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# The influence of a dust size distribution on the dust-acoustic mode

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**Abstract.** A real dusty plasma is modelled by taking into account the size distribution of the dust particles. The influence of this distribution on the dust plasma frequency is examined for a power law distribution and for a normal distribution. The first is relevant in astrophysical applications (e.g. planetary rings, comets), while the latter can be used in laboratory plasmas (e.g. dust crystals). It is shown that the dust plasma frequency for a power law distribution exceeds the plasma frequency for mono-sized dust. A normal distribution will result in an increase of the dust plasma frequency. The corrections have been quantified for these two distributions and the change to the real part of the dispersion relation for the dust-acoustic wave is discussed. © 1997 Published by Elsevier Science Ltd

## Introduction

A dusty plasma consists of charged dust grains embedded in an ambient plasma. Depending on their concentrations, one has isolated screened dust grains (dust-in-plasma) or real collective dusty plasmas where the charged dust participates in Debye screening. Reviews have been given by Goertz (1989), Northrop (1992), Mendis and Rosenberg (1994), Shukla (1996) and Verheest (1996). The study of dusty plasmas is a relatively new research topic, and recently the interest increased because of laboratory devices (Thomas and Morfill, 1996).

In the past, most papers on dusty plasmas considered mono-sized dust grains, because they are easier to treat. Aslaksen and Havnes (1992) studied the effects of a dust size distribution on the thickness of a planetary ring, while Bliokh and Yaroshenko (1985) derived the dispersion relation for oscillations in an infinite self-gravitating medium. The problem of the influence of a dust size distribution on the plasma dust frequency was also tackled by Rosenberg (1993), who concludes that the resulting plasma frequency is mostly determined by the smallest plasma particles, without really quantifying this result. Recently Brattli *et al.* (1997) looked at the effect of a dust size distribution on the damping rate of the dust-acoustic mode.

We consider two homogeneous cases: case 1 where there is mono-sized dust, and case 2 with a dust distribution and we compare the dust plasma frequency in these two cases. Because one has to make an assumption on the relation between these two cases (e.g. conservation of the total mass, conservation of number density), there are different ways to attack the problem. We follow an outline where the dust number density is the same for the two cases. This choice is the most physical one, because dust densities are easier to measure than the mass of the particles (Grün *et al.*, 1992).

We calculate both for a power law distribution (continuous and discontinuous model) as for a normal distribution of the dust grains the dust plasma frequency, and apply the results to the dust-acoustic mode (Rao *et al.*, 1990).

## Calculation and results

When we assume that all grain sizes $a \ll \lambda_D$, we can express mass and charge of a dust particle as follows:

$$m(a) = \frac{4}{3}\pi\rho a^3 \sim a^3 \tag{1}$$

$$q(a) = 4\pi\varepsilon_0 a V_0 \sim a \tag{2}$$

with $\varepsilon_0$ the vacuum permittivity, $\rho$ the mass density of the grains (assumed to be constant and equal for all grains) and $V_0$ the electric surface potential at equilibrium.

- When we assume that the dust distribution is given by a power law, for charged dust grains with radii $a$ in a given range $[a_{min}, a_{max}]$, the differential density distribution is of the form

$$n(a)\,\mathrm{d}a = Ka^{-\beta}\,\mathrm{d}a. \tag{3}$$

This kind of distribution is widely accepted in space plasmas. We find values of $\beta = 4.6$ for the F-ring of Saturn (Showalter and Cuzzi, 1993), while for the G-ring values of $\beta = 7$ and 6 were obtained (Gurnett *et al.*, 1983;

Showalter *et al.*, 1992). For cometary environments, we recall a value of $\beta = 3.4$ (McDonnell *et al.*, 1987).

First we follow the method outlined in Meuris *et al.* (1997), and describe the different dust species in a discontinuous model, where each different $q_d/m_d$ value can be seen as a different dust species. To keep the discussion tractable, we start with two different dust species, with respective grain sizes $a_1$ and $a_2$, and densities $N_{d1}$ and $N_{d2}$. The average size is logically given by

$$\bar{a} = \frac{N_{d1}a_1 + N_{d2}a_2}{N_{d1}+N_{d2}}. \tag{4}$$

We can use equations (1) and (2) to define

$$\begin{aligned} m_{d1} &= m(a_1), \quad q_{d1} = q(a_1) \\ m_{d2} &= m(a_2), \quad q_{d2} = q(a_2) \\ \overline{m_d} &= m(\bar{a}) \sim \bar{a}^3, \quad \bar{q}_d = q(\bar{a}) \sim \bar{a} \end{aligned} \tag{5}$$

and we define the following frequencies:

$$\begin{aligned} \omega_{pd1}^2 &= \frac{N_{d1}q_{d1}^2}{\varepsilon_0 m_{d1}} \\ \omega_{pd2}^2 &= \frac{N_{d2}q_{d2}^2}{\varepsilon_0 m_{d2}} \\ \overline{\omega_{pd}^2} &= \frac{(N_{d1}+N_{d2})\overline{q_d}^2}{\varepsilon_0 \overline{m_d}}. \end{aligned} \tag{6}$$

Because of equation (4), the total charge ($Q_t$) that resides on the dust grains, is the same for the case where all dust grains have the same mean radius, $Q_t = (N_{d1}+N_{d2})4\pi\varepsilon V_0 \bar{a}$ as for the dust distribution $Q_t = (N_{d1}a_1 + N_{d2}a_2)4\pi\varepsilon V_0$. We can introduce the quantities

$$\delta = \frac{a_1}{a_2}, \quad \nu = \frac{N_{d2}}{N_{d1}}. \tag{7}$$

To model a realistic power law, we have to assume that there are more grains of smaller size than larger ones, and hence $\delta < 1$, $\nu < 1$. The following ratio is investigated:

$$\begin{aligned} R_d &= \sum_d \omega_{pd}^2 / \overline{\omega_{pd}^2} \\ &= 1 + \frac{\nu(1-\delta)^2}{\delta(1+\nu)^2} > 1. \end{aligned} \tag{8}$$

This ratio is larger than one for arbitrary $\delta$ and $\nu$, and hence explicit use of two dust species increases the ratio $R_d$. Of course the same reasoning holds *a fortiori* for three or more dust species, by repeating the type of argument whenever two dust species are replaced by their averages.

- As the number of different $m_d/q_d$ ratios for the different dust grain species increases, a continuous model is reached. We define that

$$\omega_{pd}^2 = \int_{a_{\min}}^{a_{\max}} \frac{n(a)q^2(a)}{\varepsilon_0 m(a)}\,\mathrm{d}a \tag{9}$$

and

$$\bar{a} = \int_{a_{\min}}^{a_{\max}} n(a)a\,\mathrm{d}a \Big/ \int_{a_{\min}}^{a_{\max}} n(a)\,\mathrm{d}a. \tag{10}$$

Again, because of this definition, the total charge that resides on the dust grains remains constant. Following the previous calculation, we derive

$$\begin{aligned} R_c &= \omega_{pd}^2 \Big/ \frac{N_{\text{tot}}\overline{q_d}^2}{\varepsilon_0 \overline{m_d}} \\ &= \frac{(a_{\max}^{-\beta} - a_{\min}^{-\beta})(a_{\max}^{-\beta+2} - a_{\min}^{-\beta+2})}{(a_{\max}^{-\beta+1} - a_{\min}^{-\beta+1})^2} \frac{(\beta-1)^2}{\beta(\beta-2)} \\ &= \frac{(c^{-\beta}-1)(c^{-\beta+2}-1)}{(c^{-\beta+1}-1)^2} \frac{(\beta-1)^2}{\beta(\beta-2)} \end{aligned} \tag{11}$$

with $N_{\text{tot}}$ the total number density of dust grains, given by

$$N_{\text{tot}} = \int_{a_{\min}}^{a_{\max}} n(a)\,\mathrm{d}a \tag{12}$$

and $c = a_{\max}/a_{\min}$. Numerical values for $R_c(\beta, c)$ are shown in Table 1.

The ratio $R_d$ turns out to be larger than 1 in the range $(1 < \beta, 1 < c)$, which means that also for the continuous case, a power law size distribution increases the dust plasma frequency.

- For large $\beta$, the ratio goes to 1, and the influence of the dust distribution vanishes. In that case the influence of $c$ on the result may be neglected.
- However, for smaller $\beta$, the ratio $R_c$ reaches higher values and increases with $c$.

In dusty plasma experiments (dusty crystals and charging experiments) the dust size distribution is often found to be normally distributed.

$$n(a)\,\mathrm{d}a = \frac{N_{\text{tot}}}{\sqrt{\pi}\sigma\,\mathrm{Erf}\,[\varepsilon/\sigma]} \exp\left[\frac{(a-\bar{a})^2}{\sigma^2}\right]\mathrm{d}a \tag{13}$$

where $N_{\text{tot}}$ denotes the total density of dust grains, $\sigma$ the width of the distribution and $\varepsilon$ the domain $[\bar{a}-\varepsilon, \bar{a}+\varepsilon]$ in which the particle sizes can be found. We assume that $\varepsilon/\sigma > 2$, and therefore $\mathrm{Erf}[\varepsilon/\sigma] \approx 1$.

Along the same paths, we define that

$$\omega_{pd}^2 = \int_{\bar{a}-\varepsilon}^{\bar{a}+\varepsilon} \frac{n(a)q^2(a)}{\varepsilon_0 m(a)}\,\mathrm{d}a \tag{14}$$

and

**Table 1.** The expression $R_c(\beta,c)$ as a function of different $\beta$ and $c$

| $\beta$ | $c = 10^0$ | $c = 10^1$ | $c = 10^2$ | $c = 10^3$ | $c = 10^4$ |
|---|---|---|---|---|---|
| 1 | 1 | 1.53 | 4.62 | 20.9 | 118 |
| 2 | 1 | 1.41 | 2.35 | 3.46 | 4.60 |
| 3 | 1 | 1.22 | 1.32 | 1.33 | 1.33 |
| 4 | 1 | 1.12 | 1.12 | 1.12 | 1.12 |
| 5 | 1 | 1.07 | 1.07 | 1.07 | 1.07 |
| 6 | 1 | 1.04 | 1.04 | 1.04 | 1.04 |
| 7 | 1 | 1.03 | 1.03 | 1.03 | 1.03 |

**Table 2.** The expression $R_n(\sigma/\bar{a})$ as a function of different $\sigma/\bar{a}$

| $\sigma/\bar{a}$ | 5% | 10% | 20% | 30% |
|---|---|---|---|---|
| $R_n(\sigma/\bar{a})$ | 1.001 | 1.005 | 1.02 | 1.05 |

$$\bar{a} = \int_{\bar{a}-\varepsilon}^{\bar{a}+\varepsilon} n(a)a\,\mathrm{d}a \Big/ \int_{\bar{a}-\varepsilon}^{\bar{a}+\varepsilon} n(a)\,\mathrm{d}a$$

$$= \int_{\bar{a}-\varepsilon}^{\bar{a}+\varepsilon} n(a)a\,\mathrm{d}a/N_{\mathrm{tot}} \quad (15)$$

and calculate the following ratio:

$$R_n = \omega_{pd}^2 \Big/ \frac{N_{\mathrm{tot}}\overline{q_d}^2}{\varepsilon_0 \overline{m_d}}. \quad (16)$$

The numerical results are given in Table 2. We can see that although changes due to the normal distribution are small, $R_n$ turns out to be greater than 1, and hence the dust plasma frequency increases with a normal dust size distribution.

It can be shown that even for variances in the dust grain size of 30%, the resulting change in the dust plasma frequency is only 5%.

## The dust-acoustic wave

When we consider a proton–electron dusty plasma, where the plasma particles are assumed to be Boltzmann distributed, and a number of cold dust species, the dispersion relation for the dust-acoustic wave (Rao *et al.*, 1990; Shukla, 1996) is given by

$$1+\frac{1}{k^2\lambda_{\mathrm{D}}^2}-\sum_d \frac{\omega_{pd}^2}{\omega^2}+\mathrm{i}\chi_{\mathrm{Le}}+\mathrm{i}\chi_{\mathrm{Li}}+\sum_d \frac{\mathrm{i}\nu_{ch}^{di}}{k^2\lambda_{\mathrm{Di}}^2(\omega+\mathrm{i}\nu_d)}$$
$$+\sum_d \frac{\mathrm{i}\nu_{ch}^{de}}{k^2\lambda_{\mathrm{De}}^2(\omega+\mathrm{i}\nu_d)} = 0 \quad (17)$$

with $\lambda_{\mathrm{De(i)}}$ denoting the electron (ion) Debye length, while $\lambda_{\mathrm{D}}$ is given by $1/\lambda_{\mathrm{D}}^2 = 1/\lambda_{\mathrm{De}}^2 + 1/\lambda_{\mathrm{Di}}^2$. The Landau-damping is denoted by $\chi_{\mathrm{L}}$, $\nu_d$ is the charge relaxation rate originating from the variations in the effective collision cross-section due to charge perturbations at the grain surface as experienced by the unperturbed particles. Furthermore $\nu_{\mathrm{ch}}^{d\alpha}$ is the charging frequency arising from the dust charge fluctuations caused by the electrostatic disturbance. More details can be found in Shukla (1996).

It is clear that the dispersion relation equation (17) will show an additional damping due to the dust size distribution as analyzed by Brattli *et al.* (1997), but we look at the complementary case where the damping is weak. In that case the real part of the dust-acoustic wave frequency is given by

$$\omega^2 = \frac{k^2\lambda_{\mathrm{D}}^2}{1+k^2\lambda_{\mathrm{D}}^2}\sum_d \omega_{pd}^2. \quad (18)$$

The overall Debye length remains the same for a mono-sized dust, as for a dust size distribution. Indeed, because of the definition of the mean dust grain radius, the total number of charges on the dust grains remains the same for the two models. Therefore the number of free electrons, and hence the Debye length will not change. The change in phase velocity of the dust-acoustic modes relies therefore totally on the difference in dust plasma frequency, and the previous result can be used.

## Conclusions

Although in most papers, concerning dusty plasmas, the dust grains are considered as mono-sized because of treatability reasons, a real dusty plasma consists of grains of different sizes. We consider two homogeneous cases: case 1 where there is mono-sized dust, and case 2 with a dust distribution and we compare the dust plasma frequency in these two cases, when the total dust number density is constant.

We showed that the dust plasma frequency increases when a power law or a normal size distribution is taken into account. This correction is small for power law indices larger than 2, but becomes important for small $\beta$ values. For a normal distribution of dust grains, the dust plasma frequency will increase, however this correction is rather small. The correction for the real part of the phase velocity of the dust-acoustic wave can be found along the same paths. It increases when a representative power law distribution, or a normal distribution is used. The corrections have been quantified for these two distributions.

These results might be useful in astrophysical applications (e.g. planetary rings, comets), as well as in laboratory plasmas (e.g. dust crystals).

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