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Mehar's method for solving fully fuzzy linear programming problems with *L-R* fuzzy parameters

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ABSTRACT

To the best of our knowledge, there is no method in literature for solving such fully fuzzy linear programming (FLP) problems in which some or all the parameters are represented by unrestricted *L-R* flat fuzzy numbers. Also, to propose such a method, there is need to find the product of unrestricted *L-R* flat fuzzy numbers. However, there is no method in the literature to find the product of unrestricted *L-R* flat fuzzy numbers.

In this paper, firstly the product of unrestricted *L-R* flat fuzzy numbers is proposed and then with the help of proposed product, a new method (named as Mehar's method) is proposed for solving fully FLP problems. It is also shown that the fully FLP problems which can be solved by the existing methods can also be solved by the Mehar's method. However, such fully FLP problems in which some or all the parameters are represented by unrestricted *L-R* flat fuzzy numbers can be solved by Mehar's method but can not be solved by any of the existing methods.

1. Introduction

Linear programming is one of the most frequently applied operation research techniques. Although, it has been investigated and expanded for more than six decades by many researchers and from the various point of views, it is still useful to develop new approaches in order to better fit the real world problems within the framework of linear programming.

In conventional approach, parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. Usually, the value of many parameters of a linear programming model is estimated by experts. Clearly, it can not be assumed the knowledge of experts is so precise. Since, Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environments, a number of researchers have exhibited their interest to solve the FLP problems [2–8] and fully FLP problems [9–15].

On the basis of deep study of the existing methods for solving fully FLP problems, it can be concluded that there is no method in the literature for solving fully FLP problems in which some or all the parameters are represented by unrestricted *L-R* flat fuzzy numbers.

This paper is organised as follows: In Section 2, some basic definitions and arithmetic operations of *L-R* flat fuzzy numbers are presented. In Section 3, limitations of the existing method [9] are pointed out. In Section 4, product of unrestricted *L-R* flat fuzzy numbers is introduced. In Section 5, a new method, named as Mehar's method, is proposed to find the fuzzy optimal solution of fully FLP problems. In Section 6, advantages of the Mehar's method over the existing methods are discussed and to illustrate the Mehar's method numerical example is solved. Obtained results are discussed in Section 7. Conclusions are discussed in Section 8.

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2. Preliminaries

In this section, some basic definitions and arithmetic operations of *L-R* flat fuzzy numbers are presented.

2.1. Basic definitions

In this section, some basic definitions of *L-R* flat fuzzy numbers are presented.

Definition 2.1 [16]. A function $L:[0,\infty)\rightarrow[0,1]$ (or $R:[0,\infty)\rightarrow[0,1]$) is said to be reference function of fuzzy number if and only if

(i) $L(0)=1$ (or $R(0)=1$)
(ii) L (or R) is non-increasing on $[0,\infty)$.

Definition 2.2 [16]. A fuzzy number $\widetilde{A}$, defined on universal set of real numbers $\mathbb{R}$, denoted as $(m,n,\alpha,\beta)_{LR}$, is said to be an *LR* flat fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x)=\begin{cases} L(\frac{m-x}{\alpha}) & x\leqslant m,\alpha>0,\\ R(\frac{x-n}{\beta}) & x\geqslant n,\beta>0,\\ 1 & m\leqslant x\leqslant n.\end{cases}$$

Definition 2.3 [14]. An *L-R* flat fuzzy number $\widetilde{A}=(m,n,\alpha,\beta)_{LR}$ is said to be non-negative *L-R* flat fuzzy number if $m-\alpha\geqslant 0$ and is said to be non-positive *L-R* flat fuzzy number if $n+\beta\leqslant 0$.

Definition 2.4 [14]. An *L-R* flat fuzzy number $\widetilde{A}=(m,n,\alpha,\beta)_{LR}$ is said to be unrestricted *L-R* flat fuzzy number if $m-\alpha$ is a real number.

Definition 2.5 [16]. Let $\widetilde{A}=(m,n,\alpha,\beta)_{LR}$ be an *L-R* flat fuzzy number and λ be a real number in the interval $[0,1]$ then the crisp set, $A_\lambda=\{x\in X:\mu_{\widetilde{A}}(x)\geqslant\lambda\}=[m-\alpha L^{-1}(\lambda),n+\beta R^{-1}(\lambda)]$, is said to be λ-cut of $\widetilde{A}$.

Definition 2.6 [16]. Let $\widetilde{A}_1=(m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\widetilde{A}_2=(m_2,n_2,\alpha_2,\beta_2)_{LR}$, be any *L-R* flat fuzzy numbers then $\widetilde{A}_1=\widetilde{A}_2$ iff $m_1=m_2,n_1=n_2,\alpha_1=\alpha_2$ and $\beta_1=\beta_2$.

2.2. Arithmetic operations

In this section, the arithmetic operations between *L-R* flat fuzzy numbers are presented [16].

Let $\widetilde{A}_1=(m_1,n_1,\alpha_1,\beta_1)_{LR}$, $\widetilde{A}_2=(m_2,n_2,\alpha_2,\beta_2)_{LR}$ be any *L-R* flat fuzzy numbers and $\widetilde{A}_3=(m_3,n_3,\alpha_3,\beta_3)_{RL}$ be any *R-L* flat fuzzy number. Then,

(i) $\widetilde{A}_1\oplus\widetilde{A}_2=(m_1+m_2,n_1+n_2,\alpha_1+\alpha_2,\beta_1+\beta_2)_{LR}$
(ii) $\widetilde{A}_1\ominus\widetilde{A}_3=(m_1-n_3,n_1-m_3,\alpha_1+\beta_3,\beta_1+\alpha_3)_{LR}$
(iii) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-negative, then
$\widetilde{A}_1\odot\widetilde{A}_2\simeq(m_1m_2,n_1n_2,m_1\alpha_2+\alpha_1m_2-\alpha_1\alpha_2,n_1\beta_2+\beta_1n_2+\beta_1\beta_2)_{LR}$
(iv) If $\widetilde{A}_1$ is non-positive and $\widetilde{A}_2$ is non-negative, then
$\widetilde{A}_1\odot\widetilde{A}_2\simeq(m_1n_2,n_1m_2,\alpha_1n_2-m_1\beta_2+\alpha_1\beta_2,\beta_1m_2-n_1\alpha_2-\beta_1\alpha_2)_{LR}$
(v) If $\widetilde{A}_1$ is non-negative and $\widetilde{A}_2$ is non-positive, then
$\widetilde{A}_1\odot\widetilde{A}_2\simeq(n_1m_2,m_1n_2,n_1\alpha_2-\beta_1m_2+\beta_1\alpha_2,m_1\beta_2-\alpha_1n_2-\alpha_1\beta_2)_{LR}$
(vi) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-positive, then
$\widetilde{A}_1\odot\widetilde{A}_2\simeq(n_1n_2,m_1m_2,-n_1\beta_2-\beta_1n_2-\beta_1\beta_2,-m_1\alpha_2-\alpha_1m_2+\alpha_1\alpha_2)_{LR}$
(vii) $\lambda\widetilde{A}_1=\begin{cases}(\lambda m_1,\lambda n_1,\lambda\alpha_1,\lambda\beta_1)_{LR} & \lambda\geqslant 0\\ (\lambda n_1,\lambda m_1,-\lambda\beta_1,-\lambda\alpha_1)_{RL} & \lambda<0\end{cases}.$

There also exist another formula [16] for the product of such *L-R* flat fuzzy numbers in which the spreads are smaller as compared to the mean values:

(i) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-negative, then
$\widetilde{A}_1\otimes\widetilde{A}_2\simeq(m_1m_2,n_1n_2,m_1\alpha_2+\alpha_1m_2,n_1\beta_2+\beta_1n_2)_{LR}$

(ii) If $\widetilde{A}_1$ is non-positive and $\widetilde{A}_2$ is non-negative, then
$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m_1n_2, n_1m_2, \alpha_1n_2 - m_1\beta_2, \beta_1m_2 - n_1\alpha_2)_{LR}$
(iii) If $\widetilde{A}_1$ is non-negative and $\widetilde{A}_2$ is non-positive, then
$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (n_1m_2, m_1n_2, n_1\alpha_2 - \beta_1m_2, m_1\beta_2 - \alpha_1n_2)_{LR}$
(iv) If $\widetilde{A}_1$ and $\widetilde{A}_2$ both are non-positive, then
$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (n_1n_2, m_1m_2, -n_1\beta_2 - \beta_1n_2, -m_1\alpha_2 - \alpha_1m_2)_{LR}$

Remark 1. If $m = n$ then an *L-R* flat fuzzy number $(m, n, \alpha, \beta)_{LR}$ is said to be an *L-R* fuzzy number and is denoted as $(m, m, \alpha, \beta)_{LR}$ or $(n, n, \alpha, \beta)_{LR}$ or $(m, \alpha, \beta)_{LR}$ or $(n, \alpha, \beta)_{LR}$.

Remark 2. If $m = n$ and $L(x) = R(x) =$ maximize$\{0, 1 - x\}$ then an *L-R* flat fuzzy number $(m, n, \alpha, \beta)_{LR}$ is said to be a triangular fuzzy number and is denoted as (m, α, β).

Remark 3. If $m \neq n$ and $L(x) = R(x) =$ maximize$\{0, 1 - x\}$ then an *L-R* flat fuzzy number $(m, n, \alpha, \beta)_{LR}$ is said to be a trapezoidal fuzzy number and is denoted as (m, n, α, β).

Remark 4 [17]. Let $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$, $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be any *L-R* flat fuzzy numbers. Then,

(i) $\widetilde{A} \preceq \widetilde{B}$ iff $\Re(\widetilde{A}) \leqslant \Re(\widetilde{B})$
(ii) $\widetilde{A} \succeq \widetilde{B}$ iff $\Re(\widetilde{A}) \geqslant \Re(\widetilde{B})$
(iii) $\widetilde{A} \approx \widetilde{B}$ iff $\Re(\widetilde{A}) = \Re(\widetilde{B})$

where $\Re(m, n, \alpha, \beta) = \frac{1}{2}(\int_0^1 (m - \alpha L^{-1}(\lambda))\, d\lambda + \int_0^1 (n + \beta R^{-1}(\lambda))\, d\lambda), 0 \leqslant \lambda \leqslant 1$.

3. Limitations of the existing method for solving fully FLP problems

To the best of our knowledge, till now no one have defined the product of unrestricted *L-R* fuzzy numbers or *L-R* flat fuzzy numbers e.g., if $\widetilde{A}_1 = (1, 3, 4, 2)_{LR}$ and $\widetilde{A}_2 = (2, 4, 5, 3)_{LR}$ then there is neither any product rule to find the value of $\widetilde{A}_1 \odot \widetilde{A}_2$ nor to find the value of $\widetilde{A}_1 \otimes \widetilde{A}_2$. Due to non-existence of such product the existing method [9] can be used for solving fully FLP problems (P_1) and (P_2) in which all the coefficients are represented by either non-negative *L-R* fuzzy numbers or non-positive *L-R* fuzzy numbers and all the decision variables are represented by non-negative *L-R* fuzzy numbers. However, the existing method [9] can not be used to find the fuzzy optimal solution of fully FLP problems (P_3) and (P_4) in which some or all the parameters are represented by unrestricted *L-R* fuzzy numbers or unrestricted *L-R* flat fuzzy numbers.

$$\begin{aligned} &\text{Maximize (or Minimize)} \quad \sum_{j=1}^{n} (\tilde{c}_j \odot \tilde{x}_j) \\ &\text{subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij} \odot \tilde{x}_j \preceq, \approx, \succeq \tilde{b}_i, \quad i = 1, 2, \ldots, m, \end{aligned} \tag{P$_1$}$$

where $\tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{c}_j$ are non-negative or non-positive *L-R* fuzzy numbers and $\tilde{x}_j$ is a non-negative *L-R* fuzzy number.

Example 3.1. Maximize $((2, 1, 2)_{LR} \odot \tilde{x}_1 \oplus (-3, 2, 1)_{LR} \odot \tilde{x}_2)$
subject to

$$(1, 1, 2)_{LR} \odot \tilde{x}_1 \oplus (2, 1, 3)_{LR} \odot \tilde{x}_2 \preceq (20, 10, 5)_{LR}$$

$$(-2, 1, 1)_{LR} \odot \tilde{x}_1 \oplus (5, 2, 3)_{LR} \odot \tilde{x}_2 \succeq (-3, 1, 2)_{LR},$$

where $\tilde{x}_1, \tilde{x}_2$ are non-negative *L-R* fuzzy numbers and $L(x) = R(x) =$ maximum$\{0, 1 - x\}$.

$$\begin{aligned} &\text{Maximize (or Minimize)} \quad \sum_{j=1}^{n} (\tilde{c}_j \otimes \tilde{x}_j) \\ &\text{subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \preceq, \approx, \succeq \tilde{b}_i, \quad i = 1, 2, \ldots, m, \end{aligned} \tag{P$_2$}$$

where $\tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{c}_j$ are non-negative or non-positive *L-R* fuzzy numbers and $\tilde{x}_j$ is a non-negative *L-R* fuzzy number.

Example 3.2. Maximize $((1, 1, 1)_{LR} \otimes \tilde{x}_1 \oplus (2, 1, 2)_{LR} \otimes \tilde{x}_2)$
subject to

$$(4,1,0)_{LR} \otimes \tilde{x}_1 \oplus (-2,1,1)_{LR} \otimes \tilde{x}_2 \succeq (5,2,3)_{LR}$$

$$(-3,1,2)_{LR} \otimes \tilde{x}_1 \oplus (4,1,2)_{LR} \otimes \tilde{x}_2 \succeq (4,1,1)_{LR}$$

where $\tilde{x}_1, \tilde{x}_2$ are non-negative *L-R* fuzzy numbers and $L(x) = R(x) = \text{maximum}\{0, 1-x\}$.

$$\begin{aligned} &\text{Maximize (or Minimize)} \quad \sum_{j=1}^{n} (\tilde{c}_j \odot \tilde{x}_j) \\ &\text{subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij} \odot \tilde{x}_j \preceq, \approx, \succeq \tilde{b}_i, \quad i = 1,2,\ldots,m, \end{aligned} \qquad (\mathrm{P}_3)$$

where $\tilde{a}_{ij}, \tilde{x}_j, \tilde{b}_i$ and $\tilde{c}_j$ are *L-R* flat fuzzy numbers.

Example 3.3. Maximize $((4,4,0,0)_{LR} \odot \tilde{x}_1 \oplus (1,1,1,1)_{LR} \odot \tilde{x}_2)$
subject to

$$(2,5,5,2)_{LR} \odot \tilde{x}_1 \oplus (-1,5,1,2)_{LR} \odot \tilde{x}_2 \preceq (-17,45,25,46)_{LR}$$

$$(1,2,1,1)_{LR} \odot \tilde{x}_1 \oplus (3,5,2,2)_{LR} \odot \tilde{x}_2 \approx \left(11, 39, \frac{193}{5}, \frac{168}{5}\right)_{LR}$$

where $\tilde{x}_1, \tilde{x}_2$ are *L-R* flat fuzzy numbers and $L(x) = R(x) = \text{maximum}\{0, 1-x\}$.

$$\begin{aligned} &\text{Maximize (or Minimize)} \quad \sum_{j=1}^{n} (\tilde{c}_j \otimes \tilde{x}_j) \\ &\text{subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \preceq, \approx, \succeq \tilde{b}_i, \quad i = 1,2,\ldots,m, \end{aligned} \qquad (\mathrm{P}_4)$$

where $\tilde{a}_{ij}$, $\tilde{x}_j, \tilde{b}_i$ and $\tilde{c}_j$ are *L-R* flat fuzzy numbers.

Example 3.4. Maximize $((1,1,1,1)_{LR} \otimes_N \tilde{x}_1 \oplus (4,4,0,0)_{LR} \otimes_N \tilde{x}_2)$ subject to

$$(2,3,1,1)_{LR} \otimes_N \tilde{x}_1 \oplus (-3,-2,1,1)_{LR} \otimes_N \tilde{x}_2 \succeq (-27,-4,21,32)_{LR}$$

$$(1,2,1,1)_{LR} \otimes_N \tilde{x}_1 \oplus (3,5,2,2)_{LR} \otimes_N \tilde{x}_2 \approx \left(11, 39, \frac{193}{5}, \frac{168}{5}\right)_{LR}$$

where $\tilde{x}_1, \tilde{x}_2$ are *L-R* flat fuzzy numbers and $L(x) = R(x) = \text{maximum}\{0, 1-x\}$.

Remark 5. The existing methods [10–13] can be used only for solving such fully FLP problems in which some or all the coefficients are represented by triangular or trapezoidal fuzzy numbers and the decision variables are represented by non-negative triangular or trapezoidal fuzzy numbers.

Remark 6. The existing methods [14,15] can be used only for solving such fully FLP problems in which the sign of all the constraints is equality sign as well as all the coefficients are represented by triangular or trapezoidal fuzzy numbers and the decision variables are represented by non-negative triangular or trapezoidal fuzzy numbers.

4. Proposed product

In this section, to overcome the limitations of the existing methods [9–15], corresponding to the existing product rules $\odot$ and $\otimes$, presented in Section 2.2, new product rules are introduced.

4.1. New product corresponding to the existing product $\odot$

In this section, new product corresponding to the existing product $\odot$ is introduced.

Proposition 4.1. *If* $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ *and* $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ *are two L-R flat fuzzy numbers such that* $m_1 - \alpha_1 < 0$ *and* $m_1 \geqslant 0$ *then* $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_1, n'_1, \alpha'_1, \beta'_1)_{LR}$, *where,* $m'_1 = \text{minimum}\ \{m_1 m_2, n_1 m_2\}, n'_1 = \text{maximum}\{m_1 n_2, n_1 n_2\}, \alpha'_1 = \text{minimum}\ \{m_1 m_2, n_1 m_2\} - \text{minimum}\ \{m_1 n_2 + m_1 \beta_2 - \alpha_1 n_2 - \alpha_1 \beta_2, n_1 m_2 - n_1 \alpha_2 + \beta_1 m_2 - \beta_1 \alpha_2\}, \beta'_1 = \text{maximum}\ \{m_1 m_2 - m_1 \alpha_2 - \alpha_1 m_2 + \alpha_1 \alpha_2, n_1 n_2 + n_1 \beta_2 + \beta_1 n_2 + \beta_1 \beta_2\} - \text{maximum}\ \{m_1 n_2, n_1 n_2\}$.

Proof. Let $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two L-R flat fuzzy numbers such that $m_1 - \alpha_1 < 0$ and $m_1 \geqslant 0$ then using the Definition 2.5, $A_{1\lambda} = [m_1 - \alpha_1 L^{-1}(\lambda), n_1 + \beta_1 R^{-1}(\lambda)]$ and $A_{2\lambda} = [m_2 - \alpha_2 L^{-1}(\lambda), n_2 + \beta_2 R^{-1}(\lambda)]$. Since $m_1 - \alpha_1 < 0$ and $m_1 \geqslant 0$ so $m_1 - \alpha_1 L^{-1}(\lambda) \leqslant 0$ for $\lambda \geqslant L(\frac{m_1}{\alpha_1})$ and $m_1 - \alpha_1 L^{-1}(\lambda) \geqslant 0$ for $\lambda \leqslant L(\frac{m_1}{\alpha_1})$ and $n_1 + \beta_1 R^{-1}(\lambda) \geqslant 0$ for all λ, so to find the product of $\widetilde{A}_1$ and $\widetilde{A}_2$ there is need to consider the following five cases:

Case (i) If $m_2 - \alpha_2 \geqslant 0$ then $m_2 - \alpha_2 L^{-1}(\lambda) \geqslant 0$ and $n_2 + \beta_2 R^{-1}(\lambda) \geqslant 0$ for all λ so the following two subcases may arise to find the product of $A_{1\lambda}$ and $A_{2\lambda}$:

(a) If $m_1 - \alpha_1 L^{-1}(\lambda) \geqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = \left[(m_1 - \alpha_1 L^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))\right]$$

Putting $\lambda = 1$, we get:

$$A_{1\lambda}A_{2\lambda} = [m_1 m_2, n_1 n_2]. \tag{2}$$

(b) If $m_1 - \alpha_1 L^{-1}(\lambda) \leqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = \left[(m_1 - \alpha_1 L^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))\right]$$

Putting $\lambda = 0$, we get:

$$A_{1\lambda}A_{2\lambda} = [m_1 n_2 + m_1 \beta_2 - \alpha_1 n_2 - \alpha_1 \beta_2, n_1 n_2 + n_1 \beta_2 + \beta_1 n_2 + \beta_1 \beta_2]. \tag{3}$$

Now combining (2) and (3) we get:

$$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m_1'', n_1'', \alpha_1'', \beta_1'')_{LR}$$

where, $m_1'' = m_1 m_2, n_1'' = n_1 n_2, \alpha_1'' = m_1 m_2 - m_1 n_2 + m_1 \beta_2 - \alpha_1 n_2 - \alpha_1 \beta_2, \beta_1'' = n_1 n_2 + n_1 \beta_2 + \beta_1 n_2 + \beta_1 \beta_2 - n_1 n_2$.

Case (ii) If $m_2 - \alpha_2 < 0, m_2 \geqslant 0$ then $m_2 - \alpha_2 L^{-1}(\lambda) \geqslant 0$ for $\lambda \leqslant L(\frac{m_2}{\alpha_2})$, $m_2 - \alpha_2 L^{-1}(\lambda) \leqslant 0$ for $\lambda \geqslant L(\frac{m_2}{\alpha_2})$ and $n_2 + \beta_2 R^{-1}(\lambda) \geqslant 0$ for all λ then the four subcases may arise to find the product of $A_{1\lambda}$ and $A_{2\lambda}$ but since we want to find the product of $A_{1\lambda}$ and $A_{2\lambda}$ corresponding to $\lambda = 0$ and $\lambda = 1$ so there is need to consider only the following two subcases:

(a) If $m_1 - \alpha_1 L^{-1}(\lambda) \leqslant 0$ and $m_2 - \alpha_2 L^{-1}(\lambda) \leqslant 0$ then

$$\begin{aligned} A_{1\lambda}A_{2\lambda} = \Big[&minimum\{(m_1 - \alpha_1 L^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda))\}, \\ &maximum\{(m_1 - \alpha_1 L^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))\}\Big] \end{aligned}$$

Putting $\lambda = 0$, we get:

$$\begin{aligned} A_{1\lambda}A_{2\lambda} = [&minimum\{m_1 n_2 + m_1 \beta_2 - \alpha_1 n_2 - \alpha_1 \beta_2, n_1 m_2 - n_1 \alpha_2 + \beta_1 m_2 - \beta_1 \alpha_2\}, maximum\{m_1 m_2 - m_1 \alpha_2 - \alpha_1 m_2 \\ &+ \alpha_1 \alpha_2, n_1 n_2 + n_1 \beta_2 + \beta_1 n_2 + \beta_1 \beta_2\}]. \end{aligned} \tag{4}$$

(b) If $m_1 - \alpha_1 L^{-1}(\lambda) \geqslant 0$ and $m_2 - \alpha_2 L^{-1}(\lambda) \geqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = [(m_1 - \alpha_1 L^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))]$$

Putting $\lambda = 1$, we get:

$$A_{1\lambda}A_{2\lambda} = [m_1 m_2, n_1 n_2]. \tag{5}$$

Now combining (4) and (5) we get:

$$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m_2'', n_2'', \alpha_2'', \beta_2'')_{LR}$$

where, $m_2'' = m_1 m_2, n_2'' = n_1 n_2, \alpha_2'' = m_1 m_2 -$minimum$\{m_1 n_2 + m_1 \beta_2 - \alpha_1 n_2 - \alpha_1 \beta_2 - m_1 m_2, n_1 m_2 - n_1 \alpha_2 + \beta_1 m_2 - \beta_1 \alpha_2\}, \beta_2'' =$ maximum$\{m_1 m_2 - m_1 \alpha_2 - \alpha_1 m_2 + \alpha_1 \alpha_2, n_1 n_2 + n_1 \beta_2 + \beta_1 n_2 + \beta_1 \beta_2\} - n_1 n_2$.

Case (iii) If $m_2 < 0, n_2 \geqslant 0$ then $m_2 - \alpha_2 L^{-1}(\lambda) \leqslant 0$ and $n_2 + \beta_2 R^{-1}(\lambda) \geqslant 0$ for all λ so the following two subcases may arise to find the product of $A_{1\lambda}$ and $A_{2\lambda}$:

(a) If $m_1 - \alpha_1 L^{-1}(\lambda) \geqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = [(n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))]$$

Putting $\lambda = 1$, we get:

$$A_{1\lambda}A_{2\lambda} = [n_1 m_2, n_1 n_2]. \tag{6}$$

(b) If $m_1 - \alpha_1 L^{-1}(\lambda) \leqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = [minimum\{(m_1 - \alpha_1 L^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda))\}, maximum\{(m_1 - \alpha_1 L^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))\}]$$

Putting $\lambda = 0$, we get:

$$A_{1\lambda}A_{2\lambda} = [minimum\{m_1 n_2 + m_1\beta_2 - \alpha_1 n_2 - \alpha_1\beta_2, n_1 m_2 + \beta_1 m_2 - n_1\alpha_2 - \beta_1\alpha_2\}, maximum\{m_1 m_2 - m_1\alpha_2 - \alpha_1 m_2 + \alpha_1\alpha_2, n_1 n_2 + n_1\beta_2 + \beta_1 n_2 + \beta_1\beta_2\}]. \tag{7}$$

Now combining (6) and (7) we get:

$$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m_3'', n_3'', \alpha_3'', \beta_3'')_{LR}$$

where, $m_3'' = n_1 m_2, n_3'' = n_1 n_2, \alpha_3'' = n_1 m_2 -$minimum$\{m_1 n_2 + m_1\beta_2 - \alpha_1 n_2 - \alpha_1\beta_2, n_1 m_2 + \beta_1 m_2 - n_1\alpha_2 - \beta_1\alpha_2\}, \beta_3'' =$maximum$\{m_1 m_2 - m_1\alpha_2 - \alpha_1 m_2 + \alpha_1\alpha_2, n_1 n_2 + n_1\beta_2 + \beta_1 n_2 + \beta_1\beta_2\} - n_1 n_2$.

Case (iv) If $n_2 < 0, n_2 + \beta_2 \geqslant 0$ then $m_2 - \alpha_2 L^{-1}(\lambda) \leqslant 0$ for all λ and $n_2 + \beta_2 R^{-1}(\lambda) \leqslant 0$ for $\lambda \leqslant R(-\frac{n_2}{\beta_2})$ and $n_2 + \beta_2 R^{-1}(\lambda) \geqslant 0$ for $\lambda \geqslant R(-\frac{n_2}{\beta_2})$ so the four subcases may arise to find the product of $A_{1\lambda}$ and $A_{2\lambda}$ but since we want to find the product of $A_{1\lambda}$ and $A_{2\lambda}$ corresponding to $\lambda = 0$ and $\lambda = 1$ so there is need to consider only the following two subcases:

(a) If $m_1 - \alpha_1 L^{-1}(\lambda) \geqslant 0$ and $n_2 + \beta_2 R^{-1}(\lambda) \leqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = [(n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (m_1 - \alpha_1 L^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))]$$

Putting $\lambda = 1$, we get:

$$A_{1\lambda}A_{2\lambda} = [n_1 m_2, m_1 n_2]. \tag{8}$$

(b) If $m_1 - \alpha_1 L^{-1}(\lambda) \leqslant 0$ and $n_2 + \beta_2 R^{-1}(\lambda) \geqslant 0$ then
$A_{1\lambda}A_{2\lambda} = [$ minimum$\{(m_1 - \alpha_1 L^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda))\}$,maximum$\{(m_1 - \alpha_1 L^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (n_1 + \beta_1 R^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))]$
Putting $\lambda = 0$, we get:

$$A_{1\lambda}A_{2\lambda} = [minimum\{m_1 n_2 + m_1\beta_2 - \alpha_1 n_2 - \alpha_1\beta_2, n_1 m_2 - n_1\alpha_2 + \beta_1 m_2 - \beta_1\alpha_2\}, maximum\{m_1 m_2 - m_1\alpha_2 - \alpha_1 m_2 + \alpha_1\alpha_2, n_1 n_2 + n_1\beta_2 + \beta_1 n_2 + \beta_1\beta_2\}]. \tag{9}$$

Now combining (8) and (9) we get:
$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m_4'', n_4'', \alpha_4'', \beta_4'')_{LR}$where, $m_4'' = n_1 m_2, n_4'' = m_1 n_2, \alpha_4'' = n_1 m_2 -$minimum$\{m_1 n_2 + m_1\beta_2 - \alpha_1 n_2 - \alpha_1\beta_2, n_1 m_2 - n_1\alpha_2 + \beta_1 m_2 - \beta_1\alpha_2\}, \beta_4'' =$maximum$\{m_1 m_2 - m_1\alpha_2 - \alpha_1 m_2 + \alpha_1\alpha_2, n_1 n_2 + n_1\beta_2 + \beta_1 n_2 + \beta_1\beta_2\} - m_1 n_2$.

Case (v) If $n_2 + \beta_2 < 0$ then $m_2 - \alpha_2 L^{-1}(\lambda) \leqslant 0$ and $n_2 + \beta_2 R^{-1}(\lambda) \leqslant 0$ for all λ so the following two subcases may arise:

(a) If $m_1 - \alpha_1 L^{-1}(\lambda) \geqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = [(n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (m_1 - \alpha_1 L^{-1}(\lambda))(n_2 + \beta_2 R^{-1}(\lambda))]$$

Putting $\lambda = 1$, we get:

$$A_{1\lambda}A_{2\lambda} = [n_1 m_2, m_1 n_2]. \tag{10}$$

(b) If $m_1 - \alpha_1 L^{-1}(\lambda) \leqslant 0$ then

$$A_{1\lambda}A_{2\lambda} = [(n_1 + \beta_1 R^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda)), (m_1 - \alpha_1 L^{-1}(\lambda))(m_2 - \alpha_2 L^{-1}(\lambda))]$$

Putting $\lambda = 0$, we get:

$$A_{1\lambda}A_{2\lambda} = [n_1 m_2 - n_1\alpha_2 + \beta_1 m_2 - \beta_1\alpha_2, m_1 m_2 - m_1\alpha_2 - \alpha_1 m_2 + \alpha_1\alpha_2]. \tag{11}$$

Now combining (10) and (11) we get:

$$\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m''_5, n''_5, \alpha''_5, \beta''_5)_{LR}$$

where, $m''_5 = n_1m_2, n''_5 = m_1n_2, \alpha''_5 = n_1m_2 - n_1m_2 - n_1\alpha_2 + \beta_1m_2 - \beta_1\alpha_2, \beta''_5 = m_1m_2 - m_1\alpha_2 - \alpha_1m_2 + \alpha_1\alpha_2 - m_1n_2$.

Combining the results of all five cases the following result is obtained:

If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two *L-R* flat fuzzy numbers such that $m_1 - \alpha_1 < 0, m_1 \geqslant 0$ and $\widetilde{A}_2$ is any *L-R* flat fuzzy number, then $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_1, n'_1, \alpha'_1, \beta'_1)_{LR}$, where, $m'_1 =$ minimum$\{m_1m_2, n_1m_2\}, n'_1 =$ maximum$\{m_1n_2, n_1n_2\}, \alpha'_1 =$ minimum$\{m_1m_2, n_1m_2\}-$minimum$\{m_1n_2 + m_1\beta_2 - \alpha_1n_2 - \alpha_1\beta_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2 - \beta_1\alpha_2\}$, $\beta'_1 =$maximum$\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2 + \alpha_1\alpha_2, n_1n_2 + n_1\beta_2 + \beta_1n_2 + \beta_1\beta_2\}-$maximum$\{m_1n_2, n_1n_2\}$. □

Proposition 4.2. *If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two L-R flat fuzzy numbers such that $m_1 < 0$ and $n_1 \geqslant 0$ then $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_2, n'_2, \alpha'_2, \beta'_2)_{LR}$, where, $m'_2 =$ minimum $\{m_1n_2, n_1m_2\}, n'_2 =$ maximum $\{m_1m_2, n_1n_2\}, \alpha'_2 =$ minimum$\{m_1n_2, n_1m_2\}-$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2 - \alpha_1\beta_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2 - \beta_1\alpha_2\}, \beta'_2 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2 + \alpha_1\alpha_2, n_1n_2 + n_1\beta_2 + \beta_1n_2 + \beta_1\beta_2\}-$ maxi-mum $\{m_1m_2, n_1n_2\}$.*

Proof. Similar to Proposition 4.1. □

Proposition 4.3. *If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two L-R flat fuzzy numbers such that $n_1 < 0$ and $n_1 + \beta_1 \geqslant 0$ then $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_3, n'_3, \alpha'_3, \beta'_3)_{LR}$, where, $m'_3 =$ minimum $\{m_1n_2, n_1n_2\}, n'_3 =$ maximum $\{n_1m_2, m_1m_2\}, \alpha'_3 =$ minimum $\{m_1n_2, n_1n_2\}-$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2 - \alpha_1\beta_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2 - \beta_1\alpha_2\}, \beta'_3 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2 + \alpha_1\alpha_2, n_1n_2 + n_1\beta_2 + \beta_1n_2 + \beta_1\beta_2\}-$ maximum $\{n_1m_2, m_1m_2\}$.*

Proof. Similar to Proposition 4.1. □

Proposition 4.4. *If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two L-R flat fuzzy numbers such that $n_1 + \beta_1 < 0$ then $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_4, n'_4, \alpha'_4, \beta'_4)_{LR}$, where, $m'_4 =$ minimum $\{m_1n_2, n_1n_2\}, n'_4 =$ maximum $\{m_1m_2, n_1m_2\}, \alpha'_4 =$ minimum $\{m_1n_2, n_1n_2\}-$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2 - \alpha_1\beta_2, n_1n_2 + n_1\beta_2 + \beta_1n_2 + \beta_1\beta_2\}, \beta'_4 =$ maximum $\{n_1m_2 - n_1\alpha_2 + \beta_1m_2 - \beta_1\alpha_2, m_1m_2 - m_1\alpha_2 - \alpha_1m_2 + \alpha_1\alpha_2\}-$ maxi-mum $\{m_1m_2, n_1m_2\}$.*

Proof. Similar to Proposition 4.1. □

Proposition 4.5. *If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two L-R flat fuzzy numbers such that $m_1 - \alpha_1 \geqslant 0$ then $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_5, n'_5, \alpha'_5, \beta'_5)_{LR}$, where, $m'_5 =$ minimum $\{m_1m_2, n_1m_2\}, n'_5 =$ maximum $\{m_1n_2, n_1n_2\}, \alpha'_5 =$ minimum $\{m_1m_2, n_1m_2\}-$ minimum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2 + \alpha_1\alpha_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2 - \beta_1\alpha_2\}, \beta'_5 =$ maximum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2 - \alpha_1\beta_2, n_1n_2 + n_1\beta_2 + \beta_1n_2 + \beta_1\beta_2\}-$ maxi-mum $\{m_1n_2, n_1n_2\}$.*

Proof. Similar to Proposition 4.1. □

4.2. New product corresponding to the existing product ⊗

In this section, new product corresponding to the existing product ⊗ is introduced.

Proposition 4.6. *If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two L-R flat fuzzy numbers such that $m_1 - \alpha_1 \geqslant 0$ then $\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m'_1, n'_1, \alpha'_1, \beta'_1)_{LR}$, where, $m'_1 =$ minimum $\{m_1m_2, n_1m_2\}, n'_1 =$ maximum $\{m_1n_2, n_1n_2\}, \alpha'_1 =$ minimum $\{m_1m_2, n_1m_2\}-$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}, \beta'_1 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\}-$ maximum $\{m_1n_2, n_1n_2\}$.*

Proof. The proposed results may be obtained by considering the following five cases:

Case (i) Neglecting the terms $\alpha_1\beta_2$ and $\beta_1\beta_2$ from the results obtained in Case (i) of Proposition 4.1, we get $A_{1\lambda}A_{2\lambda} = [m_1m_2, n_1n_2]$ for $\lambda = 1$ and $A_{1\lambda}A_{2\lambda} = [m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1n_2 + n_1\beta_2 + \beta_1n_2]$ for $\lambda = 0$. Combining the both, we get

$$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m''_1, n''_1, \alpha''_1, \beta''_1)_{LR}$$

where, $m''_1 = m_1m_2, n''_1 = n_1n_2, \alpha''_1 = m_1m_2 - m_1n_2 + m_1\beta_2 - \alpha_1n_2, \beta''_1 = n_1n_2 + n_1\beta_2 + \beta_1n_2 - n_1n_2$.

Case (ii) Neglecting the terms $\alpha_1\beta_2, \beta_1\alpha_2, \alpha_1\alpha_2$ and $\beta_1\beta_2$ from the results obtained in Case (ii) of Proposition 4.1, we get $A_{1\lambda}A_{2\lambda} = [$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}$, maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\}]$ for $\lambda = 0$ and $A_{1\lambda}A_{2\lambda} = [m_1m_2, n_1n_2]$ for $\lambda = 1$. Combining the both, we get

$$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m''_2, n''_2, \alpha''_2, \beta''_2)_{LR}$$

where, $m''_2 = m_1m_2, n''_2 = n_1n_2, \alpha''_2 = m_1m_2 -$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2 - m_1m_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}, \beta''_2 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\} - n_1n_2$.

Case (iii) Neglecting the terms $\alpha_1\beta_2, \beta_1\alpha_2, \alpha_1\alpha_2$ and $\beta_1\beta_2$ from the results obtained in Case (iii) of Proposition 4.1, we get $A_{1\lambda}A_{2\lambda} = [n_1m_2, n_1n_2]$ for $\lambda = 1$ and $A_{1\lambda}A_{2\lambda} = [$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 + \beta_1m_2 - n_1\alpha_2\}$, maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\}]$ for $\lambda = 0$. Combining the both, we get

$$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m''_3, n''_3, \alpha''_3, \beta''_3)_{LR}$$

where, $m''_3 = n_1m_2, n''_3 = n_1n_2, \alpha''_3 = n_1m_2 -$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 + \beta_1m_2 - n_1\alpha_2\}, \beta''_3 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\} - n_1n_2$.

Case (iv) Neglecting the terms $\alpha_1\beta_2, \beta_1\alpha_2$, $\alpha_1\alpha_2$ and $\beta_1\beta_2$ from the results obtained in Case (iv) of Proposition 4.1 we get $A_{1\lambda}A_{2\lambda} = [n_1m_2, m_1n_2]$ for $\lambda = 1$ and $A_{1\lambda}A_{2\lambda} = [$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}$, maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\}]$ for $\lambda = 0$. Combining the both, we get

$$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m''_4, n''_4, \alpha''_4, \beta''_4)_{LR}$$

where, $m''_4 = n_1m_2, n''_4 = m_1n_2, \alpha''_4 = n_1m_2 -$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}, \beta''_4 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\} - m_1n_2$.

Case (v) Neglecting the terms $\beta_1\alpha_2$ and $\alpha_1\alpha_2$ from the results obtained in Case (v) of Proposition 4.1, we get $A_{1\lambda}A_{2\lambda} = [n_1m_2, m_1n_2]$ for $\lambda = 1$ and $A_{1\lambda}A_{2\lambda} = [n_1m_2 - n_1\alpha_2 + \beta_1m_2, m_1m_2 - m_1\alpha_2 - \alpha_1m_2]$ for $\lambda = 0$. Combining the both, we get

$$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m''_5, n''_5, \alpha''_5, \beta''_5)_{LR}$$

where, $m''_5 = n_1m_2, n''_5 = m_1n_2, \alpha''_5 = n_1m_2 - n_1m_2 - n_1\alpha_2 + \beta_1m_2, \beta''_5 = m_1m_2 - m_1\alpha_2 - \alpha_1m_2 - m_1n_2$.

Combining the results of all five cases the following result is obtained:

If $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are two *L-R* flat fuzzy numbers such that $m_1 - \alpha_1 < 0, m_1 \geqslant 0$ and $\widetilde{A}_2$ is any *L-R* flat fuzzy number, then

$$\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m'_1, n'_1, \alpha'_1, \beta'_1)_{LR},$$

where, $m'_1 =$ minimum $\{m_1m_2, n_1m_2\}, n'_1 =$ maximum $\{m_1n_2, n_1n_2\}, \alpha'_1 =$ minimum $\{m_1m_2, n_1m_2\} -$ minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}, \beta'_1 =$ maximum $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\} -$ maximum $\{m_1n_2, n_1n_2\}$. □

Proposition 4.7. *If* $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ *and* $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ *are two L-R flat fuzzy numbers such that* $m_1 < 0$ *and* $n_1 \geqslant 0$ *then* $\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m'_2, n'_2, \alpha'_2, \beta'_2)_{LR}$,*where,* $m'_2 =$ *minimum*$\{m_1n_2, n_1m_2\}, n'_2 =$ *maximum* $\{m_1m_2, n_1n_2\}, \alpha'_2 =$ *minimum* $\{m_1n_2, n_1m_2\} -$ *minimum* $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}, \beta'_2 =$ *maximum* $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\} -$ *maximum* $\{m_1m_2, n_1n_2\}$.

Proof. Similar to Proposition 4.6. □

Proposition 4.8. *If* $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ *and* $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ *are two L-R flat fuzzy numbers such that* $n_1 < 0$ *and* $n_1 + \beta_1 \geqslant 0$ *then* $\widetilde{A}_1 \otimes \widetilde{A}_2 \simeq (m'_3, n'_3, \alpha'_3, \beta'_3)_{LR}$, *where,* $m'_3 =$ *minimum* $\{m_1n_2, n_1n_2\}, n'_3 =$ *maximum* $\{n_1m_2, m_1m_2\}, \alpha'_3 =$ *minimum* $\{m_1n_2, n_1n_2\} -$ *minimum* $\{m_1n_2 + m_1\beta_2 - \alpha_1n_2, n_1m_2 - n_1\alpha_2 + \beta_1m_2\}, \beta'_3 =$ *maximum* $\{m_1m_2 - m_1\alpha_2 - \alpha_1m_2, n_1n_2 + n_1\beta_2 + \beta_1n_2\} -$ *maximum* $\{n_1m_2, m_1m_2\}$.

Proof. Similar to Proposition 4.6. □

Proposition 4.9. *If* $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ *and* $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ *are two L-R flat fuzzy numbers such that* $n_1 + \beta_1 < 0$ *then* $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_4, n'_4, \alpha'_4, \beta'_4)_{LR}$, *where,* $m'_4 =$ *minimum* $\{m_1n_2, n_1n_2\}, n'_4 =$ *maximum* $\{m_1m_2, n_1m_2\}, \alpha'_4 =$ *minimum* $\{m_1n_2, n_1n_2\} -$

minimum $\{m_1n_2 + m_1\beta_2 - \alpha_1 n_2, n_1n_2 + n_1\beta_2 + \beta_1 n_2\}, \beta'_4 =$ *maxi-mum* $\{n_1m_2 - n_1\alpha_2 + \beta_1 m_2, m_1m_2 - m_1\alpha_2 - \alpha_1 m_2\}-$ *maximum* $\{m_1m_2, n_1m_2\}$.

Proof. Similar to Proposition 4.6. □

Proposition 4.10. *If* $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ *and* $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ *are two L-R flat fuzzy numbers such that* $m_1 - \alpha_1 \geqslant 0$ *then* $\widetilde{A}_1 \odot \widetilde{A}_2 \simeq (m'_5, n'_5, \alpha'_5, \beta'_5)_{LR}$,

where, $m'_5 =$ minimum $\{m_1m_2, n_1m_2\}, n'_5 =$ maximum $\{m_1n_2, n_1n_2\}, \alpha'_5 =$ minimum $\{m_1m_2, n_1m_2\}-$ minimum $\{m_1m_2 - m_1\alpha_2 - \alpha_1 m_2, n_1m_2 - n_1\alpha_2 + \beta_1 m_2\}, \beta'_5 =$ maximum $\{m_1n_2 + m_1\beta_2 - \alpha_1 n_2, n_1n_2 + n_1\beta_2 + \beta_1 n_2\}-$ maximum $\{m_1n_2, n_1n_2\}$.

Proof. Similar to Proposition 4.6. □

Remark 7. Let $\widetilde{A}_1 = (m_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{A}_2 = (m_2, \alpha_2, \beta_2)_{LR}$ be two *L-R* fuzzy numbers then to find $\widetilde{A}_1 \odot \widetilde{A}_2$ and $\widetilde{A}_1 \otimes \widetilde{A}_2$ put $m_1 = n_1, m_2 = n_2$ in the Proposition 4.1 to 4.10. To find the product of triangular fuzzy numbers or trapezoidal fuzzy numbers put $L(x) = R(x) = \text{maximum}\{0, 1 - x\}$ in the proposed product of *L-R* fuzzy numbers or *L-R* flat fuzzy numbers.

5. Proposed Mehar's method

In this section, to overcome the limitations of the existing methods [9–13], a new method, named as Mehar's method, is proposed to find the fuzzy optimal solution of fully FLP problems P_3.

The same method can also be used to find the fuzzy optimal solution of the fully FLP problems P_1, P_2 and P_4 as well as other existing FLP problems [2–8].

The steps of the Mehar's method for solving fully FLP problems P_3 are as follows:

Step 1: Assuming $\tilde{a}_{ij} = (a_{ij}, b_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, $\tilde{x}_j = (x_j, y_j, \alpha''_j, \beta''_j)_{LR}$, $\tilde{b}_i = (b_i, g_i, \gamma_i, \delta_i)_{LR}$ and $\tilde{c}_j = (p_j, q_j, \alpha'_j, \beta'_j)_{LR}$ the fully FLP problem P_3 can be written as:

$$\text{Maximize (or Minimize)} \quad \sum_{j=1}^{n}((p_j, q_j, \alpha'_j, \beta'_j)_{LR} \odot (x_j, y_j, \alpha''_j, \beta''_j)_{LR})$$

subject to

$$\sum_{j=1}^{n}(a_{ij}, b_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \odot (x_j, y_j, \alpha''_j, \beta''_j)_{LR} \preceq, \approx, \succeq (b_i, g_i, \gamma_i, \delta_i)_{LR}, \quad i = 1, 2, \ldots, m$$

where, $(x_j, y_j, \alpha''_j, \beta''_j)_{LR}$ is a *L-R* flat fuzzy number.

Step 2: Assuming $(p_j, q_j, \alpha'_j, \beta'_j)_{LR} \odot (x_j, y_j, \alpha''_j, \beta''_j)_{LR} \simeq (s_j, t_j, \alpha'''_j, \beta'''_j)_{LR}$ and $(a_{ij}, b_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \odot (x_j, y_j, \alpha''_j, \beta''_j)_{LR} \simeq (m_{ij}, n_{ij}, \gamma'_{ij}, \delta'_{ij})_{LR}$ the fully FLP problem, obtained in Step 1, can be written as:

$$\text{Maximize(or Minimize)} \sum_{j=1}^{n}(s_j, t_j, \alpha'''_j, \beta'''_j)_{LR}$$

subject to

$$\sum_{j=1}^{n}(m_{ij}, n_{ij}, \gamma'_{ij}, \delta'_{ij})_{LR} \preceq, \approx, \succeq (b_i, g_i, \gamma_i, \delta_i)_{LR}, \quad i = 1, 2, \ldots, m$$

where, $(x_j, y_j, \alpha''_j, \beta''_j)_{LR}$ is a *L-R* flat fuzzy number.

Step 3: Using the Yager's ranking approach [17], the fully FLP problem, obtained in Step 2, can be written as:

$$\text{Maximize (or Minimize)} \quad \Re\left(\sum_{j=1}^{n}(s_j, t_j, \alpha'''_j, \beta'''_j)_{LR}\right)$$

subject to

$$\Re\left(\sum_{j=1}^{n}(m_{ij}, n_{ij}, \gamma'_{ij}, \delta'_{ij})_{LR}\right) \leqslant, =, \geqslant \Re(b_i, g_i, \gamma_i, \delta_i)_{LR}, \quad i = 1, 2, \ldots, m,$$

where, $x_j \leqslant y_j, \alpha''_j \geqslant 0, \beta''_j \geqslant 0$.

Step 4: Solve the crisp linear programming problem, obtained in Step 3, to find the optimal solution $x_j, y_j, \alpha''_j, \beta''_j$ and put their values in $\tilde{x}_j = (x_j, y_j, \alpha''_j, \beta''_j)_{LR}$ to find the fuzzy optimal solution.

Step 5: Find the fuzzy optimal value of fully FLP problem by putting $\tilde{x}_j$ in $\sum_{j=1}^{n} \tilde{c}_j \odot \tilde{x}_j$.

Remark 8. In Section 5, with the help of proposed product, a new method, by modifying the existing method [9], is proposed for solving fully FLP problems with inequality constraints. On the same direction, the existing method [14,15] can also be modified for solving fully FLP problems with equality constraints.

6. Advantages of the Mehar's method over the existing method

In this section, advantages of the Mehar's method over existing method are discussed.

The main advantage of the Mehar's method over existing methods [9–13] is that fully FLP problems which can be solved by using the existing methods can also be solved by using the Mehar's method but there may exist several fully FLP problems which can not be solved by any of the existing methods [9–13] but can be solved by using the Mehar's method.

6.1. Fuzzy optimal solution of chosen fully FLP problems

To show the advantages of the Mehar's method and also to illustrate the Mehar's method the fully FLP problem, chosen in Example 3.3, which can not be solved by any of the existing methods [9–13] is solved by using the Mehar's method.

The fuzzy optimal solution of the fully FLP problem, chosen in Example 3.3, by using the Mehar's method can be obtained by using the following steps:

Step 1: Assuming $\tilde{x}_1 = (x_1, y_1, \alpha_1, \beta_1)_{LR}, \tilde{x}_2 = (x_2, y_2, \alpha_2, \beta_2)_{LR}$ the chosen fully FLP problem can be written as:

Maximize $((4,4,0,0)_{LR} \odot (x_1, y_1, \alpha_1, \beta_1)_{LR} \oplus (1,1,1,1)_{LR} \odot (x_2, y_2, \alpha_2, \beta_2)_{LR})$. subject to

$$(2,5,5,2)_{LR} \odot (x_1, y_1, \alpha_1, \beta_1)_{LR} \oplus (-1,5,1,2)_{LR} \odot (x_2, y_2, \alpha_2, \beta_2)_{LR} \preceq (-17,45,25,46)_{LR}$$

$$(1,2,1,1)_{LR} \odot (x_1, y_1, \alpha_1, \beta_1)_{LR} \oplus (3,5,2,2)_{LR} \odot (x_2, y_2, \alpha_2, \beta_2)_{LR} \approx \left(11, 39, \frac{193}{5}, \frac{168}{5}\right)_{LR}$$

where, $(x_1, y_1, \alpha_1, \beta_1)_{LR}, (x_2, y_2, \alpha_2, \beta_2)_{LR}$ are *L-R* flat fuzzy numbers.

Step 2: Using the arithmetic operations, proposed in Section 4, the fully FLP problem, obtained in Step 1, can be written as:

Maximize ((minimum $\{4x_1, 4x_1\}$, maximum $\{4y_1, 4y_1\}$, minimum $\{4x_1, 4x_1\}$-minimum $\{4x_1 - 4\alpha_1, 4x_1 - 4\alpha_1\}$, maximum $\{4y_1 + 4\beta_1, 4y_1 + 4\beta_1\}$-maximum $\{4y_1, 4y_1\})_{LR}\oplus$(minimum $\{x_2, x_2\}$, maximum $\{y_2, y_2\}$, minimum $\{x_2, x_2\}-$ minimum $\{0, 2x_2 - 2\alpha_2\}$, maximum $\{0, 2y_2 + 2\beta_2\}-$ maximum $\{y_2, y_2\})_{LR}$). subject to
(minimum $\{2x_1, 5x_1\}$, maximum $\{2y_1, 5y_2\}$, minimum $\{2x_1, 5x_1\}$-minimum $\{-3y_1 - 3\beta_1, 7x_1 - 7\alpha_1\}$, maximum$\{7y_1 + 7\beta_1, -3x_1 + 3\alpha_1\}$-maximum$\{2y_1, 5y_2\})_{LR}\oplus$(minimum $\{-y_2, 5x_2\}$, maximum $\{-x_2, 5y_2\}$, minimum $\{-y_2, 5x_2\}$-minimum$\{-2y_2 - 2\beta_2, 7x_2 - 7\alpha_2\}$, maximum $\{-2x_2 + 2\alpha_2, 7y_2 + 7\beta_2\}$-maximum$\{-x_2, 5y_2\})_{LR}$)$\preceq$(-17,45,25,46)$_{LR}$ (minimum $\{x_1, 2x_1\}$, maximum $\{y_1, 2y_1\}$, minimum $\{x_1, 2x_1\}$-minimum $\{0, 3x_1 - 3\alpha_1\}$, maximum $\{0, 3y_1 + 3\beta_1\}$-maximum $\{y_1, 2y_1\})_{LR}$ - $\oplus$(minimum $\{3x_2, 5x_2\}$, maximum $\{3y_2, 5y_2\}$, minimum$\{3x_2, 5x_2\}$-minimum $\{x_2 - \alpha_2, 7x_2 - 7\alpha_2\}$, maximum$\{y_2 + \beta_2, 7y_2 + 7\beta_2\}$-maximum $\{3y_2, 5y_2\})_{LR}\approx(11, 39, \frac{193}{5}, \frac{168}{5})_{LR}$where, $(x_1, y_1, \alpha_1, \beta_1)_{LR}, (x_2, y_2, \alpha_2, \beta_2)_{LR}$ are *L-R* flat fuzzy numbers.

Step 3: Using minimum $(a,b) = \frac{a+b}{2} - |\frac{a-b}{2}|$, maximum $(a,b) = \frac{a+b}{2} + |\frac{a-b}{2}|$ and Step 3 of the Mehar's method the fully FLP problem, obtained in Step 2, can be written as:

Maximize $(2x_1 - \alpha_1 + 2y_1 + \beta_1 + \frac{1}{2}x_2 - \frac{1}{4}\alpha_2 - \frac{1}{4}|x_2 - \alpha_2| + \frac{1}{2}y_2 + \frac{1}{4}\beta_2 + \frac{1}{4}|y_2 + \beta_2|)$. subject to

$$\frac{11}{2}y_1 + 2\beta_1 + \frac{11}{2}x_1 - 2\alpha_1 - \frac{1}{2}|3y_1 + 3\beta_1 + 7x_1 - 7\alpha_1| + \frac{9}{2}y_2 + \frac{5}{2}\beta_2 + \frac{9}{2}x_2 - \frac{5}{2}\alpha_2 - |y_2 + \beta_2 + \frac{7}{2}x_2 - \frac{7}{2}\alpha_2| - \frac{3}{2}|x_1| - \frac{1}{2}|y_2 + 5x_2| + \frac{3}{2}|y_1| + \frac{1}{2}|x_2 + 5y_2| + \frac{1}{2}|7y_1 + 7\beta_1 + 3x_1 - 3\alpha_1| + |x_2 - \alpha_2 + \frac{7}{2}y_2 + \frac{7}{2}\beta_2| \leqslant 77$$

$$3x_1 - \frac{3}{2}\alpha_1 - \frac{3}{2}|x_1 - \alpha_1| + 8x_2 - 4\alpha_2 - 3|x_2 - \alpha_2| - \frac{1}{2}|x_1| - |x_2| + 3y_1 + \frac{1}{2}|y_1| + 8y_2 + |y_2| + \frac{3}{2}\beta_1 + \frac{3}{2}|y_1 + \beta_1| + 4\beta_2 + 3|y_2 + \beta_2| = 95$$

$$x_1 \leqslant y_1, x_2 \leqslant y_2, \alpha_1 \geqslant 0, \alpha_2 \geqslant 0, \beta_1 \geqslant 0, \beta_2 \geqslant 0.$$

Table 1
Results of the chosen fully FLP problems.

Example	Fuzzy optimal value		
	Existing methods [10–13]	Existing method [9]	Proposed Mehar's method
3.1	Not Applicable	$\left(-\frac{192}{31}, \frac{128}{31}, \frac{6932}{93}\right)_{LR}$	$\left(-\frac{192}{31}, \frac{128}{31}, \frac{6932}{93}\right)_{LR}$
3.2	Not Applicable	$\left(\frac{1427}{167}, \frac{956}{167}, \frac{1427}{167}\right)_{LR}$	$\left(\frac{1427}{167}, \frac{956}{167}, \frac{1427}{167}\right)_{LR}$
3.3	Not Applicable	Not Applicable	$\left(\frac{2091}{122}, \frac{2091}{122}, \frac{583}{122}, \frac{583}{122}\right)_{LR}$
3.4	Not Applicable	Not Applicable	$\left(\frac{2579}{110}, \frac{2579}{110}, \frac{67}{110}, \frac{67}{110}\right)_{LR}$

Step 4: Solving the crisp non-linear programming problem, obtained in Step 3, the optimal solution is $x_1 = \frac{377}{122}$, $x_2 = \frac{583}{122}, y_1 = \frac{377}{122}, y_2 = \frac{583}{122}, \alpha_1 = 0, \alpha_2 = 0, \beta_1 = 0, \beta_2 = 0$. Putting these values in $\tilde{x}_1 = (x_1, y_1, \alpha_1, \beta_1)_{LR}, \tilde{x}_2 = (x_2, y_2, \alpha_2, \beta_2)_{LR}$, the fuzzy optimal solution is $\tilde{x}_1 = \left(\frac{377}{122}, \frac{377}{122}, 0, 0\right)_{LR}, \tilde{x}_2 = \left(\frac{583}{122}, \frac{583}{122}, 0, 0\right)_{LR}$.

Step 5: Putting the values of $\tilde{x}_1$ and $\tilde{x}_2$, obtained from Step 4, in $((4,4,0,0)_{LR} \odot \tilde{x}_1 \oplus (1,1,1,1)_{LR} \odot \tilde{x}_2)$ the fuzzy optimal value is $\left(\frac{2091}{122}, \frac{2091}{122}, \frac{583}{122}, \frac{583}{122}\right)_{LR}$.

7. Results and discussion

To compare the existing methods [9–13] and Mehar's method the results of the chosen fully FLP problems, obtained by using the existing methods and Mehar's method, are shown in Table 1.

The results shown in Table 1 can be explained as follows:

(i) The existing methods [10–13] can be used only for solving such fully FLP problems in which some or all the parameters are represented by triangular or trapezoidal fuzzy numbers. Since, in the fully FLP problems, chosen in Example 3.1, Example 3.2, Example 3.3 and Example 3.4 all the parameters are represented by *L-R* fuzzy numbers or *L-R* flat fuzzy numbers so none of the chosen problems can be solved by the existing methods [10–13].

(ii) The existing method [9] can be used only for solving such fully FLP problems in which some or all the parameters are represented by non-negative or non-positive *L-R* fuzzy numbers. Since, in the fully FLP problems, chosen in Example 3.1 and Example 3.2 all the parameters are represented by non-negative or non-positive *L-R* fuzzy numbers so these problems can be solved by the existing method [9]. However, in the fully FLP problems, chosen in Example 3.3 and Example 3.4, all the parameters are represented by unrestricted *L-R* flat fuzzy numbers so these problems can not be solved by the existing method [9].

(iii) The proposed Mehar's method can be used for solving such fully FLP problems in which some or all the parameters are represented by unrestricted *L-R* fuzzy numbers or *L-R* flat fuzzy numbers. So, all the chosen problems can be solved by the Mehar's method.

8. Conclusions

On the basis of the present study, it can be concluded that all the fully FLP problems which can be solved by the existing methods [9–13] can also be solved by the proposed Mehar's method. However, there exist several fully FLP problems which can be solved by the proposed Mehar's method but can not be solved by any of the existing methods [9–13]. Hence, it is better to use proposed Mehar's method as compared to the existing methods [9–13] for solving fully FLP problems.

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