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Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution

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ABSTRACT

This paper discusses full fuzzy linear programming (FFLP) problems of which all parameters and variable are triangular fuzzy numbers. We use the concept of the symmetric triangular fuzzy number and introduce an approach to defuzzify a general fuzzy quantity. For such a problem, first, the fuzzy triangular number is approximated to its nearest symmetric triangular number, with the assumption that all decision variables are symmetric triangular. An optimal solution to the above-mentioned problem is a symmetric fuzzy solution. Every FLP models turned into two crisp complex linear problems; first a problem is designed in which the center objective value will be calculated and since the center of a fuzzy number is preferred to (its) margin. With a special ranking on fuzzy numbers, the FFLP transform to multi objective linear programming (MOLP) where all variables and parameters are crisp.

1. Introduction

Concept of decision analysis in fuzzy environment was first proposed by Bellman and Zadeh [1]. Some researchers have proposed several fuzzy models [2–7]. Other kinds of FLPs have also been considered in [2,8–18]. However, in all of the above-mentioned works, those cases of FLP have been studied in which not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side or the objective function coefficients were fuzzy; or the variables were not fuzzy. In this paper, we consider a problem in which that all variables and parameters are fuzzy triangular asymmetric numbers with certain conditions. Fully fuzzified linear programming problem, solution and duality have been studied in [19]. The authors in [19] used the possibilities mean value and variance of the fuzzy numbers and considered symmetric triangular fuzzy numbers data. In this manner, the coefficient vector in the objective function or the coefficient matrix of the constraints contain fuzzy elements. We will propose the nearest symmetric triangular approximate (defuzzification approach). Defuzzification methods have been widely studied for some years and were applied to fuzzy control and fuzzy expert systems. The major idea behind these methods is to obtain a typical value from a given fuzzy set according to some specified characters (center, fuzziness, gravity, median, etc.). In this paper, we use the concept of the symmetric triangular fuzzy number and introduce an approach to defuzzify a general fuzzy quantity. The basic idea of the new method is to obtain the "nearest" symmetric triangular approximation of fuzzy numbers which is a fuzzy quantity defined in [20]. Fuzzy linear programming with a multiple objective linear programming problem (MOLPP) has been considered in [17,21,11]. For solving a full fuzzy linear programming problem, we consider the ranking of the constraints. The paper is organized as follows: In Section 2, we note symmetric triangular fuzzy numbers, then we have a multiple objective linear programming problem. This MOLPP has two objective

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functions whit ordinal preference. Then we use the lexicographic method to solve it, and because of the existence of fuzzy inequalities in properties of fuzzy numbers and the MOLPP, in Section 3 we explain a fuzzy linear programming; in Section 4 we have an example; conclusion is drawn in Section 5.

2. Preliminaries

We represent an arbitrary fuzzy number by an ordered pair of functions $\tilde{u} =: (\underline{u}(r), \overline{u}(r))$, $0 \leqslant r \leqslant 1$, which satisfy the following requirements:

- $\underline{u}(r)$ is a bounded left continuous nondecreasing function over [0,1].
- $\overline{u}(r)$ is a bounded left continuous nonincreasing function over [0,1].
- $\underline{u}(r)$ and $\overline{u}(r)$ are right continuous at 0.
- $\underline{u}(r) \leqslant \overline{u}(r)$, $0 \leqslant r \leqslant 1$.

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \leqslant r \leqslant 1$.

Definition 2.1. $C_{\tilde{u}} = Core(\tilde{u}) = \overline{u}(1) = \underline{u}(1)$; and $w^L_{\tilde{u}} = C_{\tilde{u}} - \underline{u}(0) \geqslant 0$ and $w^R_{\tilde{u}} = \overline{u}(0) - C_{\tilde{u}} \geqslant 0$ are the left and right margins of ũ.

Definition 2.2. The fuzzy number $\tilde{t} =: \left(C_{\tilde{t}} - w^L_{\tilde{t}} + w^L_{\tilde{t}} r, C_{\tilde{t}} + w^R_{\tilde{t}} - w^R_{\tilde{t}} r\right) =: \left(C_{\tilde{t}}, w^L_{\tilde{t}}, w^R_{\tilde{t}}\right), 0 \leqslant r \leqslant 1$ is an asymmetric triangular fuzzy number $\widehat{ATFN}$. As a matter of fact $C_{\tilde{t}} - w^L_{\tilde{t}} + w^L_{\tilde{t}} r = \underline{t}(r)$ and $C_{\tilde{t}} + w^R_{\tilde{t}} - w^R_{\tilde{t}} r = \overline{t}(r)$ where $C_{\tilde{t}}, w^L_{\tilde{t}}, w^R_{\tilde{t}} \in \Re$. Let $\widehat{A.S.T}$ be the set of all $\widehat{ATFN}$.

A conventional fuzzy number is the symmetric triangular fuzzy number $S[x_o,\sigma]$ where $w^L_S = w^R_S = \sigma$ centered at x_o with basis 2σ. Its parametric form is $S[x_o,\sigma] = :(x_o - \sigma + r(\sigma), x_o + \sigma - r(\sigma)): = (x_o;\sigma), 0 \leqslant r \leqslant 1$ which $x_o, \sigma \in \Re$, x_o is the center and $\sigma \geqslant 0$ is the margin of $S[x_o,\sigma]$ and it is called symmetric triangular fuzzy number ($\widehat{STFN}$).

Let $\widehat{S.T}$ be the set of all $\widehat{STFN}$.

Therefore the following properties for the $\widehat{A.S.T}$ are satisfying in the $\widehat{S.T}$.

Definition 2.3. Let $\tilde{t} = \left(C_1, w^L_1, w^R_1\right), \tilde{u} = \left(C_2, w^L_2, w^R_2\right) \in \widehat{A.S.T}$ and $k \in \Re$, by using extension principal we can define:

1. $\tilde{t} = \tilde{u}$ if and only if $C_1 = C_2$; and $w^L_1 = w^L_2$ and $w^R_1 = w^R_2$.
2. $\tilde{t} + \tilde{u} = (C_1 + C_2, w^L_1 + w^L_2, w^R_1 + w^R_2)$.
3.

$$k\tilde{t} = \begin{cases} (kC_1, kw^L_1, kw^R_1), & k \geqslant 0 \\ (kC_1, -kw^R_1, -kw^L_1), & k < 0 \end{cases} \tag{2.1}$$

Definition 2.4. For two fuzzy numbers in parametric forms $\tilde{t} = (\underline{t}(r), \overline{t}(r))$, $\tilde{u} = (\underline{u}(r), \overline{u}(r))$ we have: $\tilde{t}\tilde{u} = \tilde{h} = (\underline{h}(r), \overline{h}(r))$ where $\underline{h}(r) = Min\{\underline{t}(r)\underline{u}(r), \overline{t}(r)\overline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\}, \overline{h}(r)) = Max\{\underline{t}(r)\underline{u}(r), \overline{t}(r)\overline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\}$ for example for two positive $\widehat{A.S.T}s\tilde{t} = (C_{\tilde{t}} + w^L_{\tilde{t}}(r-1), C_{\tilde{t}} + w^R_{\tilde{t}}(1-r))$, and $\tilde{u} = (C_{\tilde{u}} + w^L_{\tilde{u}}(r-1), C_{\tilde{u}} + w^R_{\tilde{u}}(1-r))$ where $C_{\tilde{t}} - w^L_{\tilde{t}} \geqslant 0$ and $C_{\tilde{u}} - w^L_{\tilde{u}} \geqslant 0$ we have: $\tilde{t}\tilde{u} = (C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}w^L_{\tilde{u}}(r-1) + w^L_{\tilde{t}}(r-1)C_{\tilde{u}} + w^L_{\tilde{t}}w^L_{\tilde{u}}(r-1)^2, C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}w^R_{\tilde{u}}(1-r) + w^R_{\tilde{t}}(1-r)C_{\tilde{u}} + w^R_{\tilde{t}}w^R_{\tilde{u}}(1-r)^2)$. Suppose $\widetilde{A} \in E^{n\times n}$, E is the eucludian space of fuzzy numbers. $\widetilde{X} = (\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_n)^T$ and $\widetilde{Y} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)^T$ are $\widehat{ATFN}$ vectors this means that $\widetilde{X}, \widetilde{Y} \in \widehat{A.S.T}^n$. Now we have

1. $Core(\widetilde{X} + \widetilde{Y}) = Core(\widetilde{X}) + Core(\widetilde{Y})$.
2. $Core(\widetilde{A}\widetilde{X}) = Core(\widetilde{A})Core(\widetilde{X})$.
3. $\widetilde{A}(\widetilde{X} + \widetilde{Y}) = \widetilde{A}\widetilde{X} + \widetilde{A}\widetilde{Y}$.

Definition 2.5 (*Ordering on* $\widehat{S.T}$). Let $\tilde{t} = (x_{o_1}; \sigma_1)$ and $\tilde{u} = (x_{o_2}; \sigma_2)$ are $\widehat{STFN}$s. We say $\tilde{t} <^* \tilde{u}$ if and only if:

1. $x_{o_1} < x_{o_2}$ or
2. $x_{o_1} = x_{o_2}$ and $\sigma_1 > \sigma_2$.

In the case equality we have $\tilde{t} =^* \tilde{u}$ if and only if $((x_{o_1} = x_{o_2}) \wedge (\sigma_1 = \sigma_2))$.

And $\tilde{t} \leqslant^* \tilde{u}$ if and only if $(\tilde{t} <^* \tilde{u} \vee \tilde{t} =^* \tilde{u})$ it means that:

$(x_{o_1} < x_{o_2}) \vee [(x_{o_1} = x_{o_2} \wedge \sigma_1 > \sigma_2) \vee (x_{o_1} = x_{o_2} \wedge \sigma_1 = \sigma_2)]$, that is equivalent with the following relation: $(x_{o_1} < x_{o_2}) \vee [(x_{o_1} = x_{o_2} \wedge \sigma_1 \geqslant \sigma_2)]$.

Its clear that by this definition $\widehat{STFN}$s have the triple axiom. For any $\tilde{t}, \tilde{u} \in \widehat{S.T}$ we have only one of these $(\tilde{t} <^* \tilde{u}, \tilde{t} = \tilde{u}, \tilde{t} >^* \tilde{u})$.

2.1. Nearest symmetric triangular defuzzification

Let ũ be a general fuzzy number and $(\underline{u}(r), \overline{u}(r))$ be its parametric form. To obtain a symmetric triangular fuzzy number which is the nearest to ũ, we should minimize:

$$D^2(\tilde{u}, S[x_o, \sigma]) = \int_0^1 (\underline{u}(r) - \underline{S[x_o, \sigma]}(r))^2 dr + \int_0^1 (\overline{u}(r) - \overline{S[x_o, \sigma]}(r))^2 dr \tag{2.2}$$

With respect to x_o and σ. If $S[x_o,\sigma]$ minimizes $D(\tilde{u},S[x_o,\sigma])$, $S[x_o,\sigma]$ provides a defuzzification of ũ with a defuzzifier x_o and σ. In order to minimize $D(\tilde{u},S[x_o,\sigma])$, we consider

$$\frac{\partial D(\tilde{u}, S[x_o, \sigma])}{\partial \sigma} = 0 \tag{2.3}$$

$$\frac{\partial D(\tilde{u}, S[x_o, \sigma])}{\partial x_o} = 0 \tag{2.4}$$

the solution is

$$\sigma = \frac{3}{2} \int_0^1 (\overline{u}(r) - \underline{u}(r))(1 - r) dr \tag{2.5}$$

$$x_o = \frac{1}{2} \int_0^1 (\overline{u}(r) + \underline{u}(r)) dr \tag{2.6}$$

i.e., the nearest symmetric triangular defuzzification of ũ is given by the Center x_o (2.6) and fuzziness σ (2.5).

3. Full fuzzy linear programming problems

In this section we are going to reduce the following FFLP (3.7) to two crisp LPs.

$$\begin{cases} Max\left(C_{\widetilde{C}}, w^L_{\widetilde{C}}, w^R_{\widetilde{C}}\right)\left(C_{\widetilde{X}}, w^L_{\widetilde{X}}, w^R_{\widetilde{X}}\right) \\ \text{s.t.}\left(C_{\widetilde{A}}, w^L_{\widetilde{A}}, w^R_{\widetilde{A}}\right)\left(C_{\widetilde{X}}, w^L_{\widetilde{X}}, w^R_{\widetilde{X}}\right) =^* \left(C_{\tilde{b}}, w^L_{\tilde{b}}, w^R_{\tilde{b}}\right) \\ C_{\widetilde{X}} - w^L_{\widetilde{X}} \geqslant 0 \\ \left(C_{\widetilde{X}}, w^L_{\widetilde{X}}, w^R_{\widetilde{X}}\right) \in \widehat{N.S.T}^n \end{cases} \tag{3.7}$$

where $\widetilde{X} = \left(C_{\widetilde{X}}, w^L_{\widetilde{X}}, w^R_{\widetilde{X}}\right), \tilde{b} = \left(C_{\tilde{b}}, w^L_{\tilde{b}}, w^R_{\tilde{b}}\right), \widetilde{A} = \left(C_{\widetilde{A}}, w^L_{\widetilde{A}} w^R_{\widetilde{A}}\right), \widetilde{C} = \left(C_{\widetilde{C}}, w^L_{\widetilde{C}}, w^R_{\widetilde{C}}\right)$, $C_{\widetilde{X}} = Core(\widetilde{X})$, $C_{\tilde{b}} = Core(\tilde{b})$, $C_{\widetilde{A}} = Core(\widetilde{A})$, $C_{\widetilde{C}} = Core(\widetilde{C})$, $w_{\widetilde{X}}$, $w_{\tilde{b}}$, $w_{\widetilde{A}}$, $w_{\widetilde{C}}$ are the margins of $\widetilde{X}$, $\tilde{b}$, $\widetilde{A}$ and $\widetilde{C}$, respectively. $\widetilde{A} \in E^{m\times n}$, $\tilde{b} \in E^m$, $\widetilde{C}$ and $\widetilde{X} \in E^n$ and $\widetilde{A}$, $\widetilde{C}$, $\widetilde{X}$, $\tilde{b}$ are arbitrary fuzzy matrix and fuzzy number vectors. Where $C_{\widetilde{A}} - w^L_{\widetilde{A}} \geqslant 0$, $C_{\widetilde{C}} - w^L_{\widetilde{C}} \geqslant 0$ and $C_{\tilde{b}} - w^L_{\tilde{b}} \geqslant 0$. We apply the fuzzy production of two positive fuzzy asymmetric parameter numbers, and with the Eqs. (2.5) and (2.6); $\left(x_{o_{\widetilde{C}\widetilde{X}}}, \sigma_{\widetilde{C}\widetilde{X}}\right)$, $\left(x_{o_{\widetilde{A}\widetilde{X}}}, \sigma_{\widetilde{A}\widetilde{X}}\right)$, $(x_{o_{\tilde{b}}}, \sigma_{\tilde{b}})$will be the nearest symmetric triangular fuzzy numbers to $\widetilde{C}\widetilde{X}$, $\widetilde{A}\widetilde{X}$, and $\tilde{b}$, that are derived from the following relations:

$$\begin{aligned} \sigma_{\widetilde{C}\widetilde{X}} &= 3/2 \int_0^1 (C_{\widetilde{C}} C_{\widetilde{X}} + C_{\widetilde{C}} w^R_{\widetilde{X}}(1 - r) + w^R_{\widetilde{C}} C_{\widetilde{X}}(1 - r) + w^R_{\widetilde{C}} w^R_{\widetilde{X}}(1 - r)^2)(1 - r) dr - 3/2 \int_0^1 (C_{\widetilde{C}} C_{\widetilde{X}} + C_{\widetilde{C}} w^L_{\widetilde{X}}(r - 1) \\ &\quad + w^L_{\widetilde{C}} C_{\widetilde{X}}(r - 1) + w^L_{\widetilde{C}} w^L_{\widetilde{X}}(r - 1)^2)(1 - r) dr \\ &= 1/2 C_{\widetilde{C}} w^R_{\widetilde{X}} + 1/2 w^R_{\widetilde{C}} C_{\widetilde{X}} + 3/8 w^R_{\widetilde{C}} w^R_{\widetilde{X}} + 1/2 C_{\widetilde{C}} w^L_{\widetilde{X}} + 1/2 w^L_{\widetilde{C}} C_{\widetilde{X}} - 3/8 w^L_{\widetilde{C}} w^L_{\widetilde{X}}, \end{aligned} \tag{3.8}$$

$$\begin{aligned} x_{o_{\widetilde{C}\widetilde{X}}} &= 1/2 \int_0^1 (C_{\widetilde{C}} C_{\widetilde{X}} + C_{\widetilde{C}} w^R_{\widetilde{X}}(1 - r) + w^R_{\widetilde{C}} C_{\widetilde{X}}(1 - r) + w^R_{\widetilde{C}} w^R_{\widetilde{X}}(1 - r)^2) dr + 1/2 \int_0^1 (C_{\widetilde{C}} C_{\widetilde{X}} + C_{\widetilde{C}} w^L_{\widetilde{X}}(r - 1) \\ &\quad + w^L_{\widetilde{C}} C_{\widetilde{X}}(r - 1) + w^L_{\widetilde{C}} w^L_{\widetilde{X}}(r - 1)^2) dr \\ &= C_{\widetilde{C}} C_{\widetilde{X}} + 1/4 C_{\widetilde{C}} w^R_{\widetilde{X}} + 1/4 w^R_{\widetilde{C}} C_{\widetilde{X}} + 1/6 w^R_{\widetilde{C}} w^R_{\widetilde{X}} - 1/4 C_{\widetilde{C}} w^L_{\widetilde{X}} - 1/4 w^L_{\widetilde{C}} C_{\widetilde{X}} + 1/6 w^L_{\widetilde{C}} w^L_{\widetilde{X}} \end{aligned} \tag{3.9}$$

And for the constrains we have:

$$\sigma_{\widetilde{A}\widetilde{X}} = 3/2\int_0^1 (C_{\widetilde{A}}C_{\widetilde{X}} + C_{\widetilde{A}}w^R_{\widetilde{X}}(1-r) + w^R_{\widetilde{A}}C_{\widetilde{X}}(1-r) + w^R_{\widetilde{A}}w^R_{\widetilde{X}}(1-r)^2)(1-r)dr - 3/2\int_0^1 (C_{\widetilde{A}}C_{\widetilde{X}} + C_{\widetilde{A}}w^L_{\widetilde{X}}(r-1) + w^L_{\widetilde{A}}C_{\widetilde{X}}(r-1) + w^L_{\widetilde{A}}w^L_{\widetilde{X}}(r-1)^2)(1-r)dr = 1/2C_{\widetilde{A}}w^R_{\widetilde{X}} + 1/2w^R_{\widetilde{A}}C_{\widetilde{X}} + 3/8w^R_{\widetilde{A}}w^R_{\widetilde{X}} + 1/2C_{\widetilde{A}}w^L_{\widetilde{X}} + 1/2w^L_{\widetilde{A}}C_{\widetilde{X}} - 3/8w^L_{\widetilde{A}}w^L_{\widetilde{X}}, \tag{3.10}$$

$$x_{o_{\widetilde{A}\widetilde{X}}} = 1/2\int_0^1 (C_{\widetilde{A}}C_{\widetilde{X}} + C_{\widetilde{A}}w^R_{\widetilde{X}}(1-r) + w^R_{\widetilde{A}}C_{\widetilde{X}}(1-r) + w^R_{\widetilde{A}}w^R_{\widetilde{X}}(1-r)^2)dr + 1/2\int_0^1 (C_{\widetilde{A}}C_{\widetilde{X}} + C_{\widetilde{A}}w^L_{\widetilde{X}}(r-1) + w^L_{\widetilde{A}}C_{\widetilde{X}}(r-1) + w^L_{\widetilde{A}}w^L_{\widetilde{X}}(r-1)^2)dr = C_{\widetilde{A}}C_{\widetilde{X}} + 1/4C_{\widetilde{A}}w^R_{\widetilde{X}} + 1/4w^R_{\widetilde{A}}C_{\widetilde{X}} + 1/6w^R_{\widetilde{A}}w^R_{\widetilde{X}} - 1/4C_{\widetilde{A}}w^L_{\widetilde{X}} - 1/4w^L_{\widetilde{A}}C_{\widetilde{X}} + 1/6w^L_{\widetilde{A}}w^L_{\widetilde{X}}, \tag{3.11}$$

For the right hand side of the constraint we have:

$$\sigma_b = 3/2\int_0^1 (C_{\tilde b} + w^R_{\tilde b} - w^R_{\tilde b}r)(1-r)dr - 3/2\int_0^1 (C_{\tilde b} - w^L_{\tilde b} + w^L_{\tilde b}r)(1-r)dr = 1/2w^R_{\tilde b} + 1/2w^L_{\tilde b}, \tag{3.12}$$

$$x_{o_b} = \frac{1}{2}\int_0^1 ((C_{\tilde b} + w^R_{\tilde b} - w^R_{\tilde b}r) + (C_{\tilde b} - w^L_{\tilde b} + w^L_{\tilde b}r))dr = C_{\tilde b} + 1/4w^R_{\tilde b} - 1/4w^L_{\tilde b}, \tag{3.13}$$

We could view the problem (3.7) as (MOLP). In (3.14) we will have $C_{\widetilde{C}\widetilde{X}} = F_0(\widetilde{X})$ in the first criteria and in the (3.15) we will have $w_{\widetilde{C}\widetilde{X}} = F_1(\widetilde{X})$ in the second criteria. In the constraints we will have $C_{\widetilde{A}\widetilde{X}} = C_{\tilde b}$ and $w_{\widetilde{A}\widetilde{X}} = w_{\tilde b}$, noting the Definition 2.5 for ranking the fuzzy numbers, for maximizing the objective function of problem (3.7), we must solve a maximization problem for the Core and a minimization problem for the margin. Suppose that $S = \left\{\widetilde{X} \mid \widetilde{A}\widetilde{X} =^* \tilde{b}, C_{\widetilde{X}} - w^L_{\widetilde{X}} \geqslant 0, \widetilde{X} \in \widehat{N.S.T^n}\right\}$. We know the preference of core of solution respect to margins is ordinal, then by applying the lexicography rule we will have the following formal representation for the Center and Fuzziness problems:

$$\begin{cases} \text{Max} & F_0(\widetilde{X}) \\ \text{s.t.} & \widetilde{X} \in S \end{cases} \tag{3.14}$$

and the problem of fuzziness is as follows:

$$\begin{cases} \text{Min} & F_1(\widetilde{X}) \\ \text{s.t.} & \widetilde{X} \in S \\ & C_{\widetilde{C}\widetilde{X}} = a^* \end{cases} \tag{3.15}$$

where a^* is the optimal value of the objective function of (3.14). The last condition is a guarantee for satisfying the optimal solution of (3.15) in (3.14). By solving (3.5) we derive one of the pareto optimal solution. We will have the following problem for (3.14):

$$\begin{cases} \text{Max} & C_{\widetilde{C}}C_{\widetilde{X}} + 1/4C_{\widetilde{C}}w^R_{\widetilde{X}} + 1/4w^R_{\widetilde{C}}C_{\widetilde{X}} + 1/6w^R_{\widetilde{C}}w^R_{\widetilde{X}} - 1/4C_{\widetilde{C}}w^L_{\widetilde{X}} - 1/4w^L_{\widetilde{C}}C_{\widetilde{X}} + 1/6w^L_{\widetilde{C}}w^L_{\widetilde{X}} \\ \text{s.t.} & C_{\widetilde{A}}C_{\widetilde{X}} + 1/4C_{\widetilde{A}}w^R_{\widetilde{X}} + 1/4w^R_{\widetilde{A}}C_{\widetilde{X}} + 1/6w^R_{\widetilde{A}}w^R_{\widetilde{X}} - 1/4C_{\widetilde{A}}w^L_{\widetilde{X}} - 1/4w^L_{\widetilde{A}}C_{\widetilde{X}} + 1/6w^L_{\widetilde{A}}w^L_{\widetilde{X}} \\ & = C_{\tilde b} + 1/4w^R_{\tilde b} - 1/4w^L_{\tilde b}, \\ & 1/2C_{\widetilde{A}}w^R_{\widetilde{X}} + 1/2w^R_{\widetilde{A}}C_{\widetilde{X}} + 3/8w^R_{\widetilde{A}}w^R_{\widetilde{X}} + 1/2C_{\widetilde{A}}w^L_{\widetilde{X}} + 1/2w^L_{\widetilde{A}}C_{\widetilde{X}} - 3/8w^L_{\widetilde{A}}w^L_{\widetilde{X}} \\ & = 1/2w^R_{\tilde b} + 1/2w^L_{\tilde b} \\ & C_{\widetilde{X}} - w^L_{\widetilde{X}} \geqslant 0 \\ & w^L_{\widetilde{X}} \geqslant 0, w^R_{\widetilde{X}} \geqslant 0 \end{cases} \tag{3.16}$$

1. If (3.16) has a unique optimal solution $\left(C_{\widetilde{X}}, w^L_{\widetilde{X}}, w^R_{\widetilde{X}}\right)$, then it is a pareto optimal solution of (3.7).
2. If (3.16) has alternative optimal solutions, then $\left(C_{\widetilde{X}}, w^L_{\widetilde{X}}, w^R_{\widetilde{X}}\right)$ is a pareto optimal solution of (3.7) if it is optimal solution of (3.17).

If problem (3.16) has a unique optimal solution then we have obtained the optimal solution of problem (3.7), otherwise it means that problem (3.16) has alternative optimal solutions, we solve (3.17) on the optimal solutions set of the problem (3.16). The objective function of (3.17) is the minimization of the margin of the objective function of the principal problem. The constrains of (3.17) are similar to the constrains of (3.16) it means that the center and margin of the constrains of the principal problem besides an additional constraint related to the optimized value of the first problem (maximization for the center of the principal problem). If the problem (3.16) has alternative solution such that, there are more solutions with unique objective function value it means that we derive more fuzzy solutions that their cores are the same, therefore for ranking the solutions in the basis of the definition (2.5) we must solve the problem related to margin. So we must solve the following problem (3.17):

$$\begin{cases} \text{Min} \quad 1/2C_{\underset{C}{\sim}}w^R_{\underset{X}{\sim}} + 1/2w^R_{\underset{C}{\sim}}C_{\underset{X}{\sim}} + 3/8w^R_{\underset{C}{\sim}}w^R_{\underset{X}{\sim}} + 1/2C_{\underset{C}{\sim}}w^L_{\underset{X}{\sim}} + 1/2w^L_{\underset{C}{\sim}}C_{\underset{X}{\sim}} - 3/8w^L_{\underset{C}{\sim}}w^L_{\underset{X}{\sim}} \\ \text{s.t.} \quad C_{\underset{A}{\sim}}C_{\underset{X}{\sim}} + 1/4C_{\underset{A}{\sim}}w^R_{\underset{X}{\sim}} + 1/4w^R_{\underset{A}{\sim}}C_{\underset{X}{\sim}} + 1/6w^R_{\underset{A}{\sim}}w^R_{\underset{X}{\sim}} - 1/4C_{\underset{A}{\sim}}w^L_{\underset{X}{\sim}} - 1/4w^L_{\underset{A}{\sim}}C_{\underset{X}{\sim}} + 1/6w^L_{\underset{A}{\sim}}w^L_{\underset{X}{\sim}} \\ \quad = C_{\tilde{b}} + 1/4w^R_{\tilde{b}} - 1/4w^L_{\tilde{b}}, \\ \quad 1/2C_{\underset{A}{\sim}}w^R_{\underset{X}{\sim}} + 1/2w^R_{\underset{A}{\sim}}C_{\underset{X}{\sim}} + 3/8w^R_{\underset{A}{\sim}}w^R_{\underset{X}{\sim}} + 1/2C_{\underset{A}{\sim}}w^L_{\underset{X}{\sim}} + 1/2w^L_{\underset{A}{\sim}}C_{\underset{X}{\sim}} - 3/8w^L_{\underset{A}{\sim}}w^L_{\underset{X}{\sim}} \\ \quad = 1/2w^R_{\tilde{b}} + 1/2w^L_{\tilde{b}} \\ \quad C_{\underset{X}{\sim}} - w^L_{\underset{X}{\sim}} \geqslant 0 \\ \quad w^L_{\underset{X}{\sim}} \geqslant 0, w^R_{\underset{X}{\sim}} \geqslant 0 \\ \quad C_{\underset{CX}{\sim\sim}} = a^* \end{cases} \tag{3.17}$$

where a^* is the optimal objective value of (3.16).

Remark 3.1. Problem (3.7) is reduced to problems (3.16) and (3.17).

Theorem 3.1. *$\widetilde{X^*} = \left(C_{\underset{X^*}{\sim}}, w^L_{\underset{X^*}{\sim}}, w^R_{\underset{X^*}{\sim}}\right)$ is an optimal solution of (3.7) if $\left(C_{\underset{X^*}{\sim}}, w^L_{\underset{X^*}{\sim}}, w^R_{\underset{X^*}{\sim}}\right)$ is an optimal solution of (3.16) and (3.17).*

Proof. By contradiction, if $\widetilde{X^*}$ is an optimal solution of (3.16) and (3.17), and it is not the optimal solution of (3.7). By Definition 2.5, there exists a feasible solution of (3.7) , say $\widetilde{X}^\circ$, such: $\left(C_{\underset{CX^\circ}{\sim\sim}} > C_{\underset{CX^*}{\sim\sim}}\right) \wedge (w_{\underset{CX^\circ}{\sim\sim}} \leqslant w_{\underset{CX^*}{\sim\sim}})$ and we know that $\widetilde{X}^\circ$ is a feasible solution of (3.16) and (3.17) as well, that is more desirable than $\widetilde{X}^*$ and this is a contradiction or $\left(C_{\underset{CX^\circ}{\sim\sim}} \geqslant C_{\underset{CX^*}{\sim\sim}}\right) \wedge \left(w_{\underset{CX^\circ}{\sim\sim}} < w_{\underset{CX^*}{\sim\sim}}\right)$ that with the above analysis we confronting with a contradiction. □

4. Examples

Example 4.1. Consider the FFLP where $\widetilde{C} = \begin{bmatrix} (15,5,2) \\ (16,6,4) \\ (14,4,3) \\ (12,2,2) \end{bmatrix}$, $\widetilde{A} = \begin{bmatrix} (10,2,3) & (11,1,2) & (12,3,1) & (15,4,2) \\ (14,2,2) & (18,4,1) & (17,3,3) & (14,1,4) \end{bmatrix}$, $\tilde{b} = \begin{bmatrix} (411.75,140,162) \\ (539.5,154,220) \end{bmatrix}$,

The first problem is related to the Core of the solution:

$$\begin{array}{ll} \text{Max} & a = 14.25x_1 + 15.5x_2 + 11.75x_3 + 12x_4 - 2.92x_1' - 3x_2' - 2.83x_3' - 2.7x_4' + 4.1x_1'' + 4.7x_2'' + 4x_3'' + 3.3x_4'' \\ \text{s.t.} & 10.25x_1 + 11.25x_2 + 11.5x_3 + 14.5x_4 - 2.17x_1' - 2.58x_2' - 2.5x_3' - 3.08x_4' + 3x_1'' + 3.08x_2'' \\ & +3.17x_3'' + 4.08x_4'' = 411.25 \\ & 14x_1 + 17.25x_2 + 17x_3 + 14.75x_4 - 3.16x_1' - 3.83x_2' - 3.75x_3' - 3.3x_4' + 3.83x_1'' + 4.66x_2'' \\ & +4.75x_3'' + 4.16x_4'' = 556 \\ & 2.5x_1 + 1.5x_2 + 2x_3 + 3x_4 - 4.25x_1' - 5.125x_2' - 4.875x_3' - 7x_4' + 6.125x_1'' + 1.25x_2'' + 6.375x_3'' + 8.25x_4'' = 151 \\ & 2x_1 + 2.5x_2 + 3x_3 + 2.5x_4 - 6.25x_1' - 7.5x_2' - 7.375x_3' - 6.625x_4' + 7.75x_1'' + 9.375x_2'' + 9.625x_3'' + 8.5x_4'' = 187 \\ & x_1 - x_1' \geqslant 0 \\ & x_2 - x_2' \geqslant 0 \\ & x_3 - x_3' \geqslant 0 \\ & x_4 - x_4' \geqslant 0 \\ & x_1, x_2, x_3, x_4, x_1', x_2', x_3', x_4', x_1'', x_2'', x_3'', x_4'' \geqslant 0 \end{array}$$

After solving this problem, we have $a^* = 560$, $y = 0$ and $\widetilde{X}^* = \begin{bmatrix} (38.14, 10.25, 0) \\ (0, 0, 3.3091) \\ (0, 0, 0) \\ (2.65, 0, 0) \end{bmatrix}$ therefore the second problem is as follows:

$$\begin{array}{ll} \text{Min} & 3.5x_1 + 5x_2 + 3.5x_3 + 2x_4 + 5.625x_1' + 5.75x_2' + 5.5.83x_3' + 5.25x_4' + 8.25x_1'' + 9.5x_2'' + 8.125x_3'' + 6.75x_4'' \\ \text{s.t.} & 10.25x_1 + 11.25x_2 + 11.5x_3 + 14.5x_4 - 2.17x_1' - 2.58x_2' - 2.5x_3' - 3.08x_4' + 3x_1'' + 3.08x_2'' \\ & +3.17x_3'' + 4.08x_4'' = 411.25 \\ & 14x_1 + 17.25x_2 + 17x_3 + 14.75x_4 - 3.16x_1' - 3.83x_2' - 3.75x_3' - 3.3x_4' + 3.83x_1'' + 4.66x_2'' + 4.75x_3'' + 4.16x_4'' = 556 \\ & 2.5x_1 + 1.5x_2 + 2x_3 + 3x_4 - 4.25x_1' - 5.125x_2' - 4.875x_3' - 7x_4' + 6.125x_1'' + 1.25x_2'' + 6.375x_3'' + 8.25x_4'' = 151 \\ & 2x_1 + 2.5x_2 + 3x_3 + 2.5x_4 - 6.25x_1' - 7.5x_2' - 7.375x_3' - 6.625x_4' + 7.75x_1'' + 9.375x_2'' + 9.625x_3'' + 8.5x_4'' = 187 \\ & 14.25x_1 + 15.5x_2 + 11.75x_3 + 12x_4 - 2.92x_1' - 3x_2' - 2.83x_3' - 2.7x_4' + 4.1x_1'' + 4.7x_2'' + 4x_3'' + 3.3x_4'' = 560 \\ & x_1 - x_1' \geqslant 0 \\ & x_2 - x_2' \geqslant 0 \\ & x_3 - x_3' \geqslant 0 \end{array}$$

$$x_4 - x'_4 \geqslant 0$$
$$x_1, x_2, x_3, x_4, x'_1, x'_2, x'_3, x'_4, x''_1, x''_2, x''_3, x''_4 \geqslant 0.$$

The solution is $\tilde{X}^* = \begin{bmatrix} (37.47, 8.33, 0) \\ (0, 0, 3.82) \\ (0, 0, 0) \\ (2.97, 1.18, 0) \end{bmatrix}$ and the value of second problem is 226.3732. If we substitute the derived solution from the first section of this example related to Core or the second section of the example related to margin into the objective function of the principal problem, then apart from the operation errors, center of it, is equal to the optimum value of the first objective function related to Core problem and margin of it, is equal to the optimum value of the second objective function related to margin problem. It means that the optimal objective value of the principal problem is a fuzzy number and in the fuzzy format is (560,226.3,226.3).

5. Conclusion

This paper, has presented a new method to convert a FFLP into two corresponding LPs. The ordinal preference of the Core of the solution respect to marriages, cause to attention to the MOLPP technique. By using the relative productions, this technique could be applied in the subject that the coefficient matrix in the constrains or vector in the objective function, have negative entries as well. Above proofs for the case minimization analogies. Although we used from $C_{\underset{\sim}{A}} - w^L_{\underset{\sim}{A}} \geqslant 0$, $C_{\underset{\sim}{C}} - w^L_{\underset{\sim}{C}} \geqslant 0$ and $C_{\tilde{b}} - w^L_{\tilde{b}} \geqslant 0$, $C_{\underset{\sim}{X}} - w^L_{\underset{\sim}{X}} \geqslant 0$, the value of the objective functions might be not satisfied in the above conditions, it may be happens because of the fuzzy production properties (we see in Example 4.1). The FFLP is unbounded if and only if the problem related to the Center is unbounded.

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