A mathematical model for finding the optimal locations of railway stations

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Abstract:

Regarding the importance of competitive advantages among the transportation systems, improving the costumer's satisfaction is an important factor in attracting them to these systems. In this research, we focus on the effects of constructing new stations on users and a new mathematical model is proposed for this problem. In the proposed model, two simultaneous effects on customers by constructing new stations are investigated. One effect is the improvement of the demand access to the railway network and the other one is related to the increase in the travelling time for customers sitting in the train. As the first effect, the less the customer's distance to the stations, the more attractive to use this system and the second effect concerns the increased in the traveling time in new stops which may result to customer's dissatisfaction. The proposed mathematical model achieves the time optimality, furthermore; it maximizes the covered population to locate the stations. For locating the optimal site for new stations, we need to apply the population usage information. The proposed model is examined on the Mashhad's subway as a case study and the results are reported.

Keywords: *locating new railway stations, traveling time, covering model, accessibility model.*

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1. Introduction

Locating the new stations along the railway transportation network is defined as the railway station location problem and decision making about the optimal number and location of stations is an important factor in designing transportation systems, which results to increase the customer's satisfaction and number of system demand. Two aims of the railway station location problem are to increase the attraction of railway travels and the covering population by constructing new stations, which the travel attraction is investigated by considering two simultaneous effects on customers.

The former aim is related to the accessibly improvement of the population to the network and the latter one concerns the increase in traveling time.

The stop-location problem has been studied in several works since the 1960s. Finding the optimal location of the station is solved in two phases by (Samanta and Jha, 2008). In the first step the potential stops for stations are found by using a GIS algorithm and in the second phase a genetic algorithm is used to minimize the cost of optimal location of stations.

A study of the high-speed transit lines is done in which the stations are chosen by the priority based on maximizing function of saved travel cost. In order to compete by other means of transportations; the customers' sensitivity to stop at new stations is analyzed by (Repolho et al, 2012).

The station location problem in the existing network to cover all demands by minimizing the additional travelling time is considered. This problem is shown as an NP-Hard problem, especially when the distances are measured using Euclidean metric in (Schobel et al, 2009).

The problem of finding the location of stations on the network that has two lines and intersect each other by an angle α is studied to construct the minimum number of stations covering all demands in (Mammana et al, 2003), also it is assumed that the stations could be located anywhere along the railway line because the users are not covered by any of the existing stations.

Another study is done to find the station locating on an existing public transportation network such as railways or bus routes to maximize the covering population and minimize the number of stations by (Schobel, 2005). To place new stations optimally, the population covering and the access problem is solved. In the first function, the total number of stations is minimized and the customers use the nearest station by considering the limitation of users covering radiance. In the second objective function, the number of new stations is fixed and the sum of the distances between the demand points and new stations is minimized (Grob et al, 2009).

An easy way to increase the attractiveness of railway travels is introduced by constructing small stations in the existing lines, which are served by the local fleets. The accessibility and the travelling time are considered as two effects of constructing the new stations in (Hamacher et al, 2001).

The station location problem is solved on a linear transition line and distribution of the passengers is considered as uniform by (Vuchic and Newell, 1968).

A stochastic model to evaluate the performance of station indicators is designed in (Semaan and Zayed, 2010). The required data are extracted by preparing some questionnaires and interviews with technicians and experts.

None of the cited works provide a mathematical model to optimally locate the stations considering customer's satisfaction and simultaneously concerns the covered population.

This paper proposes a new mathematical model concerning on the customer's utility as well as maximizing the covered population. In the proposed model, three different models including accessibility, traveling time and the covering population models are combined. The first and second models affect the attraction of train travels. Concerning the first effect, the attractiveness of rail transportation will be increased if people live closer to the stations. On the other hand, the passengers who sit in the train are slowed down due to train's stoppings at newly constructed stations that results to decrease the attraction of train travels. The described effects highly influence the attraction of this system; therefore, the proposed mathematical model is designed to optimize the covered population as well.

This paper is organized as follows. In the next section, all the components of our model are described. In the third section, the formulation of the model is presented and its components are explained. A case study on the Mashhad's subway and the results are reported in the fourth section and the last section is consisted of the results of our research and some recommendations for the future study.

2. Mathematical model

Three different effects are considered by constructing new stations, which are proposed as a new mathematical model and explained here.

2.1. The accessibility

The network is depicted by $G=(V,E)$ consisted of V and E representing the existing stations and the line segments in plane respectively. It is assumed that P represents the finite set of user sites as the coordinate points. The set $Q=\{\, j\in R^2 : j\in S_i \text{ for some } i=1\dots L \}$ consists of the feasible points for each candidate station location along the network, which S_i represents the line segment i in the network.

Because each user prefers to use the nearest station for reaching the railway network, so the candidate stations which reduce the distances among the customers and the network could be constructed. The solution set (Y) represents the feasible subset of the stations from the Q set that should be constructed.

To consider the accessibility of potential customers, the distance among each customer points $(i\in P)$ and candidate stations should not exceed a specific radius as follows.

$$d_{ij}\le r. \tag{1}$$

A user $i\in P$ is covered by the new station from the Y set if:

$$d_{iy}\le r, \quad y\in Y. \tag{2}$$

2.2 The travel time

Users will save time if their distance to the network is reduced by constructing new stations. This result increases the attraction of the railway transportation. On the other hand, customers sitting in the train will lose time because of the additional stops at newly constructed stations. These delays are assumed different in each stations, which is shown by $delay_j$ (index j represents each individual station). The number of passengers sitting in the train and passing through the new stations is shown by w_j and is multiplied by the individual delay in each new station in order to calculate the overall additional time in each station provided that the new one is constructed ($y_j = 1$). The amount of additional travel time for all customers who are affected by new stops can be calculated as Eq. (3):

$$\sum_{j\in Q} delay_j w_j y_j . \tag{3}$$

Considering the positive effect of constructing new stations, the reduction of transit time to reach the network is calculated as below.

Step 1: Calculating the minimum distance from each user point to its nearest existing station

The minimum distance from one user point to all of the existing stations is calculated in Eq. (4) because each customer prefers to use the nearest station to reach the network and it is shown by d_i. Some factors such as income, the use of private cars or other means of transportations are not taken into account and it is assumed that all of the customers use the railway network.

$$d_i = min_{\,k\in K}\, d_{ik}, \qquad \forall i\in P. \tag{4}$$

It is assumed that the customers mainly use three means of transportation to reach the stations. Some walk, others ride a bike and the rest use buses or cars and the average speed of each means of transportation are 4 km/h, 7 km/h, and 20 km/h consequently.

Therefore, the average speed of customer's transition to the stations is assumed 5 km/h [7].

Step 2: Calculation of the traveling time

Time to reach the existing stations can be calculated as follows:

$$t_i = d_i / 5, \qquad \forall i\in P. \tag{5}$$

Moreover, the transit time to reach the new stations is calculated by:

$$T_{ij} = d_{ij} / 5 \qquad \forall i\in P, \forall j\in Q, \tag{6}$$

The distance between user i and the new station j is shown by d_{ij} and the minimum distance is not considered because one station may disapprove covering radius term, but it may not be necessarily the nearest one to the user. Calculating the shortest Euclidian distance between the user points and new stations may reduce the chance of selecting these stations, and results in lower accuracy of the model because the construction of these stations may be optimized by considering their total covered population and reduced traveling time.

Step 3: Calculation of reduced traveling time

The final function to calculate the reduced travel time is offered:

$$t_{ij} = t_i - T_{ij}, \qquad \forall i \in P, \forall j \in Q, \tag{7}$$

If the reduced traveling time got negative value, it would assume $-\infty$. Multiplying this by the variable x_{ij} results in, the objective function become $-\infty$ so it prevents to assign the costumer to the new station and as the result, the costumer would be covered by another station as a new or the existing one but this not meaning that the corresponding station should not be constructed. All the mentioned steps are handled in the primary data to prepare t_{ij} as the final input of the model. The v_i coefficient represents the number of population in site i; therefore, the total reduced time could be calculated as:

$$\sum_{i \in P} \sum_{j \in Q} v_i \, t_{ij} x_{ij}. \tag{8}$$

The station location problem is NP-Hard especially in Euclidean distances [3].

2.3. The covering population

The factors explained above affect the attraction of customers toward the railway systems.

Here we try to optimize the covering population by new stations. The covered population can be defined by the population who can access the stations by expending less time. Therefore, in order to achieve the goal, the number of customers in each traffic zone is multiplied by the related binary variable (x_{ij} is one when the travelling time is reduced by the corresponding station) and the summation of the results is calculated by:

$$\sum_{i \in P} \sum_{j \in Q} v_i \, x_{ij}. \tag{9}$$

3. Mathematical formulation

The formulation of the problem based upon three mentioned effects is presented:

$$max \quad z_1 = \sum_{i \in P} \sum_{j \in Q} v_i \, t_{ij} x_{ij} - \sum_{j \in Q} delay_j w_j y_j \tag{10}$$

$$max \quad z_2 = \sum_{i \in P} \sum_{j \in Q} v_i \, x_{ij} \tag{11}$$

s.t:

$$\sum_{j \in Q} x_{ij} \le 1 \qquad \forall i \in P \tag{12}$$

$$x_{ij} \le y_j \qquad \forall i \in P, \forall j \in Q \tag{13}$$

$$\sum_{j \in Q} y_j \le N \tag{14}$$

$$y_j, x_{ij} \in \{0,1\} \qquad \forall i \in P, \forall j \in Q \tag{15}$$

The decision variables are x_{ij} and y_j that represent assignment variables and location decision variables, respectively. The user i is assigned to the new constructed station in site j if $x_{ij} = 1$ which is meant the distance from the corresponding user to the network is reduced. The new station can be constructed on site j if $y_j = 1$.

The first objective has two terms. In the first term, the summation of the reduced travelling time for all customers, from each user point to its nearest station by constructing new stations is maximized.

The second term minimizes the summation of train delays in each new station for passengers who sit in the train. The second objective is to maximize the covering population. Constraint (12) represents that each user should be assigned to at most one new constructed station because it prevents to account the coved population coefficient v_i more than one time. Constraint (13) shows that a user can be assigned to a new candidate station if the station is constructed on site j.

One of the important factors in constructing the stations is related to the number of new stations and the goal is to maximize the covered population and the saved travel time by the minimum number of stations, so the number of new constructed stations is at most equal to N by Constraint (14), which N is a parameter should be determined. Constraint (15) expresses that x_{ij} and y_j are binary variables.

4. Case study

Here the subway network of Mashhad, a city located in the north side of Iran, to examine our model is chosen. This city is split into 253 different traffic zones and the network has 17 stations belongs to the operating first line considered as existing stations in our model and 46 stations are considered as candidate stations in the new approved lines which are selected by initial feasibility study. The operating first line is shown in Fig. (1) and Mashhad's lines with their stations are depicted in Fig. (2).

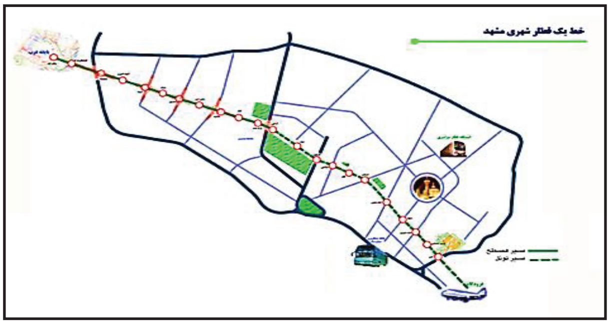

Fig. 1. Mashhad's operating first line.

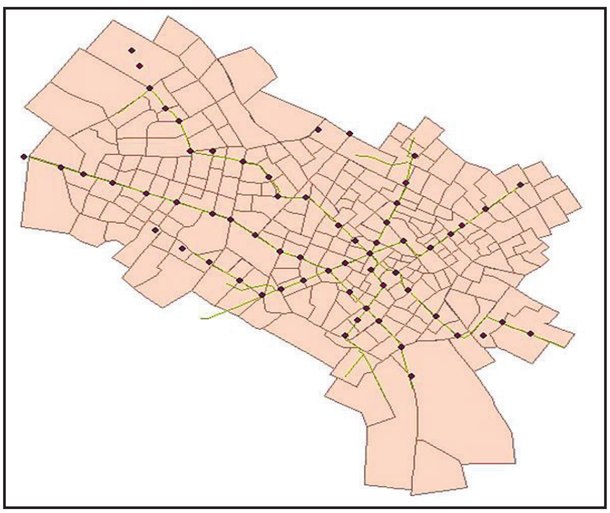

Fig. 2. Mashhad's lines and stations.

The city is split by zones to be integrated from the point of different activities; therefore the location of each zone is shown by its central as coordinate points and v_i denotes the number of population in each traffic zone. The traffic zones with their central are shown in Fig. (3). Fig. (4) illustrates the whole network including the centrality of stations and the traffic zones. The central coordinates of traffic zones and stations as well as the data of population in each zone and passing through the stations are gathered separately, also the train stop time in different stations is needed. In order to determine the optimal location of the new stations, the proposed model is solved in CPLEX 11 environment and all distances are considered as Euclidean metric.

The given radius to examine the covering radius term are considered in an interval from 500 to 800 meter which the distance from each user points to the new stations should not exceed this given radius. Otherwise the distance between the user point and a new station is considered $+\infty$; therefore, the travel time to reach the new station gets $+\infty$ and the final model input (t_{ij}) takes $-\infty$ and as the result the related user is not assigned to new station because $x_{ij} = 0$. The number of new stations is assumed between 30 and 46. The second objective function is solved by omitting the first objective to calculate its optimal value and the result is multiplied by the different coefficients to achieve a lower bound for the second objective when it is considered as a constraint and the first one is individually solved, the results are reported in table1.

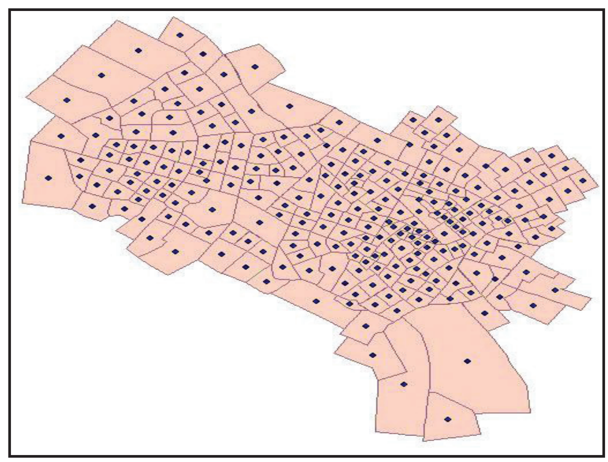

Fig. 3. Zone's centers (blue points).

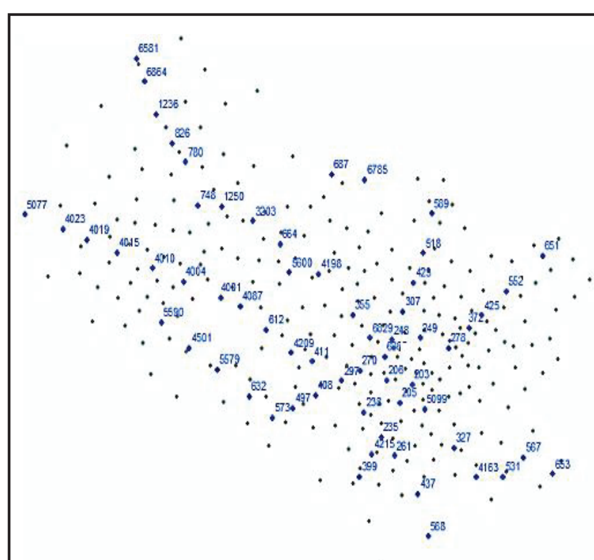

Fig. 4. The central of Mashhad's traffic zones (gray points) and the station location sites (blue points)

Considering the above assumptions, optimal locations for constructing the new stations are obtained when the coefficient α is 0.09, the lower bound of the second function is equal to 75666.15 and the optimal number of stations are 32 showing the minimum number; therefore, the location of stations are 248, 249, 278, 307, 327, 355, 372, 399, 423, 425, 497, 518, 531, 552, 589, 632, 651, 653, 664, 687, 826, 1250, 3203, 4163, 4198, 4501, 5099, 5579, 5590, 5600, 6581, and 6865. The location of stations can be found in Fig. 1.

1. Conclusion

The current methods of determining the appropriate location of stations are very time consuming and needs a great deal of cost and human resources also making decision by increasing the number of stations would be really difficult. Therefore, in this study a new mathematical model to find the optimal location of stations is proposed. The model is a combination of three components including the accessibility of the population to the network, time travelling and population coverage. On the other hand, this model is based on the customer satisfaction criterions and improves the competitive advantages of railway travels than the other means of transportation. These criterions are consisted of the reduction of passenger traveling time for reaching the railway network stations and increasing the customer's accessibility to the railway systems. The decision cost by the executive team for finding the optimal location of stations is reduced because the model can find it accurately.

This model not only can find the optimal location of new stations in new lines but also can locate new stations in the existing lines, which can be fed by the existing fleets so the best availability, the minimum time consumption and the minimum amount of costs can be achieved.

For the future study a developed model based on the operating and building costs can be generated and the covering radius dynamically can be changed by the attraction or population of each traffic zones.

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Table 1. Optimal number of Mashhad's stations with lower bound of the second objective and the optimal value of the first function.

The optimal number of new stations	The lower bound of the second objective	Optimal value of the first function (in second)
46	7604.98	2.10E+09
42	23745.03	2.02E+09
40	47298.42	1.99E+09
32	75666.15	2.42E+09
41	96121.80	2.05E+09
43	126626.90	2.29E+09
40	147978.90	2.13E+09

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