Numerical Analysis to Study the Scale Effect of Shallow Foundation on Reinforced Soils

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ABSTRACT

This research study aims at investigating the scale effect of shallow foundation on reinforced soils using finite element analysis. The finite element model was first verified by the results of laboratory model footing tests, and then was used to numerically investigate the load-settlement response of different footing sizes on reinforced soil foundations. Two different soil-reinforcement interface models were studied and three different reinforcement configurations were examined. The results of finite element analyses indicate that the scale effect of reinforced soil foundation is mainly related to the reinforcement ratio (R_r) of the reinforced zone. The difference in the bearing capacity becomes negligible as long as the reinforcement depth ratio (d/B) and the reinforcement ratio (R_r) of the reinforced zone remain constant for all footing sizes.

INTRODUCTION

Reinforced soil foundations (RSF) have been used for many years as one of the economical alternative to shallow foundation design. Several research studies have been performed to investigate the behavior and benefit of RSF. However, a review of existing literature revealed that most of experimental studies on reinforced soils were conducted using small-scale laboratory model footing tests. This raises a question on how to relate the performance of model footing tests to that of actual full-scale footings on reinforced soil foundations. The problem has been addressed in many geotechnical engineering applications, which is well- known as the “scale effect.”

The scale effect of shallow foundations on unreinforced soil is fairly well recognized and has been studied for years by many investigators (e.g., De Beer, 1965; Tatsuoka et al., 1991; Fellenius and Altaee, 1994; Ueno et al, 2001; Cerato, 2005). It is generally accepted that the scale effect of footings on coarse-grained soils is more pronounced than that on fine-grained soils due to the relatively large particle size of coarse-grained soils. The scale effect associated with a reduced-scale model tests on reinforced soil were also studied by several researchers (e.g., Das and Omar, 1994). Fakher and Jones (1996) pointed out that interpretation of model tests without taking into account the scale effect might overestimate the reinforcement benefit. Das and Omar (1994) and Elvidge and Raymond (2001) studied the scale effect by changing the footing width. The researchers in both studies reached the same conclusion, which is that an increase in footing width can increase the failure load and decrease the bearing capacity ratio (BCR), which is defined as the ratio of the bearing capacity of the RSF to that of the unreinforced soil foundation. By dimensional analysis, Fakher and Jones (1996) indicated that the stiffness of reinforcement in the model tests should be $1/n^2$ (n is the ratio of footing width in the field to that in model tests) times that of the reinforcement used in the field.

This paper aimed at studying the scale effect of reinforced soil foundations. A series of finite element analyses (FEA) on footings with different sizes was carried out using the commercial ABAQUS program to numerically study the scale effect of reinforced soil foundations. A large number of model footing tests were conducted by the authors (Chen et al., 2007, 2009) on reinforced soil foundation using square footings. Accordingly, the footings used in this study are also square footings. Two types of soils, silty clay and crushed limestone, which correspond to two different interface models, are studied here.

FINITE ELEMENT MODELING OF REINFORCED SOIL FOUNDATION

The footings used in this study were square footings. To accurately study their behaviors, three-dimensional modeling of soil foundation is usually needed from theoretical point of view. However, 3-D modeling of footings is time consuming and not practical to run multiple cases. It is common to treat square footings and circular footings with the same area as being equivalent in bearing capacity calculations (Skempton, 1951). Although, there is no theoretical justification for this assumption, it has been successfully used by different researchers (e.g. Lawton, 1995; Osman and Bolton, 2005). This assumption was verified by the authors by comparing the results of 3-D modeling of a square footing with axisymmetric modeling of an equivalent circular footing and good agreement was achieved. The authors believe that this assumption is reasonable, and therefore the following numerical analysis procedure was adopted in this study: the square footings were first converted to equivalent circular footings with the same area; axisymmetric FEA was then performed on these equivalent circular footings. The diameter of equivalent circular footings (*D*) can be calculated as:

$$D = 2B\big/\sqrt{\pi} \quad (1)$$

where *B* is the width of the square footing.

4-node bilinear axisymmetric quadrilateral solid elements (CAX4R) are used to discretize the soil, while 2-node linear axisymmetric membrane elements (MAX1) are used to discretize the reinforcement. The finite element model used in this study is illustrated in Figure 1. A rigid, perfectly rough footing is assumed in this study. The uniform vertical displacement is applied at nodal points immediately underneath the footing to model the rigid footing condition, while the corresponding horizontal displacement of these points is restrained to zero to simulate the perfect roughness of the soil-footing interface. The loading process is implemented by applying incremental footing displacement until the prescribed displacement is reached.

MATERIAL MODELS AND PARAMETERS

The soil is simulated as an isotropic elasto-perfectly plastic continuum. The yield criterion is described by the extended Drucker-Prager model with a linear form available in ABAQUS/Standard (Hibbitt, Karlsson & Sorensen, Inc., 2002):

$$f = t - p \tan \beta - d = 0 \tag{2}$$

$$t = \frac{1}{2} q \left[1 + \frac{1}{K} - \left(1 - \frac{1}{K}\right)\left(\frac{r}{q}\right)^3 \right] \tag{3}$$

where p is the mean effective stress; q is the Mises equivalent stress; r is the third invariant of deviatoric stress; K is the flow stress ratio; the ratio of the yield stress in triaxial tension to the yield stress in triaxial compression; β is the slope of the yield surface in the p-t stress plane; and d is the cohesion intercept of material in the p-t stress plane. Under triaxial condition,

$$\beta = \arctan\left[6 \sin \phi / (3 - \sin \phi)\right]; \quad \text{and} \quad d = 6c \cos \phi / (3 - \sin \phi) \tag{4}$$

The reinforcement is simulated as a membrane, which transmits in-plane force only and has no bending stiffness. A membrane element maybe the most appropriate element for the simulation of the geosynthetics (Perkins, 2001) and was used by many researchers (e.g. Dondi, 1994; Leng, 2002). The stress-strain behavior of reinforcement is modeled by a linear elastic model.

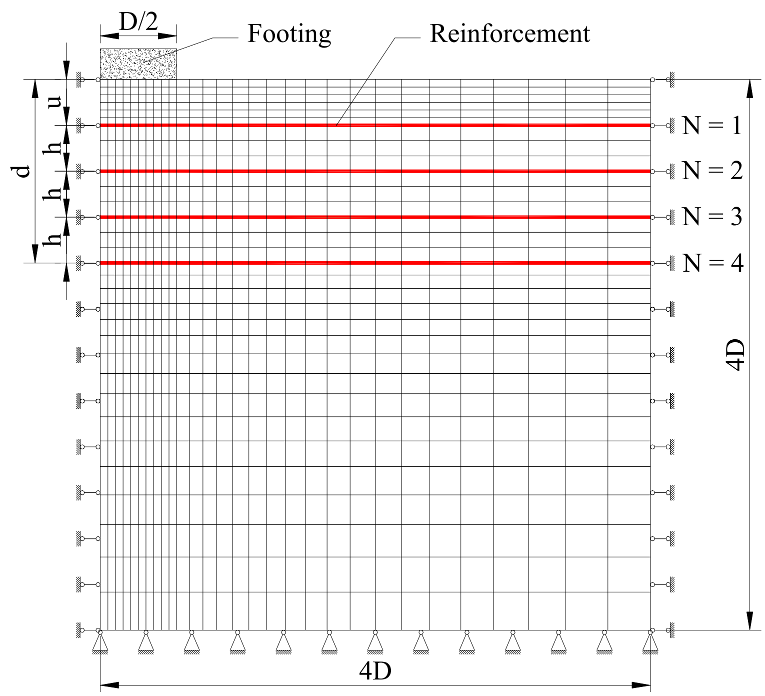


Figure 1: Finite element model of the circular footing sitting on reinforced soil

For silty clay-reinforcement interface, the "hard contact" is assumed in normal direction and no separation of surfaces is allowed once surfaces contact (i.e., the reinforcement is in contact with silty clay). The relationship between shear and normal forces is described in terms of the Coulomb friction model. An additional limit on the allowable elastic slip (γ_{crit}) is included in the Coulomb friction model. The γ_{crit} describes the interface shear stiffness, and is the limit of the relative shear displacement before the allowable interface shear stress is reached.

For crushed limestone-reinforcement interface, a full interlocking between the reinforcement and the crushed limestone surrounding it is assumed, i.e. crushed limestone and reinforcement are tied together at interface so that there is no relative motion between them. This type of contact interactions can be achieved by tied contact available in ABAQUS/standard.

The parameters used for modeling the reinforced silty clay are summarized in Table 1. Table 2 presents the parameters used for modeling the reinforced crushed limestone.

Table 1 Material and interface properties for reinforced silty clay

Materials	Model	Mechanical Properties	Elastic Modulus, E	Poisson ratio ν
Silty clay	linear Drucker-Prager	c =13 kPa, φ = 25°*	15 MPa#	0.3
Reinforcement	Linear Elastic Model	N/A	254 MPa	0.3
Interface	Hard contact & Coulomb friction model	μ= 0.6, γ_{crit} = 0.001 m&	N/A	N/A

*from large scale direct shear test, #based on Light Falling Weight Deflectometer data, &Abu-Farsakh et al. (2007)

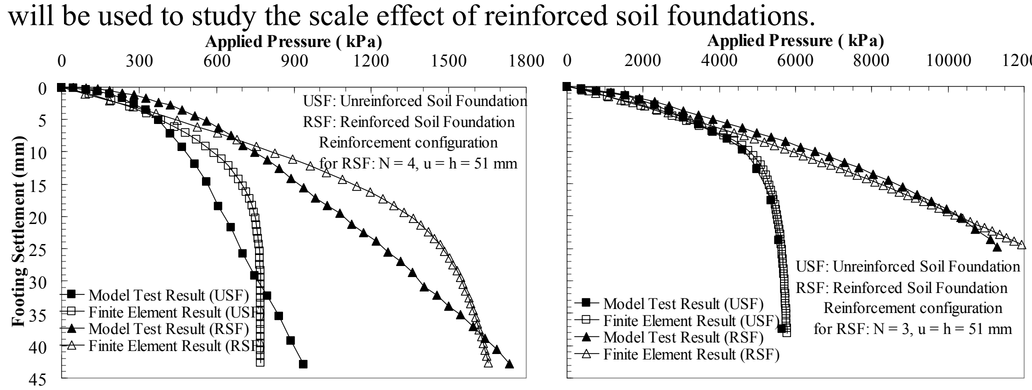
Table 2 Material and interface properties for reinforced crushed limestone

Materials	Model	Mechanical Properties	Elastic Modulus, E	Poisson ratio ν
Crushed limestone	linear Drucker-Prager	φ = 53°*	120 MPa#	0.3
Reinforcement	Linear Elastic Model	N/A	295 MPa	0.3
Interface	Tied contact	N/A	N/A	N/A

NUMERICAL RESULTS AND ANALYSIS

Verification of Finite Element Model

In order to verify the suitability of the adopted models for the soil, reinforcement, and soil-reinforcement interaction, finite element analyses were first checked against the results from laboratory model footing tests using square footings on the reinforced soils (Chen et al., 2007, 2009). Figure 2a presents the comparison between the finite element analyses and results of model footing test cases for unreinforced and reinforced silty clay soil. A comparison of the finite element analysis with the laboratory model footing tests for unreinforced and reinforced crushed limestone cases is presented in Figure 2b. The figures show that the finite element analyses have a reasonable agreement with the results of model footing test, although there are some discrepancies in the silty clay soil cases. The authors believe this comparison is acceptable for this type of laboratory tests, and therefore, the finite element models will be used to study the scale effect of reinforced soil foundations.



(a) silty clay soil **(b) crushed limestone**

Figure 2: Verification of the numerical model

Results of Finite Element Analysis

Finite element analysis was first conducted using a 152 mm (0.5 ft) wide square footing, which is the same size used in the laboratory model footing tests (Chen et al., 2007, 2009). In the following series of finite element analysis, the size of footing was increased to 3 times (B = 457 mm (1.5 ft)), 6 times (B = 914 mm (3.0 ft)), 9 times (B = 1372 mm (4.5 ft)), and 12 times (B = 1829 mm (6.0 ft)) of 152 mm wide square footing. Correspondingly, the sizes of reinforced (or influence) zone were increased by the same scale factor. The properties of soil remain the same in all models.

Three series of finite element analyses with different reinforcement layout were conducted to examine the scale effect of the model tests as described below. In all cases, the reinforcement depth ratio (*d/B*) was kept constant.

Case one: the vertical spacing ratio (*h/B*) of reinforcement layers, the number of reinforcement layers (*N*), and the tensile modulus of reinforcement (*J*) were kept constant.

Case two: the vertical spacing of reinforcement layers (*h*) and the tensile modulus of reinforcement (*J*) were kept constant, i.e increasing the number of reinforcement layers (*N*) by the same scale factors as the footing size.

Case three: the vertical spacing ratio (*h/B*) of reinforcement layers, and the number of reinforcement layers (*N*) were kept constant; and increasing the tensile modulus of reinforcement (*J*) by the same scale factor as the footing size.

The settlement of footing in all cases was expressed in a non-dimensional form of settlement ratio (*s/B*). The corresponding load settlement curves for silty clay with and without reinforcement are plotted in $q \sim s/B$ plane as shown in Figure 3; while Figure 4 depicts the load settlement curves of crushed limestone with and without reinforcement.

Results of Numerical Analyses

It can be seen from Figures 3 and 4 that the load-settlement curves of unreinforced soil for different footing sizes follow the same shape. This result indicates that if the settlement is expressed in a non-dimensional relative settlement of *s/B*, the unreinforced soil foundation has no scale effect. This numerical result is in agreement with static loading test results of Ismael (1985), Briaud and Gibbens (1994), and Fellenius and Altaee (1994).

The reinforcement ratio (R_r) is introduced to assist the analysis of scale effect of reinforced soil foundations. The reinforcement ratio (R_r) is defined here as:

$$R_r = (E_R A_R)/(E_S A_s) \quad (5)$$

where E_R is the elastic modulus of the reinforcement $=J/t_R$; J is the tensile modulus of reinforcement; A_R is the area of reinforcement per unit width = $Nt_R \times 1$; t_R is the thickness of the reinforcement; N is the number of reinforcement layers; E_s is the modulus of elasticity of soil; A_s is the area of reinforced soil per unit width = $d \times 1$; and d is the total depth of reinforced zone $=u+ (N-1)h$.

In practice, the top layer spacing u is usually selected equal to the vertical spacing h in most cases. By substituting u by h, the reinforcement ratio (R_r) can be re-written as:

$$R_r = J/(E_S h) \quad (6)$$

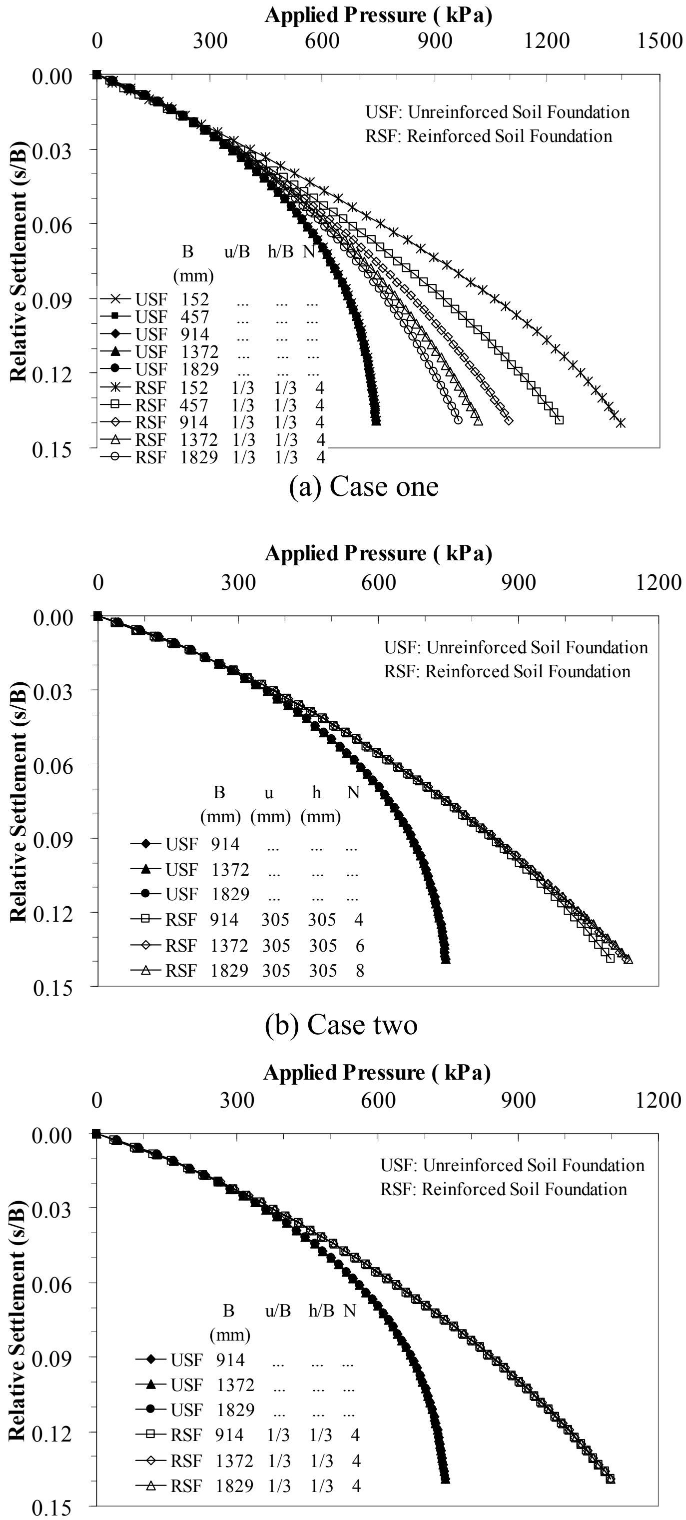
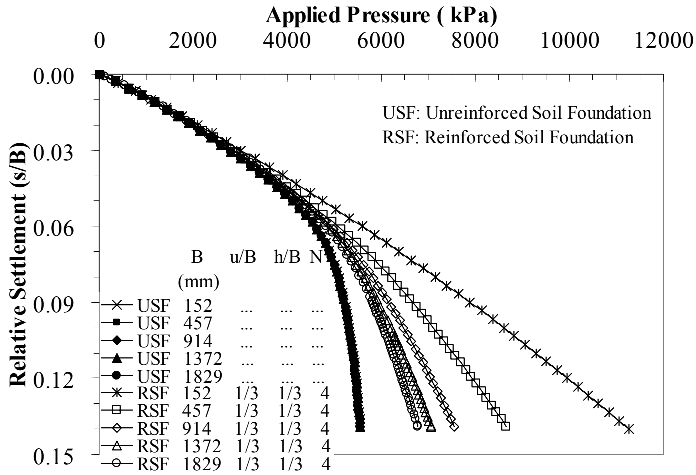
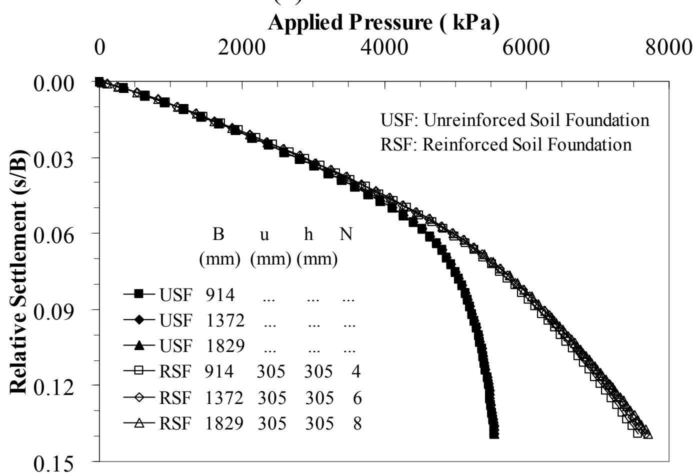


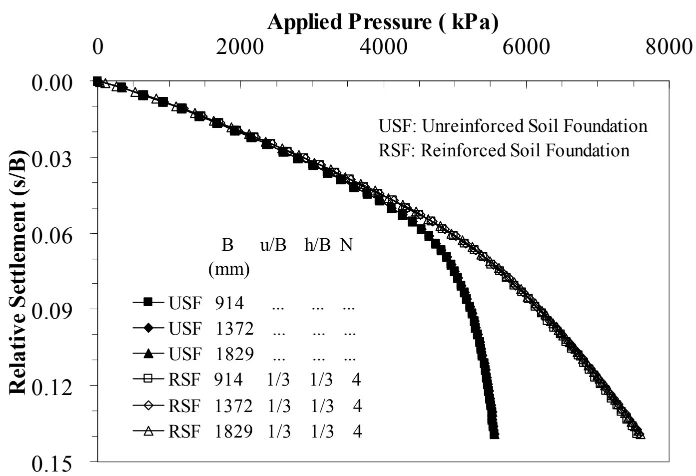
Figure 3: Pressure-settlement curves of silty clay with and without reinforcement



(a) Case one



(b) Case two

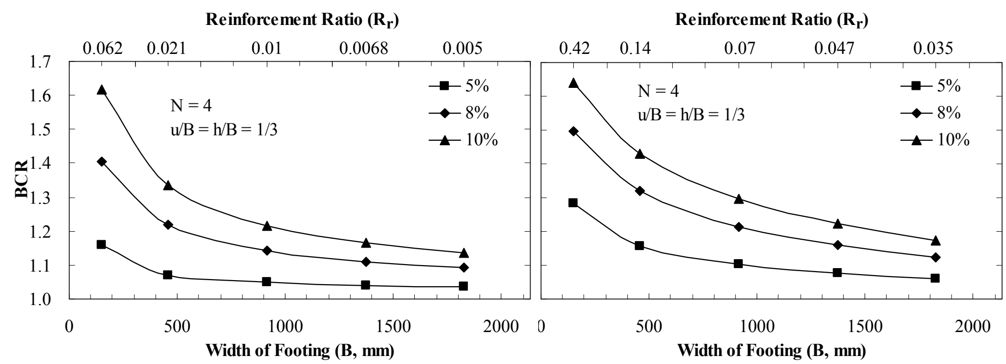


(c) Case three

Figure 4: Pressure-settlement curves of crushed limestone with and without reinforcement

It can be seen from equation (6) that for the same soil, the reinforcement ratio is proportional to the tensile modulus of reinforcements and inversely proportional to the vertical spacing of reinforcement (*h*).

For case one, the reinforcement ratio (R_r) decreases with the increase of footing size. Figure 3a shows that the bearing capacity of reinforced silty clay at the same settlement ratio (*s/B*) for case one decreases with increasing the footing size. The variations of BCRs with footing size (*B*) and reinforcement ratio (R_r) are drawn in Figure 5a for settlement ratios of *s/B*=5%, 8%, and 10%. It can be seen from the figure that the BCRs decrease with increasing footing size (*B*) and with decreasing reinforcement ratio (R_r), and become almost constant after B = 1372 mm (4.5 ft), corresponding to R_r = 0.047. Similar behavior is also obtained from finite element analysis of footings on reinforced crushed limestone, as can be seen in Figures 4a and 5b.



(a) Reinforced silty clay **(b) Reinforced crushed limestone**

Figure 5: BCR versus width of footing (*B*) and reinforcement ratio (R_r)

For case two, the reinforcement ratio (R_r) of the reinforced zone is kept the same for all footing sizes by adjusting the reinforcement spacing ratio (*h/B*). The results shown in Figures 3b and 4b indicate that the load-settlement curves of reinforced soil (silty clay and crushed limestone) for different footing sizes are very similar. The difference in bearing capacity of reinforced soil foundation at the same settlement ratio (*s/B*) is less than 3.5%. This result suggests that the load-settlement response of reinforced soil is not sensitive to the scale effect for case two if the settlement is expressed in a non-dimensional relative settlement of *s/B*.

For case three, the reinforcement ratio (R_r) of the reinforced zone is also kept constant for all footing sizes by adjusting tensile modulus of reinforcement (*J*). As shown in Figures 3c and 4c, similar results to case two are observed for case three. The difference in bearing capacity corresponding to the same settlement ratio (*s/B*) is less than 1%. Again, the result of case three suggests that the scale effect on the bearing capacity of reinforced soil is also negligible if the settlement is expressed in a non-dimensional relative settlement of *s/B*.

These results clearly indicate that the scale effect of reinforced soil foundation is directly related to the reinforcement ratio (R_r) of the reinforced zone below footings at the same reinforcement depth ratio (*d/B*). As such, if we can keep the *d/B* and R_r constant, the quantitative benefit ratio demonstrated in model test results can be extrapolated to actual full-scale reinforced soil foundations.

This is in agreement with the results of large-scale model footing tests conducted by the authors (Abu-Farsakh et al., 2008), which showed that for reinforced soil foundation sections with the same reinforcement depth ratio (*d/B*) and similar reinforcement ratio (R_r) have very similar load-settlement responses.

CONCLUSIONS

Based on the results of finite element analyses of square footing of different sizes on unreinforced and reinforced soil, the following conclusions can be drawn:

- The load-settlement curves of reinforced soil are almost identical for constant *d/B* and R_r ratios if the settlement is expressed in a non-dimensional settlement ratio of *s/B*.
- The bearing capacity of reinforced soil decreases with increasing the footing size if the reinforcement depth ratio (*d/B*), the vertical spacing ratio (*h/B*) of reinforcement layers, and hence the number of reinforcement layers (*N*) are kept constant. However, the difference in the bearing capacity becomes negligible if the reinforcement depth ratio (*d/B*) and the reinforcement ratio (R_r) remain constant for all footing sizes.
- The results of FE analyses indicate that the scale effect of RSF is mainly governed by the reinforcement ratio (R_r) of the reinforced zone. In conducting laboratory model footing tests, if we can keep the *d/B* and R_r ratios the same as those used in actual full-scale reinforced soil foundations, the quantitative benefits demonstrated in model tests can be extrapolated to actual full-scale RSFs.

It should be pointed out here that the particle size effect, which is one of the components of the scale effect in the bearing capacity of footing on granular materials (Tatsuoka et al., 1991), has not been considered in this study. Future centrifuge model tests or actual full scale tests on reinforced soils are recommended to substantiate and improve on the findings of this study.

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