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with Demand Dependent Unit Cost of Production under Two Constraints

# A Fuzzy Non-Linear Programming Economic Order Quantity Model with Demand Dependent Unit Cost of Production under Two Constraints

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A fuzzy NLP EOQ model is developed with demand dependent unit cost of production and dynamic setup cost under limited capital investment and limited storage capacity. Here fuzziness is introduced in objective function, capital system and storage area. It is solved by fuzzy non-linear technique for linear membership functions. This paper allows the modification of the Single item EOQ model in presence of fuzzy decision making process for demand dependent unit cost of production in the presence of imprecisely estimated parameters. The model is developed for the problem by employing different modeling approaches over an infinite planning horizon. It incorporates all concepts of a fuzzy arithmetic approach, the quantity ordered and the demand per unit compares both fuzzy non linear and other models. Computational algorithm using the LINGO 13.0 version software is developed to find the optimal solution and the diagrammatical representations can be obtained by MATLAB 7.8.0. (R2009a) version software. Investigation of the properties of an optimal solution allows developing an algorithm whose validity is illustrated through an example problem. Sensitivity analysis of the optimal solution is also studied with respect to changes in different parameter values and to draw managerial insights. By a comparative study of a numerical example, it demonstrates the efficiency of the available formulae in the literature to highlight the optimality of the solution technique satisfying two constraints.

**Keywords:** Fuzzy, NLP, EOQ, Budget Constraint, Storage Capacity

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## I. INTRODUCTION

Since its formulation in 1915, the square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has

been widely used and accepted by many industries, some practitioners have questioned its practical application. For several years, classical EOQ problems with different variations were solved by many researchers and had be separated in reference books and survey papers e.g. Taha [5], Urgeletti [3]. Recently, for a single product with demand related to unit price Cheng [21] and for multi products with several constraints. His

treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

Operations Research (OR) was first coined in 1940 by McClosky and Trefther in a small town, Bowdsey, in the UK. During the Second World War, this OR mathematics was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then the subject has been enlarged in importance in the field of Economics, Management Sciences, Public Administration, Behavioral Science, Social Work Commerce Engineering and different branches of Mathematics etc. But various Paradigmatic changes in science and mathematics concern the concept of uncertainty. In Science, this change has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for certainty in all its manifestations; hence uncertainty is regarded as unscientific. According to the modern view, uncertainty is considered essential to science; it is not any an unavoidable plague but has; in fact, a great utility. But to tackle non-random uncertainty no other mathematics was developed other than fuzzy set theory and showed the intention to accommodate uncertainty in the presence of random variables. Following Zadeh [11], significant contributions in this direction have been applied in many fields including production related areas. Consequently investment in introducing fuzzy is the key to avoid uncertain decision space. Many studies have modified inventory policies by considering the issues of nonrandom uncertain and fuzzy based EOQ models. Vujosevic et al. [14] presented a theoretical EOQ formula when inventory cost is fuzzy. Lee et al. [7] studied an inventory model for fuzzy demand quantity and fuzzy production quantity.

Tripathy et al. [16, 18, 19] introduced the concept and developed the framework for investing fuzzy in holding cost and setup cost in EOQ model. Tripathy et al. [17] suggested improvements to production systems by employing entropy in the fuzzy model. Pattnaik [12] extends by considering stock dependent demand rate with entropy factor in the crisp non linear EOQ model.

Sommer [2] applied fuzzy dynamic programming to an inventory and production scheduling problem in which the management wishes to fulfill a contract for providing a product and then withdraw from the market. Kacprzyk et al. [9] introduced the determination of optimal of firms from a global view point of top management in a fuzzy environment with fuzzy constraints improved on reappointments and a fuzzy goal for preferable inventory levels to be attained. Park [10] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Here, inventory costs were represented by trapezoidal fuzzy numbers (TrFN) and the EOQ model was transformed to a fuzzy optimization problem. Tripathy et al. [15] introduce the concept of promotional effort for deteriorating items in crisp instantaneous EOQ model. Pattnaik [13] extends concept of promotional effort for deteriorating items in fuzzy instantaneous replenishment model.

But Roy et al. [22], Roy et al. [23] have considered the space constraint with the objective goal in fuzzy environment and attacked the fuzzy optimization problem directly using either fuzzy non-linear or fuzzy geometric programming technique similarly Lee et al. [7] and Vujosevic et al. [14] have applied fuzzy arithmetic approach in EOQ model without constraints.

**TABLE 1. SUMMARY OF THE RELATED RESEARCHES**

| Authors                 | Demand   | Setup cost | Holding cost                                | Unit cost of production | Constraints                   | Planning horizon | Structure of the Model | Model class     |
|-------------------------|----------|------------|---|-------------------------|-------------------------------|------------------|------------------------|-----------------|
| Vujosevic et al. (1996) | Constant | Constant   | $\frac{\bar{c}_h c_p Q}{2 \times 100}$      | Constant                | No                            | Finite           | Fuzzy                  | Defuzzification |
| Tripathy et al. (2009)  | Constant | Constant   | $\frac{Hr^2 q^2}{2\lambda}$                 | Reliability and demand  | Reliability                   | Infinite         | Fuzzy                  | NLP             |
| Tripathy et al. (2011)  | Constant | Constant   | $\frac{H\lambda q^2}{2r^2}$                 | Reliability and demand  | Reliability                   | Infinite         | Fuzzy                  | NLP             |
| Tripathy et al. (2011)  | Constant | Constant   | $\frac{Hq^2}{2r^2\lambda}$                  | Reliability and demand  | Reliability                   | Infinite         | Fuzzy                  | NLP             |
| Roy et al. (1995)       | Constant | Variable   | $\frac{1}{2} C_1 q$                         | No                      | Space                         | Infinite         | Fuzzy                  | NLP             |
| Roy et al. (1997)       | Constant | Variable   | $\frac{1}{2} C_1 q$                         | Demand                  | Space                         | Infinite         | Fuzzy                  | NLP, GPP        |
| Present paper (2015)    | Constant | Variable   | $\frac{1}{2 \times 100} C_1 K D^{-\beta} q$ | Demand                  | Budget and Storage Capacities | Infinite         | Fuzzy                  | NLP             |

In this paper a single item EOQ model is developed where unit price varies inversely with demand and setup cost increases with the increase of production. In company or industry, total expenditure for production and storage area are normally limited but imprecise, uncertain, non-specificity, inconsistency vagueness and flexible. These are defined within some ranges. However, the no stochastic and ill formed inventory models can be realistically represented in the fuzzy environment. The problem is reduced to a fuzzy optimization problem associating fuzziness with the storage area and total expenditure. The optimum order quantity is evaluated by both fuzzy non linear programming (FNLP) method and the results are obtained for linear membership functions. The model is illustrated with numerical example and with the variation in tolerance limits for both shortage area and total

expenditure. A sensitivity analysis is presented. The numerical results for fuzzy and crisp models are compared. The remainder of this paper is organized as follows. In section 2, assumptions and notations are provided for the development of the model and the mathematical model is developed. In section 3, mathematical analysis of fuzzy non linear programming (FNLP) is formulated. The solution of the FNLP inventory is derived in section 4. The numerical example is presented to illustrate the development of the model in section 5. The sensitivity analysis is carried out in section 6 to observe the changes in parameters in the optimal solution. Finally section 7 deals with the summary and the concluding remarks.

## II. MATHEMATICAL MODEL

A single item inventory model with demand dependent unit price and variable setup cost under storage constraint is formulated as

$$\begin{aligned} & \text{Min } C(D, q) \\ & = C_{03}q^{v-1}D + KD^{1-\beta} + \frac{1}{2 \times 100} C_1 KD^{-\beta} q \\ & \text{s.t. } \frac{1}{2} uq \leq U \\ & Aq \leq B \\ & \forall D, q > 0 \end{aligned} \tag{1}$$

Where,

- q = number of order quantity,
- D = demand per unit time
- C<sub>1</sub> = holding cost per item per unit time.
- C<sub>3</sub> = Setup cost = C<sub>03</sub> q<sup>v</sup>,  
(C<sub>03</sub> (> 0) and v (0 < v < 1) are constants)

p = Unit production cost = KD<sup>-β</sup>, K (> 0) and β (> 1) are constants. Here lead time is zero, no back order is permitted and replenishment rate is infinite. U, u, A and B are nonnegative real numbers, U is the capital investment goal and B is the space constraint goal. The above model in a fuzzy environment is

$$\begin{aligned} & \widetilde{\text{Min}} C(D, q) \\ & = C_{03}q^{v-1}D + KD^{1-\beta} + \frac{1}{2 \times 100} C_1 KD^{-\beta} q \\ & \text{s.t. } \frac{1}{2} uq \leq \widetilde{U} \\ & Aq \leq \widetilde{B} \\ & \forall D, q > 0 \end{aligned} \tag{2}$$

(A wavy bar (~) represents fuzzification of the parameters).

### III. MATHEMATICAL ANALYSIS OF FUZZY NON LINEAR PROGRAMMING (FNLP)

A fuzzy non linear programming problem with fuzzy resources and objective are defined as

$$\begin{aligned} & \widetilde{\text{Min}} g_0(x) \\ & \text{s.t. } g_i(x) \leq \widetilde{b}_i \quad i=1, 2, 3, \dots, m \\ & g_j(x) \leq \widetilde{u}_j \quad j=1, 2, 3, \dots, m \end{aligned} \tag{3}$$

In fuzzy set theory, the fuzzy objective and fuzzy resources are obtained by their membership functions, which may be linear or nonlinear. Here μ<sub>0</sub> and μ<sub>i</sub> (i = 1, 2, ..., m) are assumed to be non increasing continuous linear membership functions for objective and resources respectively such as

$$\mu_i(g_i(x)) = \begin{cases} \mu_i(g_i(x)) & \\ 1 & \text{if } g_i(x) < b_i, \\ 1 - \frac{g_i(x) - b_i}{P_i} & \text{if } b_i \leq g_i(x) \leq b_i + P_i, \\ 0 & \text{if } g_i > b_i + P_i, \end{cases}$$

i = 0, 1, 2, ..., m.

$$\mu_j(g_j(x)) = \begin{cases} \mu_j(g_j(x)) & \\ 1 & \text{if } g_j(x) < u_j, \\ 1 - \frac{g_j(x) - u_j}{P_j} & \text{if } u_j \leq g_j(x) \leq u_j + P_j, \\ 0 & \text{if } g_j > u_j + P_j, \end{cases}$$

j = 0, 1, 2, ..., m

In this formulation, the fuzzy objective goal is b<sub>0</sub> and its corresponding tolerance is P<sub>0</sub> and for the fuzzy constraints, the goals are b<sub>i</sub>'s and their corresponding tolerances are P<sub>i</sub>'s (i = 1, 2, ..., m). To solve the problem (3), the max - min operator of Bellman et al. [20] and the approach of Zimmermann [6] are implemented.

The membership function of the decision set, μ<sub>D</sub>(x), is μ<sub>D</sub>(x) = min {μ<sub>0</sub>(x), μ<sub>1</sub>(x), ..., μ<sub>m</sub>(x)}, ∀ x ∈ X.

The min operator is used here to model the intersection of the fuzzy sets of objective and constraints. Since the decision maker wants to have a crisp decision proposal, the maximizing decision will correspond to the value of x, x<sub>max</sub> that has the highest degree of membership in the decision set.

$\mu_D (x_{max})$   
 $= \max_{x \geq 0} [\min \{ \mu_0 (x), \mu_1 (x) \dots, \mu_m (x) \}]$ . It is equivalent to solving the following crisp non linear programming problem.

$$\begin{aligned} & \text{Max } \alpha \\ & \text{s.t. } \mu_0 (x) \geq \alpha \\ & \mu_i(x) \geq \alpha \quad (i= 1, 2, \dots, m) \\ & \forall x \geq 0, \alpha \in (0, 1) \end{aligned} \tag{4}$$

A new function, i.e the Lagrangian function  $L (\alpha, x, \lambda)$  is formed by introducing  $(m + 1)$  Lagrangian multipliers  $\lambda = (\lambda_0, \lambda_1, \dots, \lambda_m)$ .

$L (\alpha, x, \lambda) = \alpha - \sum_{i=0}^m \lambda_i (g_i(x) - b_i - (1 - \alpha)P_i) - \sum_{i=0}^m \lambda_i (g_i(x) - u_i - (1 - \alpha)P_i)$ . The necessary condition of Kuhn et al. [8] for the optimal solution to this problem implies that optimal values  $x_1^*, x_2^*, x_3^*, \dots, x_n^*$  and  $\lambda_1^*, \lambda_2^*, \lambda_3^*, \dots, \lambda_n^*$  should satisfy

$$\begin{aligned} \frac{\partial L}{\partial x_j} &= 0 \\ j &= 1, 2, \dots, n \\ \frac{\partial L}{\partial \alpha} &= 0 \\ \lambda_i (g_i(x) - b_i - (1 - \alpha)P_i) &= 0 \\ \lambda_i (g_i(x) - u_i - (1 - \alpha)P_i) &= 0, \\ g_i(x) &\leq b_i + (1 - \alpha)P_i, \\ g_i(x) &\leq u_i + (1 - \alpha)P_i, \\ \lambda_i &\leq 0, i = 0, 1, \dots, m \end{aligned} \tag{5}$$

Moreover, Kuhn-Tucker's sufficient condition demands that the objective function for maximization and the constraints should be respectively concave and convex. In this formulation, it can be shown that both objective function and constraints satisfy the

required sufficient conditions. Now, solving (5), the optimal solution for the FNLP problem is obtained.

#### IV. SOLUTION OF THE PROPOSED INVENTORY MODEL

The proposed inventory model depicted by equation (2)

$$\begin{aligned} & \widetilde{Min} C (D, q) \\ & = C_{03}q^{v-1}D + KD^{1-\beta} + \frac{1}{2 \times 100} C_1KD^{-\beta}q \\ & \text{s.t. } \frac{1}{2} uq \leq \tilde{U} \\ & Aq \leq \tilde{B} \\ & \forall D, q > 0, \text{ reduces to following equation (4),} \\ & \text{Max } \alpha \\ & \text{s.t. } C_{03}q^{v-1}D + KD^{1-\beta} + \frac{1}{2 \times 100} C_1KD^{-\beta}q \\ & \leq C_0 + (1 - \alpha) P_0, \\ & \frac{1}{2} uq \leq U + (1 - \alpha)P_1 \\ & Aq \leq B + (1 - \alpha)P_2, \\ & \forall D, q > 0 \ \& \ \alpha \in (0, 1) \end{aligned} \tag{6}$$

Here, the objective goal is  $C_0$  with tolerance  $P_0$  and the capital investment constraint goal with tolerance  $P_1$  and space constraint goal is  $B$  with tolerance  $P_2$ . So, the corresponding Lagrangian function is

$$\begin{aligned} L (\alpha, D, q, \lambda_1, \lambda_2, \lambda_3) \\ = \alpha - \lambda_1 \left( C_{03}q^{v-1}D + KD^{1-\beta} + \frac{1}{2 \times 100} C_1KD^{-\beta}q - C_0 - (1 - \alpha)P_0 \right) \\ - \lambda_2 \left( \frac{1}{2}uq - U - (1 - \alpha)P_1 \right) - \lambda_3 (Aq - B - (1 - \alpha)P_2) \end{aligned}$$

From Kuhn - Tucker's necessary conditions,

$$\begin{aligned} \frac{\partial L}{\partial \alpha} = 0, \frac{\partial L}{\partial D} = 0, \frac{\partial L}{\partial q} = 0, \frac{\partial L}{\partial \lambda_1} = 0, \frac{\partial L}{\partial \lambda_2} = 0, \\ \frac{\partial L}{\partial \lambda_3} = 0, \forall \lambda_1, \lambda_2, \lambda_3 \leq 0 \end{aligned}$$

$$\frac{\partial L}{\partial \alpha} = 1 - \lambda_1 P_0 - \lambda_2 P_1 - \lambda_3 P_2 \geq 0$$

$$\frac{\partial L}{\partial D} = \lambda_1 \left( C_{03} q^{v-1} + (1 - \beta) K D^{-\beta} - \frac{1}{2 \times 100} C_1 K q \beta^{-(\beta+1)} D \right) \leq 0$$

$$\frac{\partial L}{\partial q} = \lambda_1 \left( C_{03} (v - 1) q^{(v-2)} D + \frac{1}{2 \times 100} C_1 K D^{-\beta} \right) + \frac{u}{2} \lambda_2 + A \lambda_3 \leq 0$$

$$\frac{\partial L}{\partial \lambda_1} = \left( C_{03} q^{(v-1)} D + K D^{1-\beta} + \frac{1}{2 \times 100} C_1 K D^{-\beta} q \right) - C_0 - (1 - \alpha) P_0 \geq 0$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1}{2} u q - U - (1 - \alpha) P_1 \geq 0$$

$$\frac{\partial L}{\partial \lambda_3} = A q - B - (1 - \alpha) P_2 \geq 0$$

and  $\alpha(1 - \lambda_1 P_0 - \lambda_2 P_1 - \lambda_3 P_2) = 0$

$$\lambda_1 D \left( C_{03} q^{(v-1)} + (1 - \beta) K D^{-\beta} - \frac{1}{2 \times 100} C_1 K q \beta^{-(\beta+1)} D \right) = 0$$

$$\lambda_1 q \left( C_{03} (v - 1) q^{(v-2)} D + \frac{1}{2 \times 100} C_1 K D^{-\beta} \right) + \frac{u}{2} \lambda_2 q + A \lambda_3 q = 0$$

$$\lambda_1 \left( C_{03} q^{(v-1)} D + K D^{1-\beta} + \frac{1}{2 \times 100} C_1 K D^{-\beta} q - C_0 - (1 - \alpha) P_0 \right) = 0$$

$\lambda_2 (A q - B - (1 - \alpha) P_1) = 0, \forall \alpha, D, q \geq 0$   
and  $\forall \lambda_1, \lambda_2, \lambda_3 \leq 0$ , solving these equations, optimum quantities are

$$q = \frac{B + (1 - \alpha) P_2}{A} = \frac{2(U + (1 - \alpha) P_1)}{u}$$

$$D^* = \left[ \frac{C_{03} q^{(v-1)} \pm C_{03}^2 q^{2(v-1)} - \frac{4K^2 C_1 q \beta^{(\beta-1)}}{2 \times 100}}{2(1 - \beta)K} \right]^{-1/\beta}$$

$q = f(\alpha)$  and  $D = f(q)$  where  $\alpha^*$  is a root of  $K \beta D^{*(1-\beta)} + \frac{1}{2 \times 100} C_1 K q^* D^{*-\beta} (1 + \beta) - C_0 - (1 - \alpha^*) P_0 = 0$

$$C^*(D^*, q^*) = C_{03} q^{*v-1} D^*$$

$$+ K D^{*1-\beta} + \frac{1}{2 \times 100} C_1 K D^{*-\beta} q^*$$

So, by both FNLP and NLP techniques, the optimal values of  $q^*$  and  $D^*$  and the corresponding minimum cost are evaluated for the known values of other parameters.

## V. NUMERICAL EXAMPLE

For a particular EOQ problem, let  $C_{03} = \$4$ ,  $K = 100$ ,  $C_1 = \$2$ ,  $v = 0.5$ ,  $\beta = 1.5$ ,  $u = \$0.5$ ,  $U = \$3.5$ ,  $A = 5$  units,  $B = 90$  units,  $C_0 = \$40$  and  $P_0 = \$20$  and  $P_1 = \$15$  and  $P_2 = 25$  units. For these values the optimal value of productions batch quantity  $q^*$ , optimal demand rate  $D^*$ , minimum average total cost  $C^*$  ( $D^*$ ,  $q^*$ ) and  $Aq^*$  obtained by FNLP are given in Table 2.

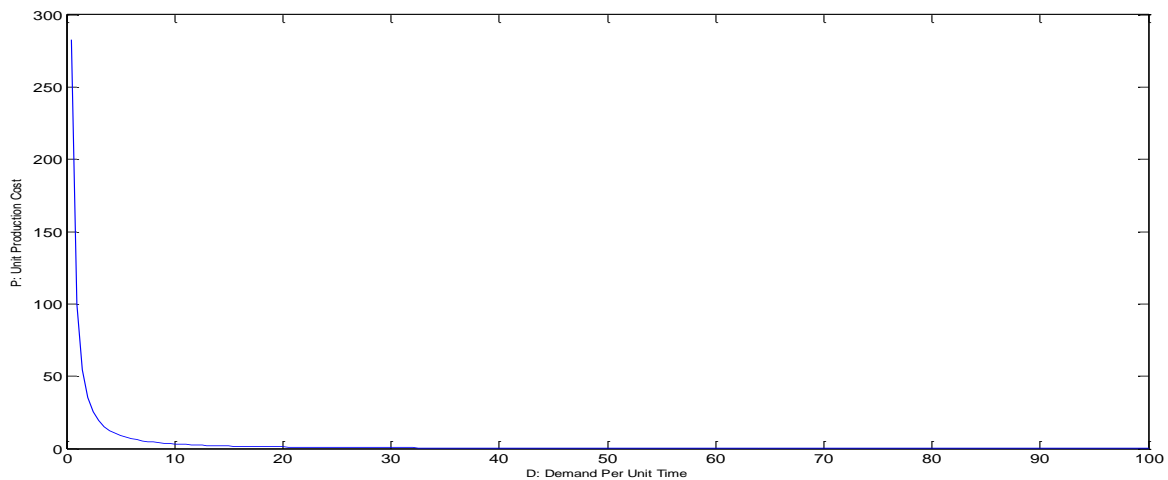
After 26 iterations Table-2 reveals the optimal replenishment policy for single item with demand dependent unit cost and dynamic setup cost. In this table the optimal numerical results of fuzzy model are compared with the results of crisp model and fuzzy model of Roy et al. (1997). The optimum replenishment quantity  $q^*$  and  $Aq^*$  are both 229.8354% and 82.45885% more than that of other crisp model and fuzzy model respectively, the optimum quantity demand  $D^*$  is 14.03017 but 9.21 and 9.81 for comparing models, hence 52.34% and 42.9972% more from the other crisp model and from the fuzzy model respectively. The minimum total average cost  $C^*(D^*, q^*)$  is 40.83059 but 54.43 and 53.93 comparing models, hence -24.98513% and -24.2930% less from other crisp and fuzzy models respectively. It permits the better use of present fuzzy model as compared to the crisp model and other fuzzy model. The results are justified and agree with the present model. It indicates the consistency of the fuzzy space of EOQ model from other models.

Fig. 1 represents the relationship between demand per unit time  $D$  and unit cost of production  $P$ . Similarly Fig. 2 shows the relationship between number of order quantity

q and variable setup cost  $C_3$  and Fig. 3 depicts number of order quantity q and average total the mesh plot of demand per unit time D, cost C.

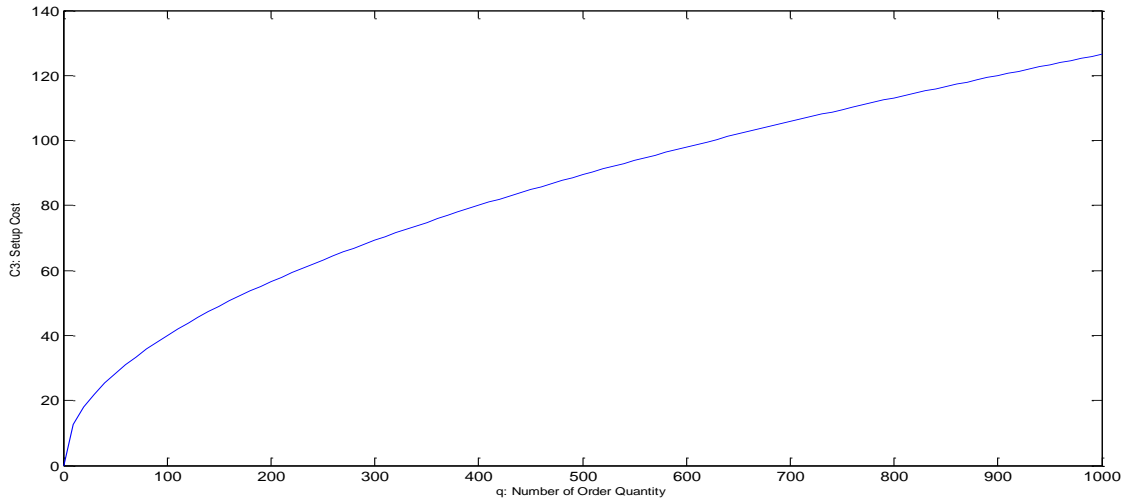
**TABLE 2. OPTIMAL VALUES FOR THE PROPOSED INVENTORY MODEL**

| Model                          | Method | Iteration | $q^*$     | $D^*$    | $C^*(D^*, q^*)$ | $\alpha^*$ | $\frac{1}{2}uq^*$ | $Aq^*$   |
|--------------------------------|--------|-----------|-----------|----------|-----------------|------------|-------------------|----------|
| Fuzzy model                    | FNLP   | 26        | 16.49177  | 14.03017 | 40.83059        | 0.9584705  | 4.1229425         | 82.45885 |
| Crisp model, Roy et al. (1997) | NLP    | -         | 5         | 9.21     | 54.43           | 1          | -                 | 50       |
| % Change                       | -      | -         | 229.8354  | 52.3363  | -24.98513       | -4.15295   | -                 | 64.9177  |
| Fuzzy model, Roy et al. (1997) | FNLP   | -         | 6.0449    | 9.8115   | 53.9324         | 0.3033     | -                 | 60.449   |
| % Change                       | -      | -         | 172.82122 | 42.9972  | -24.2930        | 216.0140   | -                 | 36.4106  |

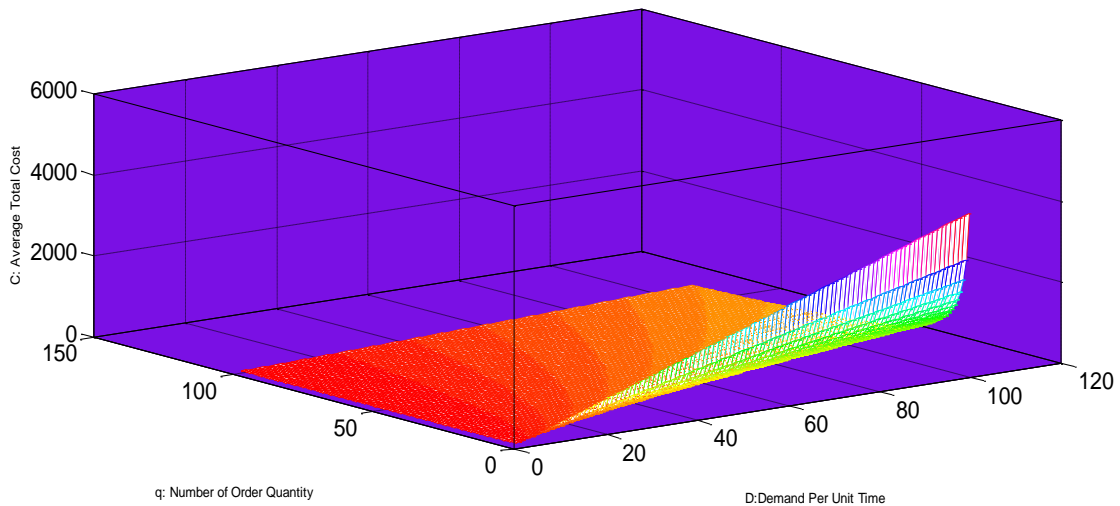


**FIGURE 1. DEMAND PER UNIT TIME D AND UNIT PRODUCTION COST P**

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**FIGURE 2. NUMBER OF ORDER QUANTITY  $q$  AND DYNAMIC SETUP COST  $C_3$**



**FIGURE 3. MESH PLOT OF DEMAND PER UNIT TIME  $D$ , NUMBER OF ORDER QUANTITY  $q$  AND AVERAGE TOTAL COST  $C$**



VI. SENSITIVITY ANALYSIS

Now the effect of changes in the system parameters on the optimal values of  $q$ ,  $D$ ,  $C$  ( $D, q$ ) and  $\frac{1}{2}uq$  and  $Aq$  when only one parameter changes and others remain unchanged the computational results are described in Table 3. As a result

$$\alpha^*, q^*, D^*, C^*(D^*, q^*), \frac{1}{2}uq^*$$

and  $Aq^*$  are less sensitive to the parameters  $P_0, P_1$  and  $P_2$ . Following Dutta et al. [1] and Hamacher et al. [4] it is observed that the effect of tolerance in the said EOQ model with the earlier numerical values and construct Table 3 for the degrees of violation  $T_0 (= (1 - \alpha)P_0)$ ,  $T_1 (= (1 - \alpha)P_1)$  and  $T_2 (= (1 - \alpha)P_2)$  for two constraints given by equation (6).

From Table 3, it is seen that: (i) For higher tolerances of  $P_0$ , the value of  $\alpha_{max}$  does not achieve 1, (ii) For higher acceptable variations  $P_0$ , the optimal solutions remain invariant and the optimal solutions are very close to the solutions ( $q^* = 16.49177, D^* = 14.03017, C^*(D^*, q^*) = 40.83059, \frac{1}{2}uq^* = 4.1229425$  and  $Aq^* = 82.45885$ ) of fuzzy model and ( $q^* = 5, D^* = 9.21, C^*(D^*, q^*) = 54.43$  and  $Aq^* = 50$ ) of the crisp model without tolerance ( $\alpha = 1$ ) respectively.

From Table 3 it is shown that: (i) For different values of  $P$ , degrees of violations  $T_0, T_1$  and  $T_2$  are never zero, i.e. different optimal solutions are obtained. (ii) As  $P_0, P_1$  and  $P_2$  increase from original values, the minimum average cost  $C^*(D^*, q^*)$  increases, decreases and remaining constant respectively, but  $q^*$  and  $D^*$  increase, decrease and remain stable respectively.

TABLE 3. SENSITIVITY ANALYSIS OF THE PARAMETERS  $P_0, P_1$  and  $P_2$

| P     | Value | Iteration | $\alpha^*$ | $q^*$  | $D^*$  | $T_0$  | $T_1$  | $T_2$  | $C^*(D^*, q^*)$ | $\frac{1}{2}uq^*$ | $Aq^*$  |
|-------|-------|-----------|------------|--------|--------|--------|--------|--------|-----------------|-------------------|---------|
|       | 25    | 24        | 0.965      | 16.239 | 13.955 | 0.9328 | 0.5597 | 0.9328 | 40.93276        | 4.0598            | 81.193  |
|       | 50    | 27        | 0.975      | 15.494 | 13.729 | 1.2447 | 0.3734 | 0.6224 | 41.24472        | 3.8735            | 77.468  |
| $P_0$ | 100   | 30        | 0.985      | 14.903 | 13.545 | 1.5049 | 0.2257 | 0.3762 | 41.50494        | 3.7258            | 74.515  |
|       | 200   | 26        | 0.992      | 14.506 | 13.418 | 1.6868 | 0.1265 | 0.2108 | 41.68675        | 3.6265            | 72.530  |
|       | 1000  | 29        | 0.998      | 14.112 | 13.291 | 1.8729 | 0.0281 | 0.0468 | 41.87295        | 3.5280            | 70.562  |
|       | 16    | 34        | 0.9599     | 16.565 | 14.051 | 0.8015 | 0.6412 | 1.0019 | 40.80148        | 4.1413            | 82.8237 |
|       | 20    | 36        | 0.9648     | 16.814 | 14.125 | 0.7034 | 0.7034 | 0.8792 | 40.70338        | 4.2035            | 84.0675 |
| $P_1$ | 23    | 38        | 0.9678     | 16.965 | 14.169 | 0.6445 | 0.7412 | 0.8057 | 40.64453        | 4.2413            | 84.8242 |
|       | 38    | 46        | 0.9772     | 17.462 | 14.312 | 0.4555 | 0.8655 | 0.5694 | 40.45552        | 4.3655            | 87.3096 |
|       | 40    | 45        | 0.9781     | 17.508 | 14.325 | 0.4385 | 0.8769 | 0.5481 | 40.43845        | 4.3770            | 87.5382 |
|       | 30    | 35        | 0.9585     | 16.492 | 14.030 | 0.8306 | 0.6230 | 1.2459 | 40.83059        | 4.1230            | 82.4589 |
|       | 40    | 29        | 0.9585     | 16.492 | 14.030 | 0.8306 | 0.6230 | 1.6612 | 40.83059        | 4.1230            | 82.4589 |
| $P_2$ | 50    | 29        | 0.9585     | 16.492 | 14.030 | 0.8306 | 0.6230 | 2.0765 | 40.83059        | 4.1230            | 82.4589 |
|       | 80    | 24        | 0.9585     | 16.492 | 14.030 | 0.8306 | 0.6230 | 3.3224 | 40.83059        | 4.1230            | 82.4589 |
|       | 100   | 26        | 0.9585     | 16.492 | 14.030 | 0.8306 | 0.6230 | 4.1530 | 40.83059        | 4.1230            | 82.4589 |

**TABLE 4. SENSITIVITY ANALYSIS OF THE PARAMETERS  
u, U, A, B, C<sub>1</sub>, C<sub>03</sub> AND K**

| Parameter             | Value | Iteration | D*       | q*       | C*(D*, q*) | % Change in C*(D*, q*) | $\frac{1}{2}uq^*$ | Aq*      |
|-----------------------|-------|-----------|----------|----------|------------|------------------------|-------------------|----------|
| <b>u</b>              | 1     | 27        | 12.46570 | 11.72621 | 43.150851  | -5.3770921             | 5.863105          | 58.63105 |
|                       | 2     | 27        | 10.91410 | 7.967578 | 45.95677   | -11.154352             | 7.967578          | 39.83789 |
|                       | 3     | 26        | 10.04213 | 6.245959 | 47.82525   | -14.625454             | 9.368939          | 31.2298  |
|                       | 5     | 25        | 8.999078 | 4.526525 | 50.42175   | -19.021871             | 11.316313         | 22.63263 |
|                       | 10    | 28        | 7.700481 | 2.858345 | 54.38897   | -24.928547             | 14.291725         | 14.29173 |
| <b>U</b>              | 4     | 31        | 14.29954 | 17.41700 | 40.47233   | 0.88519737             | 4.35425           | 87.085   |
|                       | 5     | 29        | 14.48128 | 18.05917 | 40.23666   | 1.47609170             | 4.5147925         | 90.29585 |
|                       | 20    | 29        | 14.48128 | 18.05917 | 40.23666   | 1.47609170             | 4.5147925         | 90.29585 |
|                       | 30    | 29        | 14.48128 | 18.05917 | 40.23666   | 1.47609170             | 4.5147925         | 90.29585 |
|                       | 50    | 29        | 14.48128 | 18.05917 | 40.23666   | 1.47609170             | 4.5147925         | 90.29585 |
| <b>A</b>              | 10    | 23        | 11.62476 | 9.574816 | 44.59853   | -8.4485744             | 2.90619           | 47.87408 |
|                       | 15    | 25        | 10.24085 | 6.614806 | 47.37767   | -13.818915             | 2.5602125         | 33.07403 |
|                       | 30    | 24        | 8.265069 | 3.522763 | 52.54631   | -22.295899             | 2.06626725        | 17.61382 |
|                       | 50    | 27        | 7.068062 | 2.217903 | 56.71611   | -28.0088321            | 1.7670155         | 11.08952 |
|                       | 60    | 30        | 6.685926 | 1.880896 | 58.28301   | -29.9442668            | 1.6714815         | 9.40448  |
| <b>B</b>              | 150   | 29        | 14.03017 | 16.49177 | 40.83059   | 0                      | 3.5075425         | 82.45885 |
|                       | 200   | 27        | 14.03017 | 16.49177 | 40.83059   | 0                      | 3.5075425         | 82.45885 |
|                       | 250   | 26        | 14.03017 | 16.49177 | 40.83059   | 0                      | 3.5075425         | 82.45885 |
|                       | 400   | 26        | 14.03017 | 16.49177 | 40.83059   | 0                      | 3.5075425         | 82.45885 |
|                       | 1000  | 26        | 14.03017 | 16.49177 | 40.83059   | 0                      | 3.5075425         | 82.45885 |
| <b>C<sub>1</sub></b>  | 3     | 26        | 14.24903 | 16.70619 | 40.74676   | 0.2057341492           | 3.5622575         | 83.53095 |
|                       | 4     | 32        | 14.46739 | 16.92149 | 40.66632   | 0.4039460664           | 3.6168475         | 84.60745 |
|                       | 5     | 27        | 14.68532 | 17.13767 | 40.5891    | 0.594962687            | 3.67133           | 85.68835 |
|                       | 7     | 25        | 15.12015 | 17.57268 | 40.44368   | 0.956663686            | 3.7800375         | 87.8634  |
|                       | 10    | 28        | 15.77063 | 18.23174 | 40.24615   | 1.452163747            | 3.9426575         | 91.1587  |
| <b>C<sub>03</sub></b> | 5     | 22        | 12.70034 | 18.78197 | 40.19735   | 1.575327727            | 3.175085          | 93.90985 |
|                       | 6     | 32        | 11.41916 | 19.41162 | 40.46288   | 0.908758843            | 2.85479           | 97.0581  |
|                       | 7     | 26        | 10.44439 | 19.97335 | 40.88243   | -0.126802638           | 2.6110975         | 99.86675 |
|                       | 9     | 27        | 9.043628 | 20.95129 | 41.9263    | -2.613419262           | 2.260907          | 104.7565 |
|                       | 10    | 34        | 8.518795 | 21.38577 | 42.49042   | -3.906362893           | 2.1296875         | 106.9289 |
| <b>K</b>              | 110   | 25        | 20.86033 | 18.65744 | 41.40823   | -1.394988388           | 5.2150825         | 93.2872  |
|                       | 120   | 24        | 16.66990 | 19.23358 | 39.97929   | 2.129352472            | 4.167475          | 96.1679  |
|                       | 140   | 29        | 18.79223 | 20.32908 | 39.98928   | 2.103838829            | 4.6980575         | 101.6454 |
|                       | 150   | 26        | 19.83246 | 20.85224 | 40.06344   | 1.914838067            | 4.958115          | 104.2612 |
|                       | 200   | 27        | 15.58482 | 18.65744 | 40.06637   | 1.907385171            | 3.896205          | 93.2872  |

Now the effect of changes in the system parameters on the optimal values of q, D, C (D, q),  $\frac{1}{2}uq$  and Aq when only one parameter changes and others remain unchanged the computational results are described in Table 4. As a result

- $q^*, D^*, C^*(D^*, q^*), \frac{1}{2}uq^*$  and  $Aq^*$  are highly sensitive to the parameter 'u'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$  and  $\frac{1}{2}uq^*$  are moderately sensitive to the parameter 'U'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$  and  $\frac{1}{2}uq^*$  are highly sensitive to the parameter 'A'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$  and  $\frac{1}{2}uq^*$  are insensitive to the parameter 'B'.
- $q^*, D^*, C^*(D^*, q^*), Aq^*$  and  $\frac{1}{2}uq^*$  are insensitive to the parameter 'C<sub>1</sub>'.

- $q^*, D^*, \frac{1}{2}uq^*$  and  $Aq^*$  are sensitive to the parameter ' $C_{03}$ ' but  $C^*(D^*, q^*)$  are moderately sensitive to ' $C_{03}$ '.
- $q^*, \frac{1}{2}uq^*$  and  $Aq^*$  are sensitive to the parameter ' $K$ ' but  $D^*$  and  $C^*(D^*, q^*)$  are moderately sensitive to ' $K$ '.

## VII. CONCLUSION

In constraint to Roy, the approach in this paper provides solutions better than those obtained by using properties and this paper follows real life inventory model for single item in fuzzy environment by FNLP technique. Some sensitivity analyses on the tolerance limits have been presented. The results of the fuzzy model is compared with that of crisp model which reveals that fuzzy model gives better result than the usual crisp model. Inventory modelers have so far considered auto are type of setup cost that is fixed or constant. This is rarely seen to occur in the real market. In the opinion of the author, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced / purchased may represent the tractable decision making procedure in fuzzy environment. A new mathematical model is developed and numerical example is provided to illustrate the solution procedure. The new modified EOQ model was numerically compared to the traditional EOQ model. Finally, the effect decision space was demonstrated numerically to have an adverse affect on the total average cost per unit. This method is quite general and can be extended to other similar inventory models including the ones with shortages and deteriorate items.

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