An Efficient Approach for Unit Commitment and Economic Dispatch with Combined Cycle Units and AC Power Flow

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***Abstract*—Unit Commitment and Economic Dispatch (UCED) with combined cycle (CC) units and AC power flow is an important problem to be solved by ISOs. The problem is difficult because of complicated transitions in CC units and highly non-linear AC power flows. Currently, to solve the problem, transitions among CC states are simplified and AC power flow is approximated with DC power flow. However, the resulting solution may not be consistent with actual operations of power systems. In this paper, a more operational approach of modeling UCED with CC units and AC power flow is developed. Under the frequently used assumption of low network resistance, AC power flow is represented as a monotonic function. Then, the original problem is solved by exploiting the monotonicity through the novel dynamic linearization technique and separability after relaxing coupling system-wide demand constraints. The complexity of resulting subproblems is drastically reduced and linearity is efficiently exploited by using branch-and-cut. Subproblem solutions are efficiently coordinated by our recently developed surrogate Lagrangian relaxation and convergence is guaranteed. Based on a 30-bus system, numerical results demonstrate the new approach is more computationally efficient as compared to Benders decomposition.**

***Index Terms*—Unit Commitment and Economic Dispatch, Combined Cycle Units, AC Power Flow, Branch-and-cut, Surrogate Lagrangian Relaxation**

I. INTRODUCTION

Unit Commitment and Economic Dispatch (UCED) is an important problem solved by Independent System Operators (ISOs). Combined cycle (CC) units[1] are frequently considered because of their high thermal efficiency. However, when such units are present, the UCED problem is typically difficult to solve because of complicated transitions among the combined cycle states. Additionally, when AC power flow is considered, the problem becomes highly nonlinear. To date, there are no results available to solve the UCED problem while considering AC power flow and modelling CC units to represent actual transitions accurately. When applying frequently-used methods such as Benders decomposition, the resulting MILP master problem contains all binary decision variables and it is typically difficult to solve by branch-and-cut, dynamic programming or genetic algorithm [4]. In the presence of CC units, branch-and-cut may suffer from slow convergence because constraints that model transitions among CC units treated globally within the method and affect the solution process of the entire problem.

Currently, to solve the problem, many ISOs use approximations [5]. Namely, CC units are modeled as approximate configurations, or configurations are modeled as separate resources. The resulting simplified problem can then be solved by using Benders decomposition by decomposing the problem into the master mixed-integer linear (MILP) unit commitment problem and nonlinear (NLP) economic dispatch subproblems which include the nonlinear AC power flow constraints. Generally, solving the master problem is not a trivial task. Moreover, because of simplifications, the resulting solution is not consistent with actual system operations.

Methods that have been used to solve UCED problems will be reviewed in Section II. In Section III, a more operational formulation the UCED problem with CC units and AC power flow will be presented. To efficiently solve the problem, in Section IV, the monotonicity of the AC power flow in established under the assumption of low resistance, and the monotonicity is then exploited through the novel linearization technique. The linearity of the resulting subproblems is exploited by branch-and-cut and subproblem solutions are efficiently coordinated by our recently developed surrogate Lagrangian relaxation. Numerical results presented in Section V demonstrate that the new approach is computationally efficient for a mid-size problem.

[1] As compared to conventional generators, CC units [1]-[3] are more efficient because the excess heat from combustion turbines is not wasted into the air but is used to generate more electricity by generating steam in steam turbines. However, transitions among CC states are constrained. For example, steam turbines cannot be turned on if there is not enough heat from combustion turbines.

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II. Literature Review

A. Unit Commitment and Economic Dispatch

Unit Commitment and Economic Dispatch is an important problem solved by ISOs. Historically to solve the problem, Lagrangian relaxation was used to exploit separability by relaxing system-wide system demand and transmission capacity coupling constraints and decomposing the relaxed problem into nodal or unit-wise subproblems [6]-[7]. Subproblem solutions are then coordinated by using Lagrangian multipliers. The recent trend to solve the problem is by using MIP methods that are based on branch-and-cut. Without considering combined cycle units and AC power flow, the problem can typically be solved efficiently.

B. UCED with combined cycle units and AC power flow

The UCED problem with combined cycle units and AC power flow is difficult because when transitions among combined cycle stated are modeled accurately, such transitions are difficult to follow. Moreover, AC power flow constraints are highly nonlinear. Currently, to solve the problem, approximations are used. Namely, depending on the ISO, combined cycles are modeled as approximate aggregate configurations, or configurations are modeled as separate resources [5, p. 29-30].

C. UCED with AC Power Flow

To solve the UCED problem with AC power flow, decomposition methods utilizing MIP methods have been used [4], [8]-[9]. One of the methods is Benders decomposition [4], [8]. The idea is to decompose the original problem into one MILP master problem and several NLP subproblems. The master problem is the mixed-integer linear UC problem and subproblems are economic dispatch nonlinear problems with fixed integer commitment decision variables. After solving subproblems (e.g., by using SNOPT[2]), linear Benders cuts are derived and are iteratively added to the master problem to revise commitment and dispatch thereby increasing the size of the master problem. Moreover, the master problem contains all the integer decision variables, and even without the presence of combined cycle units this creates a major bottleneck for such methods as MIP methods, dynamic programming and genetic algorithm [4].

When transitions among combined cycle units are modeled accurately, solving the master problem can lead to major difficulties for MIP solvers because constraints that model transitions among combined cycle states and cuts that are based on such constraints, frequently do not define facets of the convex hull. Moreover, cuts generated by branch-and-cut need to be valid for the entire problem, and complicated transitions within just one combined cycle unit affect the solution process of the entire problem. Without the convex hull, the method uses branch-and-bound to obtain feasible solutions thereby leading to slow convergence and a large MIP gap [10]-[12].

D. UCED with combined cycle units

Without AC power flow, the UCED problem with CC units modeled to represent actual transitions more accurately can be formulated in a linear form. However, convergence of standard MILP methods can be slow because of the reasons explained in subsection B. To solve the UCED problem with CC units, our surrogate Lagrangian relaxation [10] has been combined with branch-and-cut and the problem has been efficiently solved without [11] and with DC power flow [12]. Surrogate Lagrangian relaxation overcomes all the difficulties of standard Lagrangian relaxation. Without requiring the relaxed problem to be fully optimized, zigzagging of multipliers is alleviated and without requiring the optimal dual value for convergence, practical implementability of the method is guaranteed. Moreover, after relaxing system-side coupling constraints such as system-demand and transmission capacity constraints, and decomposing the relaxed problem into nodal or unit-wise subproblems, complexity of each subproblem is drastically reduced as compared to the original problem. As a result, the resolution of each subproblem by branch-and-cut does not affect the solution process of the entire problem thereby overcoming computational difficulties associated with existing methods mentioned in subsection C.

III. Problem Formulation

This section presents the UCED problem formulation with CC units, in which transitions are modeled to accurately represent actual operations, and AC power flow.

A. Problem Description

The UCED problem with combined cycle units and AC power flow is to commit units to satisfy system demand by minimizing the total bid cost, consisting of the total generation and the total start-up costs while satisfying unit-wise constraints including transitions among combined cycle states and satisfying system-wide constraints such as system demand and AC power flow constraints.

Consider a network with I nodes connected by L transmission lines. Each generator located at a node i submits energy bids. Each bid[3] m_i (= 1,…, M_i) at node i (= 1,.., I) will be indexed by (i,m_i). Each bid indexed by i consists of energy bidding prices c_{i,m_i}, startup costs S_{i,m_i}, maximum and minimum generation levels $p_{(i,m_i)max}$, $p_{(i,m_i)min}$, ramp rates Δ_{i,m_i} and minimum up and down times. Energy bids will be modeled for each hour by several blocks of energy with non-decreasing prices. The constraints are formulated as follows:

B. Unit-wise Constraints

To model the status of each bid (i,m_i), binary decision variables $x_{i,m_i}(t)$ will be used: $x_{i,m_i}(t) = 1$ indicates that the bid was selected, and $x_{i,m_i}(t) = 0$ otherwise. If the bid is selected, energy output $p_{i,m_i}(t)$ should satisfy:

$$x_{(i,m_i)}(t)\,p_{(i,m_i)\min} \le p_{(i,m_i)}(t) \le x_{(i,m_i)}(t)\,p_{(i,m_i)\max}. \qquad (1)$$

The startup cost $S_{i,m_i}(t)$ is incurred if and only if unit i has been turned an 'on' from an 'off' state at hour t

$$S_{(i,m_i)}(t) \ge S_{(i,m_i)}\left(x_{(i,m_i)}(t) - x_{(i,m_i)}(t-1)\right). \qquad (2)$$

Ramp-rate constraints ensure that the increase/decrease in the output of a unit within one hour does not exceed a pre-specified ramp-rate:

[2] Sparse Nonlinear OPTimizer mainly written in Fortran.

[3] Each bid corresponds to either a conventional unit, or to a combustion/steam turbine generator that comprises a combined cycle unit.

$$-\Delta_{(i,m_i)} \le p_{(i,m_i)}(t) - p_{(i,m_i)}(t-1) \le \Delta_{(i,m_i)}. \tag{3}$$

Minimum up- and down-time constraints ensure that a unit must be kept online/offline for a pre-specified number of hours. Formulation of minimum up- and down-time constraints can be found in [6] and [13].

Combined cycle units can operate at multiple configurations of combustion turbines (CTs) and steam turbines (STs) as shown in Figure 1. For example, steam turbines cannot be turned on if there is not enough heat from combustion turbines.

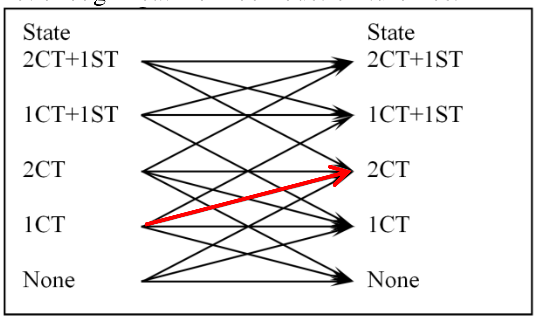

Figure 1. Transitions among the states in a combined cycle unit

Transitions among combined cycle states [1]-[3] can be modeled by using logical operators AND, OR, and $\Rightarrow$ (logical implication). A transition, which can be described as "if one combustion turbine has been online for at least one hour, then another combustion turbine can be turned on," can be modeled as

$$\begin{aligned}&(x_{CT1}(t\text{-}1)=1 \ AND \ x_{CT2}(t\text{-}1)=0 \ AND \ x_{ST}(t\text{-}1)=0) \ OR\\&(x_{CT2}(t\text{-}1)=1 \ AND \ x_{CT1}(t\text{-}1)=0 \ AND \ x_{ST}(t\text{-}1)=0) \Rightarrow\\&(x_{CT2}(t)=1 \ AND \ x_{CT1}(t)=1 \ AND \ x_{ST}(t)=0).\end{aligned} \tag{4}$$

This transition is shown on Figure 1 by a thick red arrow. Other transitions can be modeled in a similar fashion, and for brevity of explanation are not shown. Logical expressions can be linearized using the following relations [11], [12]:

a_1=1 AND a_2=0 is equivalent to $a_1+1-a_2=2$, (5)

$a=0 \Rightarrow b=0$ is equivalent to $-M \cdot a \le b \le M \cdot a$, (6)

$a_1 \le b_1 \ OR \ a_2 \le b_2$ is equivalent to
$a_1 \le b_1 + M \cdot z_1$; $a_2 \le b_2 + M \cdot z_2$; $z_1 + z_2 = 1$. (7)

where z_1 and z_2 are binary decision variables, and M is an arbitrarily big positive number.

In addition, the output of a ST is typically no more than 50% of the total CT output within one CC unit:

$$p_{ST}(t) \le \tfrac{1}{2}\left(p_{CT1}(t) + p_{CT2}(t)\right). \tag{8}$$

C. System-wide Coupling Constraints

Committed generators need to satisfy nodal load levels $P_i^D(t)$:

$$\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}(t) = \sum_{i=1}^{I} P_i^D(t). \tag{9}$$

Demand constraints (9) can be satisfied by generating power either locally or transmitting power through transmission lines. The AC power flow $f_{i,j}(t)$ in a line that connect nodes i and j is given by the formula [14]:

$$f_{i,j}(V(t),\Delta\theta(t),t) \equiv |V_i(t)||V_j(t)|\left[g_{i,j}\cos(\Delta\theta_{i,j}(t)) + b_{i,j}\sin(\Delta\theta_{i,j}(t))\right]. \tag{10}$$

Here $g_{i,j}$ is admittance, $b_{i,j}$ is reactance, $\Delta\theta_i$ is the difference between θ_i and θ_j, which are voltage phase angles at buses i and j, and $|V_i|$ and $|V_j|$ are voltage magnitudes at buses i and j. Power flows cannot exceed the transmission capacity $f_{(i,j)max}$:

$$-f_{(i,j)\max} \le f_{i,j}(V(t),\Delta\theta(t),t) \le f_{(i,j)\max}. \tag{11}$$

Moreover, at every node i, power transmitted to the node and power generated at the node, should be equal to power transmitted from the node and the power demand at the node:

$$\begin{aligned}&\sum_j f_{j,i}(V(t),\Delta\theta(t),t) + \sum_{m=1}^{M_i} p_{(i,m)}(t) =\\&\sum_j f_{i,j}(V(t),\Delta\theta(t),t) + P_i^D(t).\end{aligned} \tag{12}$$

D. The Problem Formulation

The objective is to minimize the total cost:

$$\sum_{i=1}^{I}\sum_{m=1}^{M_i}\left(\sum_{t=1}^{T} c_{(i,m_i)}\left(p_{(i,m_i)}(t),t\right) + \sum_{t=1}^{T} S_{(i,m_i)}(t)\right) \tag{13}$$

subject to constraints (1)-(12). Binary decision variables are UC decisions $x_{i,m_i}(t)$, and continuous decision variables are power levels $p_{i,m_i}(t)$, start-up costs $S_{i,m_i}(t)$, voltage magnitudes $|V_i(t)|$, phase angles $\theta_i(t)$, and power flows $f_{i,j}(t)$.

IV. Solution Methodology

This section is on the development of the solution methodology to solve the problem formulated in Section III. Surrogate Lagrangian relaxation is presented in subsection A. Monotonicity of AC power flow constraints are then be established under the assumption of low resistance, and the monotonicity is exploited through a dynamic linearization in subsection B. Resulting subproblems are then efficiently solved by using branch-and-cut. In subsection C, procedure to obtain feasible solutions will be briefly presented.

A. Surrogate Lagrangian Relaxation

To solve the problem formulated in Section III, demand constraints (9) are relaxed by introducing multipliers $\lambda(t)$ to form the Lagrangian function:

$$\begin{aligned}&L(\lambda(t),x(t),p(t),S(t),V(t),\Delta\theta(t)) =\\&\sum_{i=1}^{I}\sum_{m=1}^{M_i}\left(\sum_{t=1}^{T} c_{(i,m_i)}\left(p_{(i,m_i)}(t),t\right) + \sum_{t=1}^{T} S_{(i,m_i)}(t)\right) +\\&\sum_{t=1}^{T}\lambda(t)\left(\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}(t) - \sum_{i=1}^{I} P_i^D(t)\right).\end{aligned} \tag{14}$$

The relaxed problem is to minimize (14):

$$\min_{\substack{x(t),p(t),S(t),\\ \Delta\theta(t),V(t)}} L(\lambda(t),x(t),p(t),S(t),V(t),\Delta\theta(t)), \tag{15}$$

subject to constraints (1)-(8), (10)-(12).

After the relaxed problem is minimized, multipliers are updated based on the levels of constraint violation. Within the surrogate Lagrangian relaxation surrogate subgradient directions are used to update multipliers, and to guarantee that such directions form acute angles with directions toward the optimum, the following simple "surrogate optimality condition" has to be satisfied [10]:

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$$\tilde{L}\left(\lambda^k(t), x^k(t), p^k(t), S^k(t), V^k(t), \Delta\theta^k(t)\right) < \tilde{L}\left(\lambda^k(t), x^{k-1}(t), p^{k-1}(t), S^{k-1}(t), V^{k-1}(t), \Delta\theta^{k-1}(t)\right). \quad (16)$$

Here $\tilde{L}$ denotes the surrogate dual value, which is the Lagrangian evaluated at a solution $\{p^k(t), x^k(t), S^k(t), V^k(t), \theta^k(t)\}$. After satisfying (16), multipliers are updated as:

$$\lambda^{k+1} = \lambda^k + c^k \tilde{g}\left(p^k(t)\right), k = 0,1,\ldots. \quad (17)$$

Surrogate subgradient directions are obtained as

$$\tilde{g}\left(p^k(t)\right) = \sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}(t) - \sum_{i=1}^{I} P_i^D(t). \quad (18)$$

In order to update multipliers, the following stepsizing formula is used [10]:

$$c^k = \alpha_k \frac{c^{k-1}\left\|\tilde{g}\left(p^{k-1}\right)\right\|}{\left\|\tilde{g}\left(p^k\right)\right\|}, 0 < \alpha_k < 1, k = 1,2,\ldots. \quad (19)$$

Convergence of the method with the formula (19) was proved in [10], and one possible way to set parameters α_k is:

$$\alpha_k = 1 - \frac{1}{Mk^p}, p = 1 - \frac{1}{k^r}, M \geq 1, 0 < r < 1, k = 2,3,\ldots. \quad (20)$$

The idea of the decomposition and coordination will be extended, and through the novel dynamic linearization convergence of the combination of surrogate Lagrangian relaxation and branch-and-cut will be guaranteed.

B. Dynamic Linearization

Without relaxing all the coupling constraints, the relaxed problem is not completely decomposed. Still, following the idea of the decomposition, the objective function can be decomposed and the minimization is performed subject to unit-wise constraints, and the remaining system-wide constraints. Consider the following nodal "subproblems":

$$\min \sum_{m=1}^{M_i}\left(\sum_{t=1}^{T} c_{(i,m_i)}\left(p_{(i,m_i)}(t), t\right) + \sum_{t=1}^{T} S_{(i,m_i)}(t)\right) + \sum_{t=1}^{T}\lambda(t)\sum_{m=1}^{M_i} p_{(i,m)}(t) \quad (21)$$

subject to (1)-(8) and (10)-(12).

These subproblems are dynamically linearized and solved while avoiding local minima under the monotonicity assumption of AC power flow. In the following assumption, conditions on $\Delta\theta_{i,j}(t)$ will be developed to ensure that (10) is monotonically increasing.

Assumption 1: Differences of phase angles $\Delta\theta_{i,j}(t)$ at nodes *i* and *j* belong to the following interval:

$$\Delta\theta_{i,j}(t) \in \left[\omega_{i,j} - \pi, \omega_{i,j}\right], \quad (22)$$

where $\omega_{i,j} = \arccos\frac{g_{i,j}}{\sqrt{g_{i,j}^2 + b_{i,j}^2}}$. □

When the resistance in the network is relatively low, the interval in (22) is shifted with respect to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to the left only slightly. By using basic trigonometric reduction formulas, it is easy to see that the trigonometric part of (10) is $sin\left(\Delta\theta_{i,j}(t) - \omega_{i,j} + \frac{\pi}{2}\right)$ and it is an increasing function under Assumption 1.

Proposition 1: Under Assumption 1, AC power flow is an increasing function and it can be represented as:

$$f_{i,j}(t) = |V_i(t)||V_j(t)|\sqrt{g_{i,j}^2 + b_{i,j}^2}\sin(\Delta\theta_{i,j}(t) - \omega_{i,j} + \pi/2). \ \square \quad (23)$$

The monotonicity of the function (23) follows from the fact that it is a product of three monotonically increasing functions.

At iteration k, (23) is dynamically linearized by using values $|V_i^{k-1}(t)|$, $|V_j^{k-1}(t)|$ and $\Delta\theta_{i,j}^{k-1}(t)$ obtained at previous iterations. The sine function $sin\left(\Delta\theta_{i,j}(t) - \omega_{i,j} + \frac{\pi}{2}\right)$ is linearized by using linear terms of Taylor series:

$$\begin{aligned} &sin\left(\Delta\theta_{i,j}(t) - \omega_{i,j} + \frac{\pi}{2}\right) \sim sin\left(\Delta\theta_{i,j}^{k-1}(t) - \omega_{i,j} + \frac{\pi}{2}\right) \\ &+ sin\left(\omega_{i,j} - \Delta\theta_{i,j}(t)\right)\left(\Delta\theta_{i,j}(t) - \Delta\theta_{i,j}^{k-1}(t)\right). \end{aligned} \quad (24)$$

The resulting power flow is:

$$|V_i(t)||V_j(t)|\sqrt{(g_{i,j}^2 + b_{i,j}^2)} \times \begin{pmatrix} sin\left(\Delta\theta_{i,j}^{k-1}(t) - \omega_{i,j} + \frac{\pi}{2}\right) \\ + sin\left(\omega_{i,j} - \Delta\theta_{i,j}(t)\right)\left(\Delta\theta_{i,j}(t) - \Delta\theta_{i,j}^{k-1}(t)\right) \end{pmatrix}. \quad (25)$$

Function (25) is then linearized by fixing decision variables in a way that the resulting function is linear. For example, by fixing $|V_i(t)|$, $|V_j(t)|$ at $|V_i^{k-1}(t)|$, $|V_j^{k-1}(t)|$ the linearized power flow becomes:

$$|V_i^{k-1}(t)||V_j^{k-1}(t)|\sqrt{(g_{i,j}^2 + b_{i,j}^2)} \times \begin{pmatrix} sin\left(\Delta\theta_{i,j}^{k-1}(t) - \omega_{i,j} + \frac{\pi}{2}\right) \\ + sin\left(\omega_{i,j} - \Delta\theta_{i,j}(t)\right)\left(\Delta\theta_{i,j}(t) - \Delta\theta_{i,j}^{k-1}(t)\right) \end{pmatrix}. \quad (26)$$

In a similar fashion, other pair-wise fixings will result in linear functions. However, since (25) is a function of three variables, there are three pair-wise fixings possible. Without spending additional iterations to perform such linearizations, (25) is linearized as follows:

$$\begin{aligned} \hat{f}_{i,j}(t) = &\frac{|V_i^{k-1}(t)||V_j^{k-1}(t)|\sqrt{(g_{i,j}^2 + b_{i,j}^2)} \times \begin{pmatrix} sin\left(\Delta\theta_{i,j}^{k-1}(t) - \omega_{i,j} + \frac{\pi}{2}\right) \\ + sin\left(\omega_{i,j} - \Delta\theta_{i,j}(t)\right)\left(\Delta\theta_{i,j}(t) - \Delta\theta_{i,j}^{k-1}(t)\right) \end{pmatrix}}{3} + \\ &\frac{|V_i^{k-1}(t)||V_j(t)|\sqrt{g_{i,j}^2 + b_{i,j}^2}\, sin\left(\Delta\theta_{i,j}^{k-1}(t) - \omega_{i,j} + \frac{\pi}{2}\right)}{3} + \\ &\frac{|V_i(t)||V_j^{k-1}(t)|\sqrt{g_{i,j}^2 + b_{i,j}^2}\, sin\left(\Delta\theta_{i,j}^{k-1}(t) - \omega_{i,j} + \frac{\pi}{2}\right)}{3}. \end{aligned} \quad (27)$$

The linearized subproblems can then be formulated as:

$$\min \sum_{m=1}^{M_i}\left(\sum_{t=1}^{T} c_{(i,m_i)}\left(p_{(i,m_i)}(t), t\right) + \sum_{t=1}^{T} S_{(i,m_i)}(t)\right) + \sum_{t=1}^{T}\lambda(t)\sum_{m=1}^{M_i} p_{(i,m)}(t), \quad (28)$$

s.t. (1)-(8), (10)-(12), and

$$-f_{(i,j)max} \leq \hat{f}_{i,j}(t) \leq f_{(i,j)max}. \quad (29)$$

After iteration k, the fixed values $V^{k-1}(t)$ and $\theta^{k-1}(t)$ will be dynamically updated with $V^k(t)$ and $\theta^k(t)$.

C. Obtaining Feasible Solutions

To obtain feasible solutions, UC binary decision variables are fixed and the original problem is solved subject to all the constraints of the original problem, but instead of the AC power flow (10), equation (27) is used. After such a problem is solved, the solution is generally infeasible because resulting values corresponding to decision variables $|V_i(t)|$, $|V_j(t)|$ and $\Delta\theta_{i,j}(t)$ are different from $|V_i^{k-1}(t)|$, $|V_j^{k-1}(t)|$ and $\Delta\theta_{i,j}^{k-1}(t)$. To obtain feasible solutions, these decision variables are reset to the newly obtained values. If a solution is not found, the way multipliers are updated stays the same, but to improve convergence, stepsize re-initialization can be used along the lines of [12], and feasible solutions are searched again in a few iterations.

V. NUMERICAL TESTING

The new method is implemented by using CPLEX 12.6.0 on 64-bit windows by using a laptop with the processor Intel® Core™ i7-4910MQ CPU @ 2.90GHz, 16 GB of RAM.

Example: IEEE 30-bus system [15].

To test the new method, consider the IEEE 30-bus system that consists of 30 buses (I = 30) and 41 transmission lines (L = 41). The original data are modified so that each bus numbered 1 through 10 has exactly one combined cycle unit (M_i = 1), and each of the buses 11 and 12 has exactly one conventional generator. The following Figure 2 presents results for the problem by modeling the piece-wise cost curve by using 5 blocks of energy.

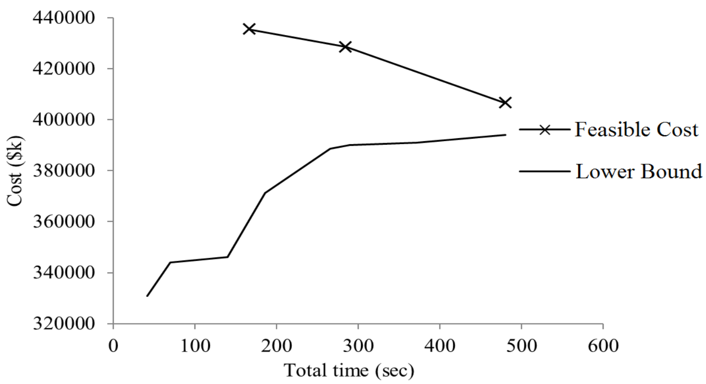

Figure 2. Solution profile for 30-bus system with 10 CC and 2 conventional units and AC power flow. Feasible costs are marked by the cross ×

As demonstrated in Figure 2, the new method obtains a feasible solution with 2.47% duality gap within 10 minutes. As motivated earlier, the efficiency of the new method is guaranteed because the complexity of each subproblem is drastically reduced, and dynamically linearized subproblems can be solved by branch-and-cut efficiently and without affecting the solution process of the entire problem. Furthermore, because of efficient coordination subproblem solutions by Lagrange multipliers, the overall method is efficient.

The comparison with widely-used branch-and-cut is impossible because the method requires the linearity of the problem formulation. Nevertheless, since most existing decomposition methods involve solving the MILP master problem, branch-and-cut can be used to solve the master problem. A representative decomposition method chosen for comparison is Benders decomposition [4]. As argued in Section II, branch-and-cut can suffer from slow convergence when CC units are present. Numerical results indicate that when solving the MILP master by branch-and-cut, the algorithm terminates after 2 hours and 49 minutes after running out of memory with the gap 8.7%.

Another comparison is based on the dynamic linearization of the original problem by using linear terms of Taylor series expansion of the AC power flow. The resulting problem is solved by using branch-and-cut. However, as discussed before, because of the difficulties mentioned before, branch-and-cut terminated after 2 hours and 46 minutes with the gap 9.17%.

VI. CONCLUSION

In this paper, an important but difficult UCED problem with AC power flow is considered, in which CC units are modeled to represent actual transitions among CC states accurately. While this accurate modeling of CC units introduces computational difficulties for existing methods, because of the efficient decomposition and coordination as well as efficient dynamic linearization, the complexity is drastically reduced and new method is very efficient.

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