Portfolio Optimization Problem by the Firefly Algorithm

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Abstract: This paper presents metaheuristics framework for solving portfolio optimization problem. Many methods and techniques exist for tackling this well-known economics and finance problem. As additional constraints are being added to the basic problem definition, traditional techniques become insufficient and optimization metaheuristics emerge as a better approach. In this paper, firefly algorithm (FA) swarm intelligence metaheuristics was applied to the portfolio optimization problem and it was tested on a set of five assets with promising results.

Key–Words: portfolio optimization problem, metaheuristic optimization, swarm intelligence, firefly algorithm (FA), nature inspired algorithms

1 Introduction

Portfolio optimization problem, also known as portfolio selection problem, is one of the most studied research topics in the field of finance and economics. Financial portfolios are collection of financial instruments (investments), all owned by the same organization or by an individual. They usually include bonds (investments in debts), stocks (investments in individual businesses), and mutual funds (pools of money from many professional investors).

In its basic definition, portfolio optimization problem is dealing with the selection of portfolio's assets (or securities) that minimizes the risk subject to the constraint that guarantees a given level of returns. Individual and institutional investors prefer to invest in portfolios rather than in a single asset because by doing this, the risk is mitigated with no negative impact on the expected returns [1].

Thus, the goal is to select a portfolio with minimum risk at defined minimal expected returns. This means reducing nonsystematic risks to zero. Alternatively, portfolio optimization problem can be defined as multi-criteria optimization in which risks have to be minimized, while, on the other hand, return has to be maximized. Unfortunately, this approach to the problem have several drawbacks [2]. First, it can be difficult to collect enough data for precise estimation of the risk and returns. Second, the estimation of return and covariance (used for defining the risk) from historical data is very prone to measurement errors [3]. Third, and finally, this model is considered to be too simplistic for practical purposes because it does not capture many properties of the real-world trading, such as maximum size of portfolio, transaction costs, preferences over assets, cost management, etc. These properties can be modeled by adding additional constraints to the basic problem formulation leading to the constrained portfolio optimization problem. Constrained problem is more complex than the unconstrained one, and belong to the class of NP-Complete problems [4].

Portfolio optimization problem can be solved using various methods and techniques. Fuzzy portfolio selection problem was successfully solved using parametric quadratic programming technique [5], and linear programming method [6]. The application of integer programming can be found in [7].

As mentioned above, constrained portfolio optimization problem adds additional real-world requirements to the basic problem formulation. Moreover, in some cases, portfolio characteristics, such is its size (number of assets in portfolio), makes the problem intractable in a reasonable amount of computational time. In those cases, exact methods can not obtain results, and the use of approximate algorithms and in particular metaheuristics in necessary.

One of the most interesting groups of metaheuristics are nature-inspired algorithms which simulate the behavior of natural systems. They can roughly be divided into two groups: evolutionary algorithms (EA) and swarm intelligence. Well-known representative

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of EA is genetic algorithms (GA) which was successfully applied on portfolio optimization problem [8].

Swarm intelligence is using principles of the collective behavior of social insect colonies and other animal groups in the search process. The key concept of swarm intelligence lies in the effect of emergent behavior of many individuals which exhibit extraordinary collective intelligence. Particle swarm optimization (PSO) is a swarm intelligence algorithm which mimics social behavior of fish schooling or bird flocking. PSO was tested on portfolio optimization problem [9]. Ant colony optimization (ACO) showed great success in solving many hard optimization problems [10], [11], [12], [13]. ACO was inspired by the foraging behavior of ants who deposit pheromone trails which help them in finding the shortest path between food sources and their nests. Artificial bee colony (ABC) metaheristics is one of the latest simulations of the honey bee swarm. In this implementation, three group of bees: employers, outlookers and scouts work together and carry exploitation and exploration processes. ABC showed outstanding results in global optimization problems [14].

In this paper, we present the firefly algorithm (FA) for portfolio optimization problem. The implementation of the FA for this problem was not found in the literature.

This paper begins with illustration of mathematical formulation of the portfolio optimization problem in Section 2. Section 3 introduces FA metaheuristics. Experimental data, problem setup and experimental results are presented in Section 4, while Section 5 concludes the paper.

2 Portfolio optimization problem

The fundamental guideline in making financial investments decisions is diversification where investors invest into different types of assets. Portfolio diversification minimizes investors' exposure to the risks while maximizing returns on portfolios.

Many methods can be applied to solving multi-objective optimization problems such is portfolio optimization. One essential method is to transform the multi-objective optimization problem into a single-objective optimization problem. This method can be further divided into two sub-types. In the first approach, one important objective function is selected for optimization, while the rest of objective functions are defined as constrained conditions. Alternatively, only one evaluation function is created by weighting the multiple objective functions [15].

The first method is defined by Markowitz and is called the standard mean-variance model [16]. In this model, the selection of risky portfolio is considered as one objective function and the mean return on an asset is considered to be one of the constraints [9]. It can be formulated as follows:

$$min\ \sigma^2_{R_p} = \sigma^2_p = \sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\omega_j Cov(\bar{R}_i\bar{R}_j) \tag{1}$$

Subject to

$$\bar{R}_p = E(R_p) = \sum_{i=1}^{N}\omega_i\bar{R}_i \geq R \tag{2}$$

$$\sum_{i=1}^{N}\omega_i = 1 \tag{3}$$

$$\omega_i \geq 0,\ \forall i \in (1, 2, ...N) \tag{4}$$

where N is the number of available assets, $\bar{R}_i$ is the mean return on an asset i and $Cov(\bar{R}_i\bar{R}_j)$ is covariance of returns of assets i and j respectively. Weight variable ω_i controls the proportion of the capital that is invested in asset i, and constraint in Eq. 3 ensures that the whole available capital is invested. In this model, the goal is to minimize the portfolio risk σ^2_p, for a given value of portfolio expected return $\bar{R}_p$.

The second method refers to the construction of only one evaluation function that models portfolio selection problem. This method comprises two distinct models: efficient frontier and sharpe ratio model [15].

In efficient frontier model, the goal is to find the different objective function values by varying desired mean return R. The best practice is to introduce new parameter $\lambda \in [0, 1]$ which is called risk aversion indicator [15]. In this case, the model is approximated to only one objective function:

$$min\ \lambda[\sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\omega_j Cov(\bar{R}_i\bar{R}_j] - (1-\lambda)[\sum_{i=1}^{N}\omega_i\bar{R}_i] \tag{5}$$

Subject to

$$\sum_{i=1}^{N}\omega_i = 1 \tag{6}$$

$$\omega_i \geq 0,\ \forall i \in (1, 2, ...N) \tag{7}$$

λ controls the relative importance of the mean return to the risk of the investor. When λ is zero, mean return of the portfolio is maximized regardless of the risk. Contrarily, when λ equals 1, risk of the portfolio is being minimized regardless of the mean return. Thus,

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with the increase of λ, the relative importance of the risk to the investor increases, and importance of the mean return decreases, and vice-versa.

With the change of the value of λ, objective function value changes also. The reason of this change is that the objective function is composed of the mean return value and the variance (risk). The dependencies between changes of λ and the mean return and variance intersections are shown on a continuous curve which is called efficient frontier in the Markowitz theory [16]. Since each point on this curve indicates an optimum, portfolio optimization problem is considered as multi-objective, but λ transforms it into single-objective optimization task.

Sharpe ratio (SR) model combines the information from mean and variance of an asset [17]. This simple model is risk-adjusted measure of mean return and can be described with the following equation [17]:

$$SR = \frac{R_p - R_f}{StdDev(p)}, \tag{8}$$

where p denotes portfolio, R_p is the mean return of the portfolio p, and R_f is a test available rate of return on a risk-free asset. $StdDev(p)$ is a measure of the risk in portfolio (standard deviation of R_p). By adjusting the portfolio weights w_i, portfolio's sharpe ratio can be maximized.

Here, we presented only the basic problem definitions. As we mentioned in the previous section, additional constraints can be applied to make the problem more realistic. For example, budget, cardinality and transaction lot constraints were successfully applied in solving portfolio optimization problem using particle swarm optimization (PSO) method in [9].

3 Implementation of the FA

FA is one of the latest swarm intelligence metaheuristics. It is inspired by the flashing behavior of fireflies. The main algorithm's principle is that each firefly moves towards the brighter firefly. Firefly's flash is used as a signaling system for attracting other fireflies. FA was first proposed for unconstrained optimization problems [18].

Three simplification rules guide the construction of the FA: each firefly attracts all other fireflies with weaker flashes (firefly's sex is neglected), attractiveness of fireflies is proportional to their brightness, while, at the other side, the brightness is reverse proportional to its distance from the light source, and the brightness of a firefly is determined, or at least affected by the distribution of the objective function.

With the increase of the distance from the lighting source, the light intensity decreases. So, light intensity follows the inverse square law:

$$I(r) = \frac{I_0}{r^2}, \tag{9}$$

where $I(r)$ is the light intensity, r is distance, and I_0 is the light intensity at the source. Besides that, the air also absorbs part of the light, and the light becomes weaker. Thus, the light absorption coefficient γ must be included in Eq. (9):

$$I(r) = \frac{I_0}{1 + \gamma r^2} \tag{10}$$

As mentioned above, attractiveness β of a firefly is proportional to its brightness (light intensity), and this can be shown in the following expression:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{11}$$

Objective function $f(x)$ is used to encode the brightness of a given firefly. It represents the light intensity at location x, as $I(x) = f(x)$.

Movement of a firefly (process of exploitation) is based upon attractiveness, and when firefly j is more attractive (brighter) than firefly i, firefly i is moving towards j:

$$x_i(t) = x_i(t) + \beta_0 r^{-\gamma r_{i,j}^2}(x_j - x_i) + \alpha(rand - 0.5), \tag{12}$$

where β_0 is attractiveness at $r = 0$, α is randomization parameter, $rand$ is random number uniformly distributed between 0 and 1, and $r_{i,j}$ is distance between fireflies i and j. This distance is calculated using Cartesian distance form:

$$r_{i,j} = ||x_i - x_j|| = \sqrt{\sum_{k=1}^{D}(x_{i,k} - x_{j,k})}, \tag{13}$$

where D is the number of problem parameters. For most problems, $\beta_0 = 0$ and $\alpha \in [0, 1]$ are adequate settings.

FA pseudo-code is shown below. Some details are omitted for simplicity.

Generate initial population of fireflies x_i,
$(i = 1, 2, 3, ..., FN)$
Light intesity I_i at point x_i is defined by $f(x)$
Define light absorption coefficient γ
Define number of iterations IN
while ($t < IN$) *do*
  for ($i = 1$ to FN) *do*
    for ($j = 1$ to i) *do*
      if ($I_j < I_i$) *then*

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Move firefly j towards firefly i in d dimension
Evaluate new solution, replace worst with better solution, and update light intensity
end if
end for
end for
Rank all fireflies, find the current best, and move them randomly
end while

In the pseudo-code above, FN is total number of fireflies in the population, IN is total number of algorithm's iterations, and t is the current iteration.

4 Problem formulation, data and results

In this section, we present portfolio optimization problem formulation used in testing FA approach, data used in the experiments and experimental results. We used the same problem formulation and data set like in [19].

4.1 Problem definition

The goal is to select weights of the each asset in the portfolio in order to maximize the portfolio's return and to minimize the portfolio's risk. We transformed multi-objective problem into single one with constraints.

The expected return of each individual security i is presented as follows:

$$E(\omega_i) = w_i * r_i, \tag{14}$$

where w_i denotes the weight of individual asset i, and r_i is the expected return of i. Total expected return of the portfolio P can be formulated as follows:

$$E(P) = \sum_{i=1}^{n} E(\omega_i), \tag{15}$$

where n is the number of securities in the portfolio P.

In our problem formulation, first goal is to maximize portfolio's expected return, and thus, the expression shown in Eq. (15) is objective function for the portfolio's return and it should be maximized.

The objective function of the portfolio variance (risk) is presented as a polynomial of second degree:

$$\sigma^2(P) = \sigma^2(w_i) = \sum_{i=1}^{n}(\omega_i^2 \sigma^2(r_i)) +$$

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} 2\omega_i \omega_j Cov(r_i, r_j), \tag{16}$$

where $\sigma^2(w_i)$ is variance of asset i, and $Cov(r_i, r_j)$ is covariance between securities i and j.

According to Eq. (15) and Eq. (16), the multi-objective function to be minimized is illustrated as:

$$H(P) = E(P) - \sigma^2(P) \tag{17}$$

Alternatively, considering individual asset i, not the whole portfolio P, it can be formulated as:

$$H(\omega_i) = E(\omega_i) - \sigma^2(\omega_i) \tag{18}$$

Problem constraints are:

$$\sum_{i=1}^{n} \omega_1 = 1 \tag{19}$$

$$\omega_i^{min} \leq \omega_i \leq \omega_i^{max} \tag{20}$$

and to reach the positive portfolio return, we used:

$$\sum_{i=1}^{n} r_i \omega_i \geq 0, \tag{21}$$

where ω_i^{min} and ω_i^{max} are minimum and maximum weights of asset i respectively.

4.2 Experimental data

For testing purposes, we used simple historical data set like in [19]. Data set is shown in Table 1.

Table 1: Data set for the experiments

Year	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
2007	-0.15	0.29	0.38	0.18	-0.10
2008	0.05	0.18	0.63	-0.12	0.15
2009	-0.43	0.24	0.46	0.42	0.15
2010	0.79	0.25	0.36	0.24	0.10
2011	0.32	0.17	-0.57	0.30	0.25

The mean return on each asset and covariance matrix are given in Tables 2 and 3 respectively.

Table 2: Mean returns for each asset

Stock 1	0.116
Stock 2	0.226
Stock 3	0.252
Stock 4	0.204
Stock 5	0.11

Table 3: Covariance matrix

	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
Stock 1	0.21728	-0.003376	-0.053492	-0.009264	0.01064
Stock 2	-0.003376	0.00253	0.008468	0.002376	-0.00456
Stock 3	-0.053492	0.008468	0.22247	-0.031128	-0.02392
Stock 4	-0.009264	0.002376	-0.031128	0.04068	0.00276
Stock 5	0.01064	-0.00456	-0.02392	0.00276	0.01675

4.3 Algorithm settings and experiment results

In this subsection, we present experimental results for testing FA for portfolio optimization problem. See subsection 4.1 for problem formulation.

Tests were performed on Intel Core 2 Duo T8500 processor @4GHz with 4GB of RAM memory, Windows 7 x64 Ultimate 64 operating system and Visual Studio 2012 with .NET 4.5 Framework. Solution number SN was set to 40, while maximum iteration number IN was set to 6000, yielding totally 240.000 objective function evaluations (40*6000). The same number of objective function evaluations was used in [20]. The algorithm was tested on 30 independent runs each starting with a different random number seed.

Since we used a set of five portfolio's assets, dimension D of a problem is 5. Each firefly in the population is a 5-dimensional vector. In initialization phase, firefly x is created using the following:

$$x_i = \omega_i^{min} + rand(0,1) * (\omega_i^{max} - \omega_i^{min}), \quad (22)$$

where $rand(0,1)$ is a random number uniformly distributed between 0 and 1.

We also used constraint handling techniques to direct the search process towards the feasible region of the search space. Equality constraints decrease efficiency of the search process by making the feasible space very small compared to the entire search space. For improving the search process, the equality constraints can be replaced by inequality constraints using the following expression [21]:

$$|h(x)| - \varepsilon \leq 0, \quad (23)$$

where $\varepsilon > 0$ is very small violation tolerance. The ε was dynamically adjusted according to the current algorithm's iteration:

$$\varepsilon(t+1) = \frac{\varepsilon(t)}{dec}, \quad (24)$$

where t is the current iteration, and dec is a value slightly larger than 1. When the value of ε reaches the predetermined threshold value, Eq. (24) is no longer applied. Summary of FA parameter set is given in Table 4.

Table 4: FA parameter set

Parameter	**Value**
Number of fireflies (FN)	40
Number of iterations (IN)	6000
Initial value for randomization parameter α	0.5
Attractiveness at $r = 0$ β_0	0.2
Absorption coefficient γ	1.0
Initial violation tolerance (ε)	1.0
Decrement (dec)	1.002
ω^{min}	0
ω^{max}	1

In experimental results, we show best, mean and worst results for objective function value, variance (risk) and average return of portfolios.

Table 5: Experimental results

	Best	**Worst**	**Mean**
Objective function	4.542	4.698	4.615
Variance	0.036	0.072	0.059
Return	0.218	0.198	0.205

In Table 6, we show portfolio weights for the best and worst results.

Table 6: Portfolio weights for best and worst results

	ω_1	ω_2	ω_3	ω_4	ω_5
Best	0.056	0.432	0.361	0.072	0.079
Worst	0.042	0.198	0.319	0.262	0.179

According to the experiment results presented in Tables 5 and 6, FA for portfolio optimization performs similar like GA approach in [19]. In [19], three variants of GA were shown: single-point, two-point and arithmetic. Arithmetic variant performed significantly better than other two variants, and also better than the FA presented in this paper. But, at the other hand, FA showed better performance than single-point and two-point variants of the GA presented in [19].

5 Conclusion

In this paper, FA for portfolio optimization problem was presented. The algorithm was tested on a set of five assets, like GA in [19]. The experimental results show that the FA metaheuristics has potential for solving this problem.

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