{\underline{Y}^*_{ik}(t)}{\underline{Y}_{ik}}. \tag{27}$$

Let $\underline{U}_{ik}(t) = \min_{i \in \mathcal{P}} \min_{u_{ik} \in \mathcal{U}_i} U_{ik}(t)$.

C. Decide $A_{ik}(t)$ From $U_{ik}(t)$ and $L(t)$

Each agent decides on an initial up/down pattern $A^0_{ik}(t)$ by $L(t)$ as follows:

$$A^0_{ik}(t) = \begin{cases} 1 & \text{if } U_{ik}(t) \geq L(t) \\ 0 & \text{otherwise.} \end{cases} \tag{28}$$

Note that $A^0_{ik}(t)$ does not always satisfy constraints (24) and (25).

D. Satisfy Minimum Uptime/Downtime Constraints

The procedure `SatMinUDTimeConst` makes an up/down pattern satisfy the minimum uptime/downtime constraints (24) and (25).

Let us find the up/down pattern that is close to A^0_{ik} and satisfies (24) and (25). The procedure `StartStop` seeks up/down patterns by waking up suspending units; therefore, the procedure `SatMinUDTimeConst` also changes the up/down pattern by waking up suspending units.

The first step makes the pattern satisfy the minimum downtime constraint. Because the procedure only wakes up suspending units, a unit must be awakened for the time periods in which the unit violates the minimum downtime constraint. A modified up/down pattern $A^1_{ik}(t)$ that satisfies the minimum downtime constraint is obtained as follows:

$$A^1_{ik}(t) = \begin{cases} 1 & \text{if } \exists \tau : \big[A^0_{ik}(\tau - 1) < A^0_{ik}(\tau) \\ & \quad \wedge\ X_{ik}(\tau - 1) < T^{\text{down}}_{ik} \\ & \quad \wedge\ t \geq \tau - X_{ik}(\tau - 1) \\ & \quad \wedge\ t \leq \tau - 1\big] \\ A^0_{ik}(t) & \text{otherwise.} \end{cases} \tag{29}$$

To satisfy the minimum uptime constraint, a unit is awakened at certain times. Fig. 4 shows the recursive function `DFS` that decides the times when the unit is awakened. An up/down pattern that satisfies the minimum uptime/downtime

procedure `DFS` (d, A)

$U_{\text{total}} = F(A)$;
if ($d = |\mathcal{V}|$) {
  if (`Sat`$(A) \wedge U_{\text{total}} < U_{\text{best}}$) {
    $U_{\text{best}} = U_{\text{total}}$; $A_{\text{best}} = A$; }
  return; }
if ($U_{\text{total}} \geq U_{\text{best}}$) **return** ;
`DFS`$(d+1, A)$;
`DFS`$(d+1, A|_{A(t_{d+1})=1})$;
return A_{best} ;

Fig. 4. Recursive function `DFS` to satisfy minimum uptime constraint.

constraints is obtained by $A = \texttt{DFS}(0, A^1_{ik})$, where $\mathcal{V} = \{t \mid A^1_{ik}(t) = 0\}$, $U_{\text{best}} = \infty$, and $A_{\text{best}} = A^1_{ik}$.

The evaluation function $F(A_{ik})$ of up/down pattern A_{ik} at line 1 in DFS is as follows:

$$F(A_{ik}) = \sum_{t \in T} \left| A_{ik}(t) - A^0_{ik}(t) \right| \cdot |L(t) - U_{ik}(t)|. \quad (30)$$

By using this evaluation function, the unit tends to be awakened at the times with larger $U_{ik}(t)$, and the number of changed time periods tends to be small.

At lines 3–5 in DFS, the incumbent is updated, where function Sat(A) returns true if and only if A satisfies the minimum uptime/downtime constraints. Line 6 bounds redundant search. Lines 7 and 8 call DFS recursively, where $A|_{A(t_{d+1})=1}$ at line 8 means that the value of A at time t_{d+1} is set to 1. Therefore, the up/down pattern with less modification is searched first.

The procedure SatMinUDTimeConst always terminates in finite time due to the finite number of combinations.

E. Execute Walras

Using the up/down pattern decided in the previous step, the DUCP is solved with a Walrasian auction. Two possibilities exist as the end status of the auction.

1) The Walrasian auction converges.
2) The bidding strategy of an agent is unsolvable.

When the Walrasian auction converges, the up/down and generation-level patterns represent the solution.

The bidding strategy becomes unsolvable when a certain number of units does not operate. If $L(t) = \underline{U_{ik}}(t)$, then the procedure terminates unsuccessfully because no more units exist to be awakened. Otherwise, $L(t)$ is updated.

F. Update $L(t)$

$L(t)$ is updated as follows:

$$L(t) = \max_i \max_{k \in \mathcal{S}_i(t)} U_{ik}(t) \quad (31)$$

where $\mathcal{S}_i(t)$ is the set of suspending units of agent i at time t. However, the unit that is awakened by StaMinUDTimeConst is not included in $\mathcal{S}_i(t)$. As noted before, $L(t)$ can be updated with the distributed election algorithm through the cooperation of agents.

Because $L(t)$ decreases monotonicity and becomes $\underline{U_{ik}}(t)$ in finite steps, the procedure StartStop always terminates if the Walrasian auction terminates.

V. Computational Evaluation

A. Experimental Conditions

The proposed method has been implemented with Java; the bidding strategies of agents are solved by using the MATLAB Optimization Toolbox.

We consider five kinds of agents: 1) F1; 2) F2; 3) B1; 4) H1; and 5) H2. Agents F1 and F2 both own a gas turbine and a gas boiler and behave as suppliers. Agents B1 and H2 own a gas boiler; agent H1 owns a gas turbine and a gas boiler. B1, H1, and H2 behave as customers. The scheduling horizon is one day, that is, $|T| = 24$, and Figs. 5 and 6 depict electrical

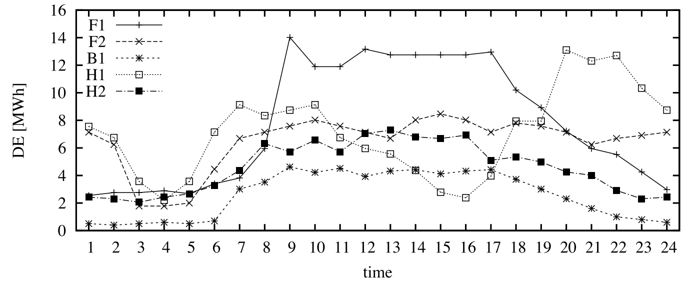

Fig. 5. Electrical demand of each agent.

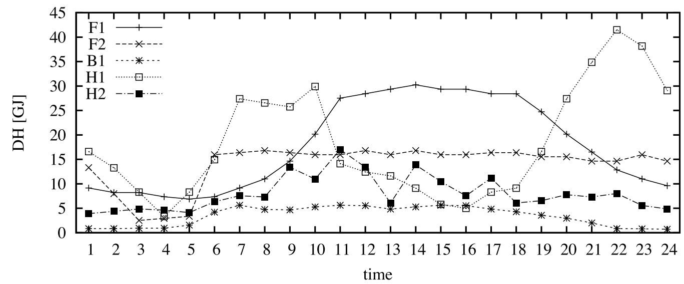

Fig. 6. Thermal demand of each agent.

TABLE I
GROUP COMPOSITIONS

agent	G1	G2	G3	G4	G5	G6	G7
F1	x	x	x	x	x	x	x
F2	x	x	x	x	x	x	x
B1	x			x	x		x
H1		x		x		x	x
H1			x		x	x	x

and thermal demands, respectively. We consider seven kinds of groups (G1–G7) composed of these agents. Table I shows the composition of the groups.

In this experiment, the energy cost $\text{EC}_i(t)$ in the cost $\text{TC}_i(t)$ is replaced by a nonlinear function $EU_i(t)$, as follows:

$$\begin{aligned} EU_i(t) &= \alpha_{\text{BE}^e}(t)\text{BE}^e_i(t) + \alpha_{\text{BG}}(t)\text{BG}_i(t) \\ &\quad + \alpha_{\text{BE}}(t) \cdot k \cdot \left(\exp\left(\frac{\text{BE}_i(t)}{k} \right) - 1 \right) \\ &\quad + \alpha_{\text{BH}}(t) \cdot k \cdot \left(\exp\left(\frac{\text{BH}_i(t)}{k} \right) - 1 \right) \\ &\quad - \alpha_{\text{SE}}(t) \cdot k \cdot \ln\left(\frac{\text{SE}_i(t)}{k} + 1 \right) \\ &\quad - \alpha_{\text{SH}}(t) \cdot k \cdot \ln\left(\frac{\text{SH}_i(t)}{k} + 1 \right) \end{aligned} \quad (32)$$

where k is a parameter. When the energy cost $\text{EC}_i(t)$ is used, an obtained solution satisfies the first-order necessary condition of the group cost minimization problem. Therefore, it may be a global minimum or a local minimal.[1] A necessary condition for the convergence of the tatônnement procedure is strict convexity of the optimization problem of agents [19]. Equation (32) is a solution to make the problem strictly convex; however, an equilibrium point reached under (32) is no more optimal, and a tatônnement procedure does not always

[1] Strictly speaking, it is minimal or maximal.

TABLE II
PARAMETERS OF EACH AGENT

		F1	F2	B1	H1	H2
p_{BA}	[GJ/10^{2}m^3]	4.5	4.5	4.5	4.5	5.0
b_{BA}		0.95	1.1	0.96	0.95	0.87
d_{BA}	[GJ]	1.0	1.0	0.5	0.4	1.0
$\overline{OH_{BA}}$	[GJ]	10.0	5.0	7.0	15.0	17.0
$\underline{OH_{BA}}$	[GJ]	1.0	0.1	0.1	1.0	1.0
Cos_{BA}	[10^{3}yen]	0.3	0.3	0.2	0.3	0.3
T^{up}_{BA}		1	1	1	1	1
T^{down}_{GT}		1	1	1	1	1
p_{GT_E}	[MWh/10^{2}m^3]	0.7	0.75	—	0.5	—
b_{GT_E}		0.9	0.85	—	0.9	—
d_{GT_E}	[MWh]	2.0	2.5	—	0.4	—
p_{GT_H}	[GJ/10^{2}m^3]	1.2	1.8	—	1.2	—
b_{GT_H}		0.87	0.85	—	0.9	—
d_{GT_H}	[GJ]	3.3	6.0	—	1.0	—
$\overline{OE_{GT}}$	[MWh]	20.0	10.0	—	15.0	—
$\underline{OE_{GT}}$	[MWh]	1.0	1.0	—	5.0	—
Cos_{GT}	[10^{3}yen]	0.8	0.8	—	0.8	—
T^{up}_{GT}		2	2	—	2	—
T^{down}_{GT}		2	2	—	2	—

reach equilibrium. Because the objective of this paper is to show a distributed method of deciding the up/down pattern of units through a Walrasian auction, the optimality of the equilibrium point is beyond the paper's scope. Our experience shows that a tatônnement procedure using (32) reaches equilibrium with a higher probability than the tatônnement procedure using $\mathrm{EC}_i(t)$.

The input–output characteristic of a gas boiler is expressed by

$$\mathrm{OH}_{\mathrm{BA}} = p_{\mathrm{BA}}(\mathrm{IG}_{\mathrm{BA}})^{b_{\mathrm{BA}}} - d_{\mathrm{BA}}$$

and that of a gas turbine is expressed by

$$\mathrm{OE}_{\mathrm{GT}} = p_{\mathrm{GT}_E}(\mathrm{IG}_{\mathrm{GT}})^{b_{\mathrm{GT}_E}} - d_{\mathrm{GT}_E}$$
$$\mathrm{OH}_{\mathrm{GT}} = p_{\mathrm{GT}_H}(\mathrm{IG}_{\mathrm{GT}})^{b_{\mathrm{GT}_H}} - d_{\mathrm{GT}_H}.$$

Table II shows the parameters of each unit. From these parameters, the efficiency of gas boilers is about 90%, and the electrical and thermal generation efficiency levels of gas turbines are about 30% and 20%, respectively.

The unit prices of the energies purchased from outside are α_{BE^e}=10.39[10^{3}yen/MWh] and α_{BG}=2.86[10^{3}yen/10^{2}m^3], respectively. The initial value of $L(t)$ is 0.8 for all time, which is decided through preliminary experiments.

Two comparison methods are used: 1) individual optimization (IOP) and 2) `StartStop` by fixed threshold (SS-FIX).

The IOP does not use energy trading within the group, and each agent decides on its plan that uses only its own units and energy outside of the group. Gas boilers in F1, F2, and H1 operate only when necessary; other units operate every time. The IOP is used to observe the effect of energy trading on the group.

The SS-FIX decides on operating units, using the given and fixed $L(t)$, and it does not update $L(t)$. The value of $L(t)$ is the same for every time t. $L(t) = 0$ represents a situation where every unit is in operation every time, and energy trading within the group is used.

TABLE III
COST RATIO TO IOP OF GROUP G1

agent	SS-FIX L = 0.0	0.2	0.4	0.6	0.8	StartStop
F1	0.959	0.957	0.957	0.957	0.957	0.957
F2	1.000	0.985	0.986	0.986	0.986	0.985
B1	0.643	0.640	0.640	0.640	0.639	0.641
total	0.909	0.903	0.902	0.902	0.902	0.903

TABLE IV
COST RATIO TO IOP OF GROUP G2

agent	SS-FIX L = 0.0	0.2	0.4	0.6	0.8	StartStop
F1	0.972	0.971	0.970	0.973	-	0.969
F2	1.002	0.988	0.988	0.983	-	0.983
H1	0.950	0.949	0.948	0.943	-	0.949
total	0.974	0.968	0.968	0.966	-	0.966

TABLE V
COST RATIO TO IOP OF GROUP G3

agent	SS-FIX L = 0.0	0.2	0.4	0.6	0.8	StartStop
F1	0.921	0.917	-	-	-	0.918
F2	0.989	0.972	-	-	-	0.972
H2	0.695	0.693	-	-	-	0.692
total	0.870	0.863	-	-	-	0.862

TABLE VI
COST RATIO TO IOP OF GROUP G4

agent	SS-FIX L = 0.0	0.2	0.4	0.6	0.8	StartStop
F1	0.921	0.923	-	-	-	0.924
F2	0.991	0.976	-	-	-	0.975
B1	0.671	0.667	-	-	-	0.665
H1	0.973	0.969	-	-	-	0.967
total	0.919	0.914	-	-	-	0.913

TABLE VII
COST RATIO TO IOP OF GROUP G5

agent	SS-FIX L = 0.0	0.2	0.4	0.6	0.8	StartStop
F1	0.848	0.843	-	-	-	0.841
F2	0.957	0.941	-	-	-	0.941
B1	0.758	0.753	-	-	-	0.749
H2	0.778	0.774	-	-	-	0.772
total	0.845	0.838	-	-	-	0.836

TABLE VIII
COST RATIO TO IOP OF GROUP G6

agent	SS-FIX L = 0.0	0.2	0.4	0.6	0.8	StartStop
F1	0.886	-	-	-	-	0.887
F2	0.978	-	-	-	-	0.961
H1	0.720	-	-	-	-	0.713
H2	0.984	-	-	-	-	0.979
total	0.895	-	-	-	-	0.888

B. Results and Discussion

Tables III–IX show the costs incurred by agents and the total costs; the value is a cost ratio by the IOP. The "-" in the tables means that the auction terminates unsuccessfully due to a shortage of thermal energy. Note that the tables show another advantage of the proposed method; that is, the cost incurred by each agent can be derived. The problem of how to allocate

TABLE IX
COST RATIO TO IOP OF GROUP G7

agent	SS-FIX					StartStop
	$L = 0.0$	0.2	0.4	0.6	0.8	
F1	0.823	-	-	-	-	0.814
F2	0.944	-	-	-	-	0.928
B1	0.772	-	-	-	-	0.764
H1	0.795	-	-	-	-	0.792
H2	0.993	-	-	-	-	0.986
total	0.877	-	-	-	-	0.868

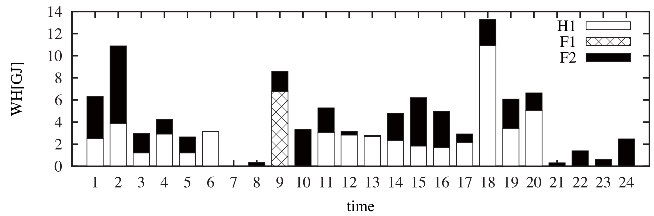

Fig. 7. Waste thermal energy in G2 in the case of IOP.

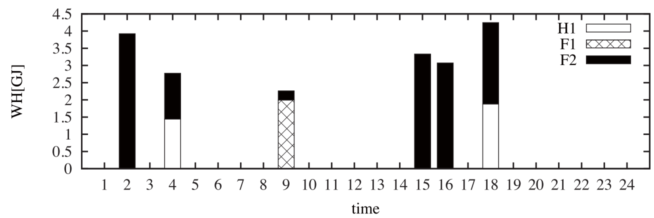

Fig. 8. Waste thermal energy in G2 in the case of SS-FIX($L = 0.2$).

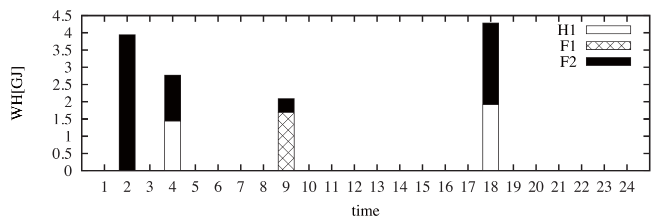

Fig. 9. Waste thermal energy in G2 in the case of StartStop.

the reduced cost to agents arises in a centralized EMS, but the cost incurred by each agent is derived at the time of finding a solution in the DEMS.

SS-FIX and StartStop reduce the energy cost compared with IOP because of the energy trading within the group. StartStop reduces the cost by 10.9% on average. Figs. 7–9 depict waste thermal energy in G2. As shown in Fig. 7, a large amount of thermal energy is wasted because no energy trading is used in the case of IOP, and an imbalance of demands exists. In the case of SS-FIX or StartStop, waste thermal energy is greatly reduced because of the energy trading.

SS-FIX decides the up/down pattern with a fixed $L(t)$, but the values where a solution can be obtained vary, depending on the group. Therefore, it seems difficult to decide on the up/down patterns with a fixed $L(t)$.

On the other hand, `StartStop` succeeds in deciding on the up/down patterns by updating $L(t)$ until a solution could be found. Figs. 10 and 11 depict the up/down patterns of the gas boiler of F1 and the gas turbine of H1 in G2, respectively. Thermal energy is wasted at time periods 15 and 16 in SS-FIX; it is not wasted in StartStop because StartStop suspends the

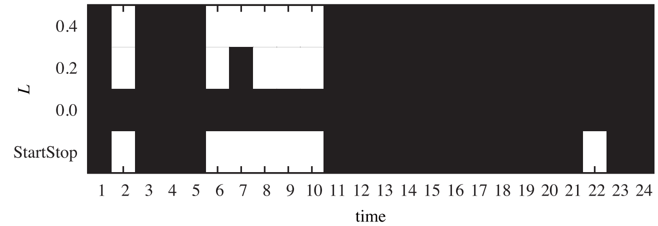

Fig. 10. Up/down patterns of gas boiler of F1 in G2, where it is up when it is black.

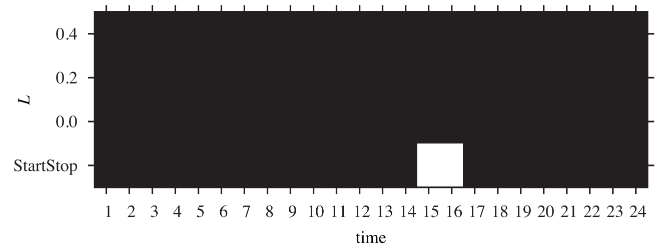

Fig. 11. Up/down patterns of gas turbine of H1 in G2.

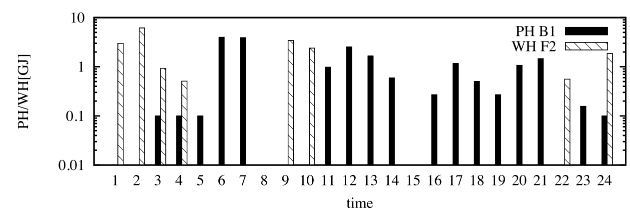

Fig. 12. Produced thermal energy by the gas boiler of B1 and wasted thermal energy of F2 in G1.

gas turbine of H1, as shown in Fig. 11. H1 has small electrical and thermal demand at time periods 15 and 16. In the case of StartStop, the agent suspends the gas turbine and purchases electrical and thermal energy within the group, so no thermal energy is wasted by F2.

However, the decision procedure of up/down patterns in StartStop is similar to a procedure to find the first feasible solution in the best-first search. Different from the best-first search, StartStop terminates when a feasible solution is found; thus, there is a problem in the viewpoint of optimality. For example, the gas boiler of B1 in G1 operates at time periods 3, 4, and 24, but the generation level is at the minimum output (see Fig. 12). Moreover, F2 wastes thermal energy at these time periods. Therefore, if B1 suspends the gas boiler and purchases thermal energy within the group at time period 3, the waste thermal energy by F2 can be reduced. This is because the proposed method decides on the up/down pattern by using the relaxed solution $Y^{*}_{ik}(t)$. Generally, an integer solution near the relaxed solution is not always optimal; thus, the proposed method does not assure optimality. Thus, StartStop does not assure optimality of the up/down patterns; further investigation is required.

In this paper, transmission and ramping constraints are ignored to simplify the problem. The proposed method can be applied even if the ramping constraint is considered because each agent can find its plan under the ramping constraint. When we consider the transmission constraint, additional markets to decide on the amount transmission must be introduced.

VI. Conclusion

In this paper, we have studied the distributed version of the UCP. Especially, we have addressed the problem of deciding on the unit commitment in a distributed manner. Since it is an NP-hard problem, finding the optimal solution is difficult. We have proposed a Walrasian auction-based method for the DUCP. The proposed method is able to decide on both up/down and generating-level patterns of units by introducing the decision procedure for up/down patterns in the tatônnement procedure of a Walrasian auction.

We have evaluated the proposed method through computational experiments using several scenarios. The results show that it is possible to reduce energy consumption through energy trading within the group, as well as to decide on up/down patterns adaptively, depending on the group's configuration.

The proposed method decides on up/down patterns by using the relaxed solution. Generally, an integer solution near the relaxed solution is not always optimal; thus, the proposed method does not assure optimality. The investigation for optimization is left as an issue for the future.

Appendix

Suppose that two agents exist, $\boldsymbol{x}_i$, $i \in \{1, 2\}$ is a decision variable vector, agent 1 is a supplier, and agent 2 is a customer. Let $f_i(\boldsymbol{x}_i)$, $\boldsymbol{h}_i(\boldsymbol{x}_i)$, and $\boldsymbol{g}_i(\boldsymbol{x}_i)$ be the objective function, equality constraints, and inequality constraints of agent i, respectively. Let M be the set of markets, and $x_i^{(m)}$ be the corresponding decision variable of agent i. Then a whole problem (WP) considered in this paper is formulated as follows:

$$\begin{aligned} \text{(WP)} \quad \min \quad & f_1(\boldsymbol{x}_1) + f_2(\boldsymbol{x}_2) \\ \text{subject to} \quad & \boldsymbol{h}_1(\boldsymbol{x}_1) = \boldsymbol{0}, \boldsymbol{h}_2(\boldsymbol{x}_2) = \boldsymbol{0} \\ & \boldsymbol{g}_1(\boldsymbol{x}_1) \le \boldsymbol{0}, \boldsymbol{g}_2(\boldsymbol{x}_2) \le \boldsymbol{0} \\ & x_1^{(m)} = x_2^{(m)} \quad (m \in M). \end{aligned}$$

The Walrasian auction method divides (WP) into two problems, a supplier's problem (SP) and a customer's problem (CP), as follows:

$$\begin{aligned} \text{(SP)} \quad \min \quad & f_1(\boldsymbol{x}_1) - \sum_{m\in M} \alpha_m x_1^{(m)} \\ \text{subject to} \quad & \boldsymbol{h}_1(\boldsymbol{x}_1) = \boldsymbol{0}, \quad \boldsymbol{g}_1(\boldsymbol{x}_1) \le \boldsymbol{0} \\ \text{(CP)} \quad \min \quad & f_2(\boldsymbol{x}_2) + \sum_{m\in M} \alpha_m x_2^{(m)} \\ \text{subject to} \quad & \boldsymbol{h}_2(\boldsymbol{x}_2) = \boldsymbol{0}, \quad \boldsymbol{g}_2(\boldsymbol{x}_2) \le \boldsymbol{0} \end{aligned}$$

where α_m is the price in the market m.

Let $\boldsymbol{x}^* = (\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)^{\mathrm{T}}$ be an optimal solution of (WP). The Lagrange function of (WP) is as follows:

$$\begin{aligned} L(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= f_1(\boldsymbol{x}_1) + f_2(\boldsymbol{x}_2) + \sum_{i\in\{1,2\}} \sum_{s} \lambda_{is} h_{is}(\boldsymbol{x}_i) \\ &\quad + \sum_{i\in\{1,2\}} \sum_{t} \mu_{it} g_{it}(\boldsymbol{x}_i) + \sum_{m\in M} \lambda_m \left(x_2^{(m)} - x_1^{(m)}\right) \\ &= F_1(\boldsymbol{x}_1, \boldsymbol{\lambda}_1, \boldsymbol{\mu}_1) + F_2(\boldsymbol{x}_2, \boldsymbol{\lambda}_2, \boldsymbol{\mu}_2) \\ &\quad + \sum_{m\in M} \lambda_m \left(x_2^{(m)} - x_1^{(m)}\right) \end{aligned}$$

where $F_1(\boldsymbol{x}_1, \boldsymbol{\lambda}_1, \boldsymbol{\mu}_1) = f_1(\boldsymbol{x}_1) + \sum_s \lambda_{1s} h_{1s}(\boldsymbol{x}_1) + \sum_t \mu_{1t} g_{1t}(\boldsymbol{x}_1)$, $F_2(\boldsymbol{x}_2, \boldsymbol{\lambda}_2, \boldsymbol{\mu}_2) = f_2(\boldsymbol{x}_2) + \sum_s \lambda_{2s} h_{2s}(\boldsymbol{x}_2) + \sum_t \mu_{2t} g_{2t}(\boldsymbol{x}_2)$.

From the first-order necessary condition, there exists $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$ for $\boldsymbol{x}^*$, satisfying the following conditions:

$$\begin{aligned} \nabla_{\boldsymbol{x}_1} L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= \nabla_{\boldsymbol{x}_1} F_1(\boldsymbol{x}_1{}^*, \boldsymbol{\lambda}_1{}^*, \boldsymbol{\mu}_1{}^*) \\ &\quad - \sum_{m\in M} \lambda_m{}^* \nabla_{\boldsymbol{x}_1} x_1^{(m)*} = \boldsymbol{0} \\ \nabla_{\boldsymbol{x}_2} L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= \nabla_{\boldsymbol{x}_2} F_2(\boldsymbol{x}_2{}^*, \boldsymbol{\lambda}_2{}^*, \boldsymbol{\mu}_2{}^*) \\ &\quad + \sum_{m\in M} \lambda_m{}^* \nabla_{\boldsymbol{x}_2} x_2^{(m)*} = \boldsymbol{0} \\ \nabla_{\boldsymbol{\lambda}_1} L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= \boldsymbol{h}_1(\boldsymbol{x}_1^*) = \boldsymbol{0} \\ \nabla_{\boldsymbol{\lambda}_2} L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= \boldsymbol{h}_2(\boldsymbol{x}_2^*) = \boldsymbol{0} \\ \frac{\mathrm{d}}{\mathrm{d}\lambda_m} L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= x_1^{(m)*} - x_2^{(m)*} = 0 \quad (m \in M) \\ \boldsymbol{\mu}_i^* \ge \boldsymbol{0}, \ \boldsymbol{g}_i(\boldsymbol{x}_i^*) &\le \boldsymbol{0}, \ \boldsymbol{g}_i(\boldsymbol{x}_i^*)\boldsymbol{\mu}_i^* = \boldsymbol{0} \quad (i \in \{1, 2\}). \end{aligned}$$

Let $\hat{\boldsymbol{x}} = (\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{x}}_2)^{\mathrm{T}}$ be an equilibrium solution by using a Walrasian auction, and $\hat{\boldsymbol{\alpha}}$ be the equilibrium price. Let $L_1^{(\mathrm{SP})}(\cdot)$ be the Lagrange function of (SP). Because the equilibrium solution is an optimal solution of (SP), $\hat{\boldsymbol{\lambda}}_1$ and $\hat{\boldsymbol{\mu}}_1$ satisfying the following conditions must exist:

$$\begin{aligned} \nabla_{\boldsymbol{x}_1} L_1^{(\mathrm{SP})}\left(\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{\lambda}}_1, \hat{\boldsymbol{\mu}}_1\right) &= \nabla_{\boldsymbol{x}_1} F_1\left(\hat{\boldsymbol{x}_1}, \hat{\boldsymbol{\lambda}_1}, \hat{\boldsymbol{\mu}_1}\right) \\ &\quad - \sum_{m\in M} \hat{\alpha}_m \nabla_{\boldsymbol{x}_1} \hat{x}_1^{(m)} = \boldsymbol{0} \\ \nabla_{\boldsymbol{\lambda}_1} L_1^{(\mathrm{SP})}\left(\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{\lambda}}_1, \hat{\boldsymbol{\mu}}_1\right) &= \boldsymbol{h}_1(\hat{\boldsymbol{x}}_1) = \boldsymbol{0} \\ \hat{\boldsymbol{\mu}}_1 \ge \boldsymbol{0}, \boldsymbol{g}_1(\hat{\boldsymbol{x}}_1) &\le \boldsymbol{0}, \boldsymbol{g}_1(\hat{\boldsymbol{x}}_1)\hat{\boldsymbol{\mu}}_1 = \boldsymbol{0}. \end{aligned}$$

In the same way, let $L_2^{(\mathrm{CP})}(\cdot)$ be the Lagrange function of (CP), then $\hat{\boldsymbol{\lambda}}_2$, $\hat{\boldsymbol{\mu}}_2$ satisfying the following conditions must exist:

$$\begin{aligned} \nabla_{\boldsymbol{x}_2} L_2^{(\mathrm{CP})}\left(\hat{\boldsymbol{x}}_2, \hat{\boldsymbol{\lambda}}_2, \hat{\boldsymbol{\mu}}_2\right) &= \nabla_{\boldsymbol{x}_2} F_2\left(\hat{\boldsymbol{x}_2}, \hat{\boldsymbol{\lambda}_2}, \hat{\boldsymbol{\mu}_2}\right) \\ &\quad + \sum_{m\in M} \hat{\alpha}_m \nabla_{\boldsymbol{x}_2} \hat{x}_2^{(m)} = \boldsymbol{0} \\ \nabla_{\boldsymbol{\lambda}_2} L_2^{(\mathrm{CP})}\left(\hat{\boldsymbol{x}}_2, \hat{\boldsymbol{\lambda}}_2, \hat{\boldsymbol{\mu}}_2\right) &= \boldsymbol{h}_2(\hat{\boldsymbol{x}}_2) = \boldsymbol{0} \\ \hat{\boldsymbol{\mu}}_2 \ge \boldsymbol{0}, \boldsymbol{g}_2(\hat{\boldsymbol{x}}_2) &\le \boldsymbol{0}, \ \boldsymbol{g}_2(\hat{\boldsymbol{x}}_2)\hat{\boldsymbol{\mu}}_2 = \boldsymbol{0}. \end{aligned}$$

Because each market is balanced at the equilibrium point, the following equation must hold:

$$\hat{x}_1^{(m)} = \hat{x}_2^{(m)} \quad (m \in M).$$

Therefore, $\hat{\boldsymbol{x}}$ satisfies the first-order necessary condition of (WP). ■

References

[1] N. P. Padhy, "Unit commitment," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 1196–1205, May 2004.

[2] T. Miyamoto *et al.*, "An energy trading system with consideration of CO_2 emissions," *Electr. Eng. Jpn.*, vol. 162, no. 4, pp. 54–63, Mar. 2008.

[3] T. Miyamoto, "Distributed energy management using the market-oriented programming," in *Energy Management*, F. M. Perez, Ed. Vukovar, Croatia: Intech, 2010, pp. 17–38.

[4] K. Yakire *et al.*, "An application of the market-oriented programming to energy trading decision method in distributed energy management systems," *IEEJ Trans. Electron. Inf. Syst.*, vol. 128, no. 1, pp. 24–31, Jan. 2008.

[5] T. Miyamoto, Y. Sugimoto, K. Mori, S. Kitamura, and T. Yamomoto, "On energy trading decision methods in distributed energy management systems with multiple customers," *IEEJ Trans. Electron. Inf. Syst.*, vol. 130-C, no. 2, pp. 178–185, Feb. 2010.
[6] M. P. Wellman, "A market-oriented programming environment and its application to distributed multicommodity flow problems," *J. Artif. Intell. Res.*, vol. 1, pp. 1–23, Jul. 1993.
[7] T. Nagata, M. Ohno, H. Sasaki, and H. Fujita, "A multi-agent approach to unit commitment," *Electr. Eng. Jpn.*, vol. 141, no. 2, pp. 41–47, Nov. 2002.
[8] T. Li and M. Shahidehpour, "Price-based unit commitment: A case of Lagrangian relaxation versus mixed integer programming," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 2015–2025, Nov. 2005.
[9] E. Mashhour and S. M. Moghaddas-Tafreshi, "Bidding strategy of virtual power plant for participating in energy and spinning reserve markets," *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 949–956, May 2011.
[10] K. H. Chung, B. H. Kim, and D. Hur, "Multi-area generation scheduling algorithm with regionally distributed optimal power flow using alternating direction method," *Int. J. Elect. Power Energy Syst.*, vol. 33, no. 9, pp. 1527–1535, Nov. 2011.
[11] B. Ramachandran, S. K. Srivastava, C. S. Edrington, and D. A. Cartes, "An intelligent auction scheme for smart grid market using a hybrid immune algorithm," *IEEE Trans. Ind. Electron.*, vol. 58, no. 10, pp. 4603–4612, Oct. 2011.
[12] A. Pantoja and N. Quijano, "A population dynamics approach for the dispatch of distributed generators," *IEEE Trans. Ind. Electron.*, vol. 58, no. 10, pp. 4559–4567, Oct. 2011.
[13] H. S. V. S. Kumar Nunna and S. Doolla, "Multiagent-based distributed-energy-resource management for intelligent microgrids," *IEEE Trans. Ind. Electron.*, vol. 60, no. 4, pp. 1678–1687, Apr. 2013.
[14] D. Sharma, A. Trivedi, D. Srinivasan, and L. Thillainathan, "Multi-agent modeling for solving profit based unit commitment problem," *Appl. Soft Comput. J.*, vol. 13, no. 8, pp. 3751–3761, Aug. 2013.
[15] A. L. Motto and F. D. Galiana, "Equilibrium of auction markets with unit commitment: The need for augmented pricing," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 798–805, Aug. 2002.
[16] P. F. Correia, "Decentralised unit commitment in a market structure: Problem formulation and solution advancement," *IEE Proc. Gen. Transm. Distrib.*, vol. 153, no. 1, pp. 121–126, Jan. 2006.
[17] W. Gatterbauer and M. Ilić, "Counterexamples to commonly held assumptions on unit commitment and market power assessment," in *Engineering IT-Enabled Sustainable Electricity Services*, M. Ilić, L. Xie, and Q. Liu, Eds. New York, NY, USA: Springer, 2013, pp. 261–271.
[18] R. Sioshansi, S. Oren, and R. O'Neill, "The cost of anarchy in self-commitment-based electricity markets," in *Competitive Electricity Markets: Design, Implementation and Performance*, F. P. Sioshansi, Ed. Oxford, U.K.: Elsevier, 2008, pp. 245–266.
[19] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Belmont, MA, USA: Athena Scientific, 1997.
[20] R. Hanson, "Combinatorial information market design," *Inf. Syst. Front.*, vol. 5, no. 1, pp. 107–119, Jan. 2003.
[21] H. P. Williams, *Model Building in Mathematical Programming*, 5th ed. West Sussex, U.K.: Wiley, 2013.
[22] G. Tel, *Introduction to Distributed Algorithms*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 2000.

Toshiyuki Miyamoto (S'95–M'97) received the B.E. and M.E. degrees in electronic engineering and the Dr. Eng. degree in electrical engineering from Osaka University, Osaka, Japan, in 1992, 1994, and 1997, respectively.

From 1997 to 2003, he was a Research Assistant with the Graduate School of Engineering, Osaka University. In 2000 and 2001, he was a Visiting Researcher with the Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA. He is currently an Associate Professor with the Division of Electrical, Electronic, and Information Engineering, Osaka University. His current research interests include the theory and applications of concurrent systems and multiagent systems.

Dr. Miyamoto is a member of the Institute of Electronics, Information, and Communication Engineers, the Institute of Systems, Control, and Information Engineers, and the Society of Instrument and Control Engineers.

Kazuyuki Mori (M'98) received the B.E. and M.E. degrees in industrial administration from the Tokyo University of Science, Tokyo, Japan, in 1985 and 1987, respectively, and the Dr. Eng. degree in electrical engineering from Osaka University, Osaka, Japan, in 1998.

He joined Mitsubishi Electric Corporation, Amagasaki, Japan, in 1987, where he is currently engaged in research on discrete event systems, production scheduling, systems optimization, and energy solution. He is currently a Chief Researcher of the Advanced Technology Research and Development Center, Mitsubishi Electric Corporation.

Dr. Mori is a member of the Institute of Electrical Engineers in Japan, the Society of Instrument and Control Engineers, and the Institute of Systems, Control and Information Engineers.

Shoichi Kitamura received the B.E. and M.E. degrees in biophysical engineering and the Dr. Eng. degree in electrical engineering from Osaka University, Osaka, Japan, in 2000, 2002, and 2013, respectively.

He joined the Advanced Technology Research and Development Center, Mitsubishi Electric Corporation, Amagasaki, Japan, in 2002, where he was engaged in research on the factory energy management system and currently engaged in research on the smart grid and smart community-related technologies.

Dr. Kitamura is a member of the Institute of Electrical Engineers of Japan.

Yoshio Izui (M'80) received the B.E., M.E., and Dr. Eng. degrees in electrical engineering from the University of Tokyo, Tokyo, Japan, in 1981, 1983, and 1986, respectively.

He joined Mitsubishi Electric Corporation, Amagasaki, Japan, in 1986. He is currently affiliated with the Advanced Technology Research and Development Center, where he is engaged in research and management on the smart gird and smart community-related technologies.

Dr. Izui is a member of the Institute of Electrical Engineers of Japan, the Institute of Electronics, Information, and Communication Engineers, the Information Processing Society of Japan, the Society of Instrument and Control Engineers, the Institute of Systems, Control, and Information Engineers, and the Japanese Neural Network Society.