LIMITATION OF ELECTROSTATIC CHARGING OF DUST PARTICLES IN A PLASMA

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Abstract. We show that in a dusty plasma consisting of a plasma (density n and temperature T) and dust grains (density N and radius a) the charge on a grain is not given by its free-space value $Q_o = Va$, where V is its surface potential. Instead the charge is reduced by a factor $1 + x$, with $x = 4\pi Na\lambda_D^2$, where λ_D is the plasma Debye length. Except in the optically thin E and G rings this factor is large. Usually electromagnetic forces on dust particles in Saturn's ring system are too small to produce observable effects. The current carried by dust particles moving relative to the plasma with a speed w is to a good approximation given by $j = NQw$. Thus magnetic perturbations by the F ring should be much smaller than previously estimated.

Introduction

The discovery of fine structure in the Saturnian ring system has motivated a vigorous research into the motion of charged dust particles embedded in a magnetized plasma (e.g., Hill and Mendis, 1982a,b; Mendis et al., 1982; Grün et al., 1983; Goertz and Morfill, 1983). It was shown by several authors that when the charge to mass ratio of a dust grain (Q/m) is large, interesting phenomena are to be expected which are caused by the influence of magnetic and electric forces on the grain. For example, Hill and Mendis (1982c) suggest that the F ring, composed of micron-sized grains, could carry a current of the order of 10^5 A which would significantly alter the planetary magnetic (dipole) field. Houpis and Mendis (1983) discuss tearing instabilities in a dusty plasma and relate this to the ringlet structure of the main Saturnian rings. Goertz and Morfill (1983) show how the currents in a dusty plasma cloud could polarize the cloud and cause its radial motion, which they relate to the rapid growth of the spokes in Saturn's ring. The motion of charged dust particles has been discussed in detail by Hill and Mendis (1979, 1980) and by Northrop and Hill (1982, 1983a,b). Their work has laid the foundation of what has been called "gravito-electrodynamics."

In all theories the value of Q/m is of crucial importance because it must be large for electromagnetic effects to be important. It has usually been assumed that the charge on a dust particle is given by its free-space value

$$Q_o = Va$$

where V is its surface potential and a its radius. For a micron-sized particle with a surface potential of 40 V, the free-space surface charge is about 30,000 electrons. This is a large value and sufficient to justify the interest in electromagnetic effects. However, we will show below that this value applies only for a single dust grain. If many dust grains are present, the charge on a single grain is reduced, usually by a large factor.

This reduction is basically due to the fact that the potential V on the surface of an individual dust grain is not only due to the charge on this grain alone but also due to all other dust grains in the vicinity. For a sufficiently dense packing of the dust, the average charge on a dust particle may be less than one electron, i.e., most grains will be uncharged (Goertz and Morfill, 1983).

In addition, the question of whether the relative motion of dust grains and plasma constitutes a current is not as straightforward as usually assumed. One must consider the fact that the charged dust particle is surrounded by a plasma which screens the electric field of the grain. Within about a Debye sphere, the plasma is not charge neutral but carries a small charge density which, when integrated over all space, exactly cancels the charge on the grain. Since for a slow moving grain the Debye sphere always forms around it (even though the plasma electrons and ions do not move with the grain), one expects a reduction of the total current (conduction plus displacement current). One can show that a single dust grain moving in a plasma will not produce a magnetic perturbation far away ($\gg \lambda_D$) from it. On the other hand, a very densely packed cloud of moving (velocity w) charged grains would constitute a current ($j = NQw$). We will derive an equation for the current carried by an assembly of charged grains moving through a stationary plasma and show that the current density cannot reach arbitrarily large values even when the number density of the grains (N) becomes large. This is due to the reduction of Q which occurs when N is large. Thus in most cases the magnetic perturbations produced by the dust in Saturn's ring system are negligibly small.

The Average Charge on a Dust Grain

When a dust particle is immersed in a plasma, its surface potential V will adjust to such a value that the total current to the dust grain is zero. This "floating" potential is given implicitely by the equation

$$F_e(V) - F_i(V) - F_\nu(V) = 0 \tag{1}$$

where F_e is the plasma-electron flux, F_i the plasma-ion flux, and F_ν is the photoelectron flux. The potential is normalized to zero at infinity. For a dense neutral plasma the surface

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potential of a dust grain in free space will be negative with

$$V = \alpha\, kT_e/e \qquad (2)$$

where $\alpha \approx -2.51$ if photoelectrons can be neglected. If photoelectrons are important, the value of $|\alpha|$ is reduced. Also if the dust grains trap many charges, the surrounding plasma is not neutral and the floating potential must be evaluated numerically. A determination of the floating potential of the grains requires solving for the plasma potential (Φ_p) and the grain potential (V) through Poisson's equation,

$$\nabla^2\Phi_p + 4\pi e(n_i - n_e) = -4\pi \sum_j Q_j\, f(\vec{r}-\vec{r}_j) \qquad (3)$$

where the source function is given by:

$$f(\vec{r}-\vec{r}_j) = \begin{cases} \dfrac{3}{4\pi a^3} & ; \quad |\vec{r}-\vec{r}_j| < a \\ 0 & ; \quad |\vec{r}-\vec{r}_j| > a \end{cases} .$$

The particle densities for a negative grain potential are given by the Boltzmann's distributions of the plasma electrons and constant ion densities (if collisions are neglected)

$$\begin{aligned} n_e(\Phi_p) &= n_o \cdot \exp(e\Phi_p/kT_e) \\ n_i(\Phi_p) &= n_o \end{aligned} \qquad (4)$$

together with the condition of current equilibrium to the grains [equation (1)].

In the above, n_i and n_e are the ion and electron number densities, n_o is the plasma density for $\Phi_p = 0$, Q_j is the total electronic charge accumulated on the jth grain, and T_i ($= T_e$) is the ion temperature. A complete treatment of this set of equations which depicts the charged grains as discrete points arranged in a three-dimensional lattice which in turn is immersed in a continuum plasma requires numerical integration if $e\Phi_p/kT \gtrsim 1$. However, we will linearize equation (4). The solution of equation (3) is then given by the integral of the Green's function

$$\Phi_p(\vec{r}) = \sum_j \int Q_j\, f(\vec{r}\,'-\vec{r}_j)\, \frac{e^{-|\vec{r}-\vec{r}\,'|/\lambda_D}}{|\vec{r}-\vec{r}\,'|}\, d\tau' \qquad (5)$$

where the integration is over the volume of the dusty plasma. For $e\Phi_p/kT > 1$ the exact solution involves a Green's function which falls off somewhat less rapidly than the exponential (Parker, 1980). We now make the reasonable assumption that the radius of each grain is the same (a) and that it is much smaller than the Debye length λ_D. We also assume that all grains have the same charge (Q) which should be a good approximation for the grains inside the dust cloud but not true for the grains on its surface. The integral is complicated to solve exactly. But the following picture will simplify the calculations. We put the center of our coordinate system at the center of the ith grain. Then the potential on the surface of this grain is equal to

$$\begin{aligned} V_i = \Phi_p(\vec{r} = a) &= Q\, \frac{e^{-a/\lambda_D}}{a}\, \frac{3}{2} \left(\frac{\lambda_D}{a}\right)^3 \\ &\times \left[\left(\frac{a}{\lambda_D}\right)\left(e^{a/\lambda_D} + e^{-a/\lambda_D}\right) - \left(e^{a/\lambda_D} - e^{-a/\lambda_D}\right)\right] \\ &+ \sum_{j\neq i} \int Q\, f(\vec{r}\,'-\vec{r}_j)\, \frac{e^{-|r - r'|/\lambda_D}}{|\vec{r}-\vec{r}\,'|}\, d\tau' \quad . \end{aligned} \qquad (6)$$

The first term is the surface potential due to the charge on the ith grain itself. For $a/\lambda_D \ll 1$, it reduces to $Q\, e^{-a/\lambda_D}/a$. The second term expresses the contribution from all other grains. We can calculate this integral approximately by noting that beyond a minimum distance R (nearest neighbor distance) the distribution of dust grains is almost homogeneous and the sum can be replaced by an integral.

$$\begin{aligned} &\sum_{j\neq i} \int Q\, f(\vec{r}\,'-\vec{r}_j)\, \frac{e^{-|\vec{r}-\vec{r}\,'|/\lambda_D}}{|\vec{r}-\vec{r}\,'|}\, d\tau' \\ &\approx Q \int_R^\infty N\, \frac{e^{-r/\lambda_D}}{r}\, 4\pi r^2\, dr \end{aligned} \qquad (7)$$

where N is the uniform dust density. Thus the surface potential of the ith grain (which is equal to the surface potential of all other grains) is given by

$$V = Q\, \frac{e^{-a/\lambda_D}}{a}\, [1 + x] \approx \frac{Q}{a}\, [1 + x] \qquad (8)$$

where:

$$x = 4\pi N a \lambda_D^2\, e^{-R/\lambda_D}(1 + R/\lambda_D) \quad . \qquad (9)$$

We have, of course, made the approximation $a \ll \lambda_D$. Thus the average charge on a dust grain is reduced from the value it would have if it were the only dust particle in the plasma ($Q_o = Va\, e^{a/\lambda_D} \simeq Va$) by a factor $1 + x$.

$$Q = \frac{Va\, e^{a/\lambda_D}}{1 + x} = \frac{Q_o}{1 + x} \quad . \qquad (10)$$

This reduction may be understood by considering that the charge on each grain is proportional to the difference between the grain potential and the average plasma potential between the grains. This plasma potential is not zero because it is the superposition of the (screened) potentials due to all the grains in the plasma. For a negative grain charge the potential due to an individual grain will be negative everywhere with its magnitude decreasing as $e^{-r/\lambda_D}/(r/a)$. Thus the potential of one individual grain is very small once $r \gg a$. However, if there are many dust grains (large dust density N), the sum of many small potentials gives rise to an average plasma potential which is significantly different from zero.

The factor x can be very large as the following example shows. In the F ring with an optical depth $\tau = N\pi\, a^2\, h = 1$ where $h \approx 10$ km is the thickness of the ring, we have for $a = 10^{-4}$ cm $N \approx 30\ \mathrm{cm}^{-3}$. Using a plasma density of $10^2\ \mathrm{cm}^{-3}$ (Frank et al., 1980) and $T_e = 4$ eV, we get $\lambda_D = (kT_e/4\pi ne^2)^{1/2} = 150$ cm. The nearest neighbor distance is $R = (3/4\pi N)^{1/3} \approx 0.2$ cm $\ll \lambda_D$. This gives a correction factor of x = 800. Thus instead of a free charge of Q_o = Va of 7,000 electrons, only about 9 electrons reside on each grain.

In the E ring we have $\tau = 10^{-6}$, h = 10 km, and $x = 3 \times 10^{-6}$. Here the charge is not affected by the presence of other dust particles. However, the optical depth of the E ring is so small that any electromagnetic effects would be extremely difficult to observe.

In the spokes (τ = 0.1, h = 30 km, λ_D = 100 cm, a = 0.5 μ), we have x = 25. The average charge on a dust grain would reach an equilibrium value of about 200 electrons. However, according to Goertz and Morfill (1983) the spoke dust particles will leave the dense plasma clouds long before they reach their equilibrium charge. Because of the low optical depth ($\tau \approx 10^{-3}$), the electrostatic charging of the D ring particles (and hence the current carried by the dust particles) as discussed in Ip and Mendis (1983) is not affected by this limitation.

In Figure 1 we show the variation of the factor $Q/Q_o = (1+x)^{-1}$ as a function of optical depth for a typical dust grain size and electron density. Since optically observable effects require a certain optical depth ($\tau > 10^{-1}$), the charge on a grain in optically visible rings is always considerably less than the free-space value. It seems that except in the optically thin E and D rings electromagnetic effects are usually not very important for determining the dust motion.

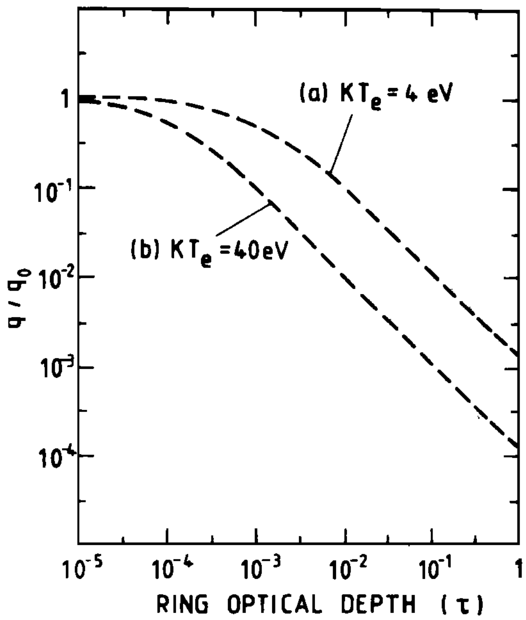

Fig. 1. The ratio of dust charge Q to the free-space value (Q_o = Va) as a function of optical depth. The dust cloud has a height of 10 km and consists of 1 micron grains. The plasma density is $10^2\ \mathrm{cm}^{-3}$.

We note, as an interesting aside, that the total charge density trapped on the dust particles can never exceed the free-electron density times the factor $\alpha(NQ < |\alpha| ne)$.

Current Carried by the Relative Motion of Dust Particles

After derivation of the electrostatic charge on a dust grain as a function of the Debye length and the inter-particle distance, the next question to ask is perhaps what is the corresponding current carried by such charged dust under similar assumptions.

It has been suggested by Hill and Mendis (1982) that the Keplerian motion of charged dust particles relative to the corotating plasma in the F ring constitutes a current of 10^5 A, which would cause a noticeable perturbation in the magnetic field. In their treatment they have neglected the screening effect and have thus overestimated the charge in the dust. In addition, they have not included the displacement current. In fact, if dust particles are small ($a \ll R$), the conduction current is only nonzero at the instantaneous position of the dust particle. Everywhere else (i.e., over the largest part of the volume), the current is a displacement current.

$$j_D = \frac{1}{4\pi}\frac{\partial E}{\partial t} \quad . \tag{11}$$

This current varies in space and time due to the inhomogeneous distribution of the electric field. For the purpose of calculating a magnetic field perturbation, however, one needs to know a spatial and temporal average of this current.

Consider a sphere of radius R. In a time T = 2R/w, where w is the velocity of the dust particles relative to the plasma, one grain passes through the sphere. The electric field due to the other dust particles is nearly constant in time over the sphere and does thus not constitute a displacement current. To calculate the current we average the effect of the one dust grain passing through the center of the sphere

$$\vec{j} = \frac{1}{4\pi TV}\iint_0^T \frac{\partial \vec{E}}{\partial t}\, dV\, dt \tag{12}$$

where V is the volume of the sphere. The integration is found to be

$$\vec{j} = NQ\vec{w}\, f(R/\lambda_D) \tag{13}$$

with

$$f(z) = \frac{1}{z}\left[(1-e^{-z}) + (1-e^{-z}(1+z))\right] \quad . \tag{14}$$

The function f(z) is shown in Figure 2. For $z \ll 1$ $f(z) = 1$, whereas for large values of z, $f(z) \to 1/z$. The usual formula $j = NQw$ used by Hill and Mendis (1982c) and Goertz and Morfill (1983) thus applies only when $z \ll 1$. This condition is equivalent to the normal plasma condi-

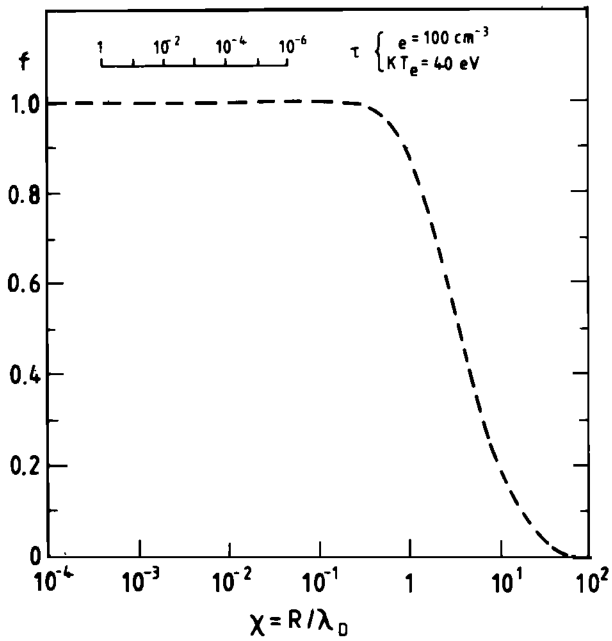

Fig. 2. The function f(z) described in the text. The insert indicates an equivalent optical depth scale corresponding to z for a plasma with n = 10^2 cm^{-3} and kT_e = 40 eV, assuming a height of 10 km.

tion $N\lambda_D^3 \gg 1$, i.e., that there may be many dust particles in a Debye sphere. This condition is amply satisfied in the F ring and for the spokes. In the E ring we have R = 250 cm, i.e., $R/\lambda_D \sim 2$ and $f(z) \sim 0.73$, which is still close to 1. Thus j = NQw is a good approximation to the current density carried by the dust particles everywhere in Saturn's magnetosphere.

Combining equations (2), (9), (10), and (13) for z << 1, we find an interesting upper limit to the current density.

$$j < \alpha\, enw \quad . \tag{15}$$

Thus the current density of a dusty plasma can never exceed a value which is given not by the dust density but by the surrounding plasma density.

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