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Finite time position synchronised control for parallel manipulators using fast terminal sliding mode

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A new finite time position synchronised control approach for parallel manipulators is proposed using a fast terminal sliding mode (TSM). By developing a novel synchronisation and coupling position error, a non-singular fast TSM is proposed in coupling position error space. The proposed controller can guarantee position error and synchronisation error converge to zero in a finite time simultaneously without requiring the explicit using system dynamic model. The corresponding stability analysis is presented to lay a foundation for theoretical understanding to the underlying issues as well as safe operation for real systems. An illustrative example is demonstrated in support of the effectiveness of the proposed approach.

Keywords: synchronised control; coupling position error; finite time stability; Lyapunov stability; terminal sliding mode control; parallel manipulator

1. Introduction

Parallel manipulators have been widely studied due to their higher accuracy, higher stiffness and higher load-carrying capacity in comparison with serial manipulator. By virtue of their merits, parallel manipulators can be used as actuators for high-precision operation of large payload, such as flight simulator, astronomical telescopes and precision machining (Dasgupta and Mruthyunjaya 2000; Merlet 2000). In spite of their smart configurations, trajectory tracking control algorithm is essential and indispensable for achieving high-precision operation in industrial applications, because the control algorithm directly affects the efficiency and success of manipulator operations. In the system and control community, the parallel manipulator is a typical multi-input multi-output (MIMO) non-linear system, which can serve as a test bed for high-performance controller (Kim, Cho and Lee 2005). So far, much attention has been attracted in research and application of trajectory tracking control of parallel manipulators. Generally, the tracking control of parallel manipulators can be classified as two kinds of approaches (Su, Sun, Lu and Mills 2006): model-free control approaches (Chiacchio, Pierrot, Sciavicco and Siciliano 1993; Amirat, Francois, Pontnau and Dafaoui 1996; Ghorbel, Chetelat, Gunawardana and Longchamp 2000) and mode-based control approaches (Lin and McInroy 2003; Kim et al. 2005; Zhu, Tao, Yao and Cao 2008). The position of synchronised control issue is not addressed in these traditional control approaches, that is, these control schemes only concern the asymptotical convergence of position tracking errors but are not concerned with how these errors converge to equilibrium point.

Considering the closed-loop kinematic chain mechanism that comprises parallel manipulators, all actuated joints should be controlled to move in a synchronous manner. Poor synchronisation of these actuated joints will result in diminished tracking accuracy or even damage the manipulator. To improve the trajectory tracking accuracy of parallel manipulators, the so-called position of synchronised control approach has been developed, such as, model-free approach (Su et al. 2006; Sun, Lu, Mills and Wang 2006) and model-based approach (Lu 2005; Lu, Mills and Sun 2006). By explicitly using synchronisation error and coupling error technique (Sun 2003; Rodriguez-Angeles and Nijmeijer 2004), the traditional synchronised control approach can stabilise the tracking error of each actuated joint to equilibrium point asymptotically in a synchronous manner. From an experimental comparison study of these synchronised control approaches, (Lu, Mills and Sun 2007)

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demonstrate that model-based synchronised control can achieve better performance than mode-free ones but the structure of model-based synchronised controller is much more complex than the model-free one due to the complexity of the parallel manipulator dynamic model. It is not an easy job to estimate the dynamic model of parallel manipulator in practice and the complexity controller structure will lead extensive online calculations. Though the model-free synchronised controller is simple and can be implemented easily, the operation precision needs to be further improved. It should be noticed that all of the aforementioned control approaches proposed for parallel manipulator can only achieve asymptotic stability, which requires infinite time to converge to equilibrium point. To get fast convergence rate, control gains need to be greatly increased in asymptotic stability controller design. The high gain request is undesirable and cannot be implemented in practice. Consequently, it is important to develop a fast convergent synchronised control approach accommodating both theory and applications for parallel manipulator tracking control.

Terminal sliding mode control (TSMC) is a finite time stability control approach, which offers some superior properties such as finite time convergence, and strong robustness to systems uncertainties (Feng, Yu and Man 2002; Hong, Xu and Huang 2002). This control approach is particularly useful for high-precision control as it speeds up the converge rate near an equilibrium point. Some achievements have been acquired for serial manipulators tracking control (Man and Yu 1997; Yu 1998; Barambones and Etxebarria 2002; Feng et al. 2002; Yu, Yu, Shirinzadeh and Man 2005). However, most of the existing TSMC are mode-based without addressing the position synchronisation issue. Though singularity is addressed explicitly by literatures (Feng et al. 2002; Yu et al. 2005), these TSMC cannot deliver fast convergence as system states are far away from the equilibrium point on the terminal sliding mode (TSM). Yu and Man (2002) present a fast TSMC scheme for single-input single-output (SISO) system, which can achieve fast and finite time convergence both at distance from and at a close range of the equilibrium. This approach cannot be directly used for a parallel manipulator due to its MIMO dynamical nature. Note that the aforementioned TSMC are all model-based approaches, which need intensive online computation. Recently, a new fast non-singularity TSMC (Zhao, Li and Gao 2008) has been developed for serial manipulator, which does not require the explicit use of robot dynamic model. This new TSMC is simple and applicable. However, position synchronisation does not accommodate in this control approach. It must be extended for parallel position synchronised control.

In this study, a new finite time position synchronised control is proposed for parallel manipulator with fast TSMC. In light of the synchronisation principle for mechanical systems (Sun 2003; Sun, Shao and Feng 2007), a novel position coupling error is proposed for parallel manipulator synchronised control. By incorporating this novel coupling error into the new fast TSMC approach (Zhao et al. 2008), a new finite time position synchronised control is developed for a parallel manipulator. The proposed approach can guarantee that the position tracking error and synchronisation error converge to zero in a finite time, namely, this approach can regulate position tracking while coordinating the actuated joints of parallel manipulator to move in a synchronous manner. It should be noted that the proposed approach is different from the existing synchronised position control and traditional TSMC. In comparison with existing synchronised position control for parallel manipulator, by defining a new synchronisation error and coupling position error, the proposed approach can achieve finite time stability, which is desired by industrial applications, while all of the existing synchronised position control (Lu 2005; Lu et al. 2006; Su et al. 2006; Sun et al. 2006) can only achieve asymptotical stability. In comparison with traditional TSMC, the proposed approach does not require explicit use of a dynamic model in controller design and addresses position synchronisation and fast convergence on TSM explicitly for parallel manipulator—while the traditional TSMC (Man and Yu 1997; Yu 1998; Barambones and Etxebarria 2002; Feng et al. 2002; Yu et al. 2005) is a model-based approach developed for a serial manipulator, it does not address the position synchronisation and fast convergence on TSM. The proposed approach has higher precision due to the position synchronisation, and TSM techniques are explicitly employed in the controller design. It should be noticed that the control structure of the proposed approach is much simpler than the model-based synchronised control approach, which makes the propose approach implement easily in practice.

In summary, this study takes into the following two considerations. The first is, for applications, the proposed approach may offer an alternative, but more effective position synchronised control for parallel manipulator. The second is, for theory, TSMC and synchronised control have been important and challenging topics in theoretical studying. Recently, TSMC (Yu 1998; Barambones and Etxebarria 2002; Feng et al. 2002; Yu et al. 2005;) and mechanical synchronised control (Blekhman, Fradkov, Nijmeijer and Pogromsky 1997; Huijberts, Nijmeijer and Willems 2000; Sun 2003) have been extensively studied. Hopefully, to establish a basis for further development,

the study can provide a new insight and application incentive in aspect of the theoretical development.

The rest of this article is organised as follows. In Section 2, the problem formulation is given. In Section 3, the main results of this article are presented. The finite time position synchronised controller is developed for parallel manipulators and the corresponding stability is also presented. In Section 4, an illustrative example is described to initially validate the proposed approach. Finally, in Section 5, some concluding remarks are given.

2. Problem formulation

In this section, the finite time position synchronised control problem will be formulated. It includes a dynamic model of parallel manipulator, the definition of synchronisation error, new coupling position error and non-singular TSM.

2.1. *Dynamic model of parallel manipulator*

Figure 1 illustrates a general parallel manipulator. The manipulator has a moving platform, a base platform and six chains. Each chain is composed of a prismatic joint and two spherical joints with only the prismatic joint actuated (the chain is also called leg in this article). This manipulator is categorised as 6-SPS type, where S and P represent prismatic and spherical joints, respectively. The 6-SPS presents the general configuration of parallel manipulators. The controller developed for it can be easily applied to other types of parallel manipulator.

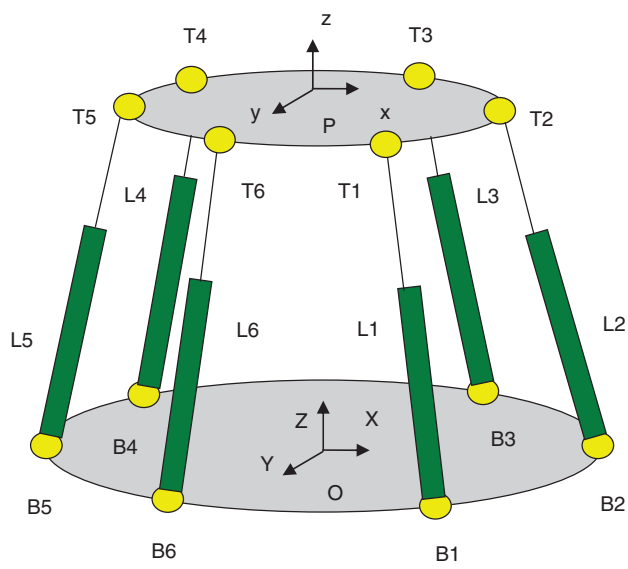

Figure 1. Coordinates of 6-SPS parallel manipulator.

As shown in Figure 1, the inertial frame $O-XYZ$ is fixed at the base platform with its origin at the geometry centre of the base platform, the body-fixed frame (moving frame) $P-xyz$ is attached to the mass origin of the moving platform. The 6 degrees of freedom (6DOF) are translations along the X, Y, Z axes and the rotations about the axis $O-X$, $O-Y$, $O-Z$.

The work space coordinates of the mass centre of moving platform can be written as

$$P = \begin{bmatrix} X & Y & Z & \alpha & \beta & \gamma \end{bmatrix}^T \tag{1}$$

where X, Y, Z represent the translations and α, β, γ represent the rotations. $q = [q_1, \ldots, q_n]^T$ is length of legs, which represents generalised coordinates. The relationship between P and q is

$$\dot{P}(t) = (J(t))^{-1}\dot{q}(t) \tag{2}$$

where $J(t)$ is a Jacobian matrix.

By using the natural orthogonal complement method, the dynamic model of parallel manipulator can be derived in joint space (Lu et al. 2006; Sun et al. 2006):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{3}$$

where $q \in \mathbb{R}^{n\times 1}$ is the generalised coordinate, $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal force coefficient matrix, $G(q) \in \mathbb{R}^{n\times 1}$ is the gravity force vector, $\tau \in \mathbb{R}^{n\times 1}$ is the actuating force vector. This dynamic model has the following properties that will be used in controller design (Lu et al. 2006; Su et al. 2006; Sun et al. 2006):

(P1) The inertia matrix $M(q)$ is a symmetric and positive definite matrix and satisfies $\underline{m}I \le M \le \bar{m}I$ for some constants $\underline{m}, \bar{m} > 0$, where $I \in \mathbb{R}^{n\times n}$ is an identity matrix.

(P2) $1/2\,\dot{M}(q) - C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is always skew symmetric matrix for all $t \ge 0$.

The technical purpose of this study is to design a finite time position synchronised controller with TSM without using a dynamic model explicitly. By incorporating synchronisation error and coupling position error into the TSMC, the proposed approach can guarantee position error and the synchronisation error converges to zero in a finite time simultaneously. To achieve this purpose, synchronisation error, coupling position error and fast TSM will be developed as follows.

2.2. *Synchronisation error and coupling position error*

To further improve position tracking accuracy of parallel manipulator, a special kinematic relationship

is maintained among their closed-loop chains. It also means that the motion of actuated joints needs to be coordinated in tracking operation. The key to coordinate actuated joints is position synchronisation. According to the particular problem, synchronisation of physical systems can be illustrated by a synchronisation function (Blekhman et al. 1997; Huijberts et al. 2000). Suppose that the actuated joints are subject to the following synchronisation function (Sun et al. 2006).

$$\begin{aligned} f(q_1(t), q_2(t), \ldots, q_n(t)) : c_1(t)q_1(t) \\ = c_2(t)q_2(t) = \cdots = c_n(t)q_n(t) \end{aligned} \tag{4}$$

where $c_i(t)$ denotes coupling coefficient of the i-th actuated joint. This synchronisation function represents a new task requirement in kinematics. Since Equation (4) also holds for all desired generalised coordinates $q_i^d(t)$, $i = 1, \ldots, n$, that is

$$\begin{aligned} f\big(q_1^d(t), q_2^d(t), \ldots, q_n^d(t)\big) : c_1(t)q_1^d(t) \\ = c_2(t)q_2^d(t) = \cdots = c_n(t)q_n^d(t) \end{aligned} \tag{5}$$

According to Equations (4) and (5), the following synchronisation goal is defined

$$c_1(t)e_1(t) = c_2(t)e_2(t) = \cdots = c_n(t)e_n(t) \tag{6}$$

where $e_i(t) = q_i(t) - q_i^d(t)$ is the position error of i-th actuated joint.

This synchronisation goal means that in controller design it addresses not only the convergence of position errors $e_i(t)$ but also how these errors converge to zero. The kinematic relationship among actuated joints defined by (6) must be held. Coupling coefficient $c_i(t)$ is determined in terms of kinematic characteristic of parallel manipulators, which considers the kinematic relationship among actuated joints.

The synchronisation error can be defined from the results of Su et al. (2006): when the ratio of the actual coordinate of each actuated joint to its desired value is equal to those of all other actuated joints, the parallel manipulator will move in a synchronous manner and the desired pose of the moving platform is maintained. Then, $c_i(t) = \frac{1}{q_i^d(t)}$ $i = 1, \ldots, n$, expression (6) can be written as

$$\frac{e_1(t)}{q_1^d(t)} = \frac{e_2(t)}{q_2^d(t)} = \cdots = \frac{e_n(t)}{q_n^d(t)} \tag{7}$$

It is obvious that $c_i(t) \to \infty$ as $q_i^d(t) \to 0$, $c_i(t)$ will be singularity at this time. The singularity will deteriorate control performance. It is undesired in controller design. If these points, $q_i^d = 0$, can be avoided in joint position trajectory plan, then singularity of $c_i(t)$ can be avoided.

It is convenient and convincing that the synchronisation sub-goal can be divided into n sub-goals, such as $\frac{e_{i-1}(t)}{q_{i-1}^d(t)} = \frac{e_i(t)}{q_i^d(t)} = \frac{e_{i+1}(t)}{q_{i+1}^d(t)}$, with the boundary condition that when $i = 1$, $i - 1 = n$ and when $i = n$, $i + 1 = 1$. Then, the position synchronisation error can be defined as

$$\begin{cases} \varepsilon_1(t) = 2\dfrac{e_1(t)}{q_1^d(t)} - \dfrac{e_2(t)}{q_2^d(t)} - \dfrac{e_6(t)}{q_6^d(t)} \\ \varepsilon_2(t) = 2\dfrac{e_2(t)}{q_2^d(t)} - \dfrac{e_1(t)}{q_1^d(t)} - \dfrac{e_3(t)}{q_3^d(t)} \\ \qquad\vdots \\ \varepsilon_6(t) = 2\dfrac{e_6(t)}{q_6^d(t)} - \dfrac{e_5(t)}{q_5^d(t)} - \dfrac{e_1(t)}{q_1^d(t)} \end{cases} \tag{8}$$

where ε_i denotes synchronisation error of i-th actuated joint. Obviously, if the synchronisation error $\varepsilon_i = 0$ for all $i = 1, \ldots, n$ the synchronisation goal (7) can be achieved automatically. Rewrite (8) in matrix format as

$$\varepsilon = \begin{bmatrix} \dfrac{2_1}{q_1^d(t)} & -\dfrac{1}{q_2^d(t)} & 0 & \cdots & -\dfrac{1}{q_6^d(t)} \\ -\dfrac{1}{q_1^d(t)} & \dfrac{2}{q_2^d(t)} & -\dfrac{1}{q_3^d(t)} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\dfrac{1}{q_4^d(t)} & \dfrac{2}{q_5^d(t)} & -\dfrac{1}{q_6^d(t)} \\ -\dfrac{1}{q_1^d(t)} & 0 & \cdots & -\dfrac{1}{q_5^d(t)} & \dfrac{2}{q_6^d(t)} \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_5(t) \\ e_6(t) \end{bmatrix}$$

$$\varepsilon = Te \tag{9}$$

where $\varepsilon = [\varepsilon_1(t), \ldots, \varepsilon_n(t)]^T$, $e = [e_1(t), \ldots, e_n(t)]^T$ and $T \in \mathbb{R}^{n \times n}$ is synchronisation transformation matrix.

If the appropriate desired trajectory $q_i^d(t)$ is selected, it can guarantee that $q_i^d(t) \neq 0$, that is, $c_i = 1/q_i^d$ is bounded. Then, the synchronisation error will be non-singular always.

Remark 1: The synchronisation error proposed by this article (9) is different from the existing synchronisation error developed for parallel manipulators. In Su et al. (2006), it addresses the set-point tracking issue, the coupling coefficient $1/q_i^d$ is a constant, while the proposed approach addresses trajectory tracking, the coupling coefficient $1/q_i^d(t)$ is time varying. In Sun et al. (2006), the synchronisation error only addresses the kinematic relationship of every two neighbouring actuated joints, while the proposed synchronisation addresses the kinematic relationship between each actuated joint and its two adjacent actuated joints. Then, the proposed synchronisation error is more convincing than Sun et al. (2006). In Lu et al. (2006),

due to the forward Jacobian being employed in its definition, the synchronisation errors are model based and very complex, while the proposed synchronisation error is model free and convenient for implementation. It is known that the model-based approach will lead to heavy online computation. In summary, the proposed synchronisation error is convenient and convincing for industrial application.

To design a controller that guarantees finite time convergence of both position error e and synchronisation error ε, the following coupling position error is defined

$$E = e + \gamma\varepsilon \tag{10}$$

where $E \in \mathbb{R}^n$ is the coupling position error, and $\gamma \in \mathbb{R}^{n\times n}$ is a diagonal positive definite control gain matrix. The higher gain γ, the more enhanced the synchronisation control will be (Sun et al. 2007). Therefore, it should take balance in selection of γ. Substituting (9) into (10), yields

$$E = (I + \gamma T)e \tag{11}$$

where $I \in \mathbb{R}^{n\times n}$ is a identity matrix. Through selecting the matrix γ and planning trajectory appropriately, one can guarantee the matrix $(I + \gamma T)$ to be non-singularity. It is obvious that $E = 0$ implies $e = 0$ and $\varepsilon = 0$.

Remark 2: It should be noticed that the proposed coupling position error is different from existing ones. In Sun et al. (2006), the position error and synchronisation error are not linearly coupled in coupling position error for the use of integration of synchronisation error $\int \varepsilon \, \mathrm{d}t$. As a result, coupling position error convergence to zero does not necessarily lead to convergence to zero of position error and synchronisation error. Then it cannot be employed in developing TSM-based position synchronised controller for parallel manipulator. The proposed coupling position error can guarantee both position error and synchronisation error to converge to zero at same time, which is appropriate for a TSM-based position synchronised controller. In Lu (2005) and Lu et al. (2006) coupling position error is model based, which will result in intensive online computational work. The proposed coupling position error is model free and simple for applications. In summary, the proposed coupling position error is linear composition of position error and synchronisation error, it can make position error and synchronisation error converge to zero simultaneously. In the following context, it can be seen that this new coupling position is very useful for design of finite time position synchronised controller with TSM.

To design a finite time synchronised controller with TSM, it is important to define TSM in coupling position error space. The following part of this section will address this issue.

2.3. *Fast non-singular TSM and control objective*

For simplicity but not losing generality, the following notions are introduced (Haimo 1986)

$$\mathrm{sig}(y^*)^\eta = \left[|y_1^*|^\eta \mathrm{sign}(y_1^*) \cdots |y_n^*|^\eta \mathrm{sign}(y_n^*)\right]^T$$

where $y \in \mathbb{R}^n$ is a column vector, η is a real number, sign$(\cdot)$ is signum function.

To avoid singularity, the following definitions of $E_r = [E_{r1}, \ldots, E_{rn}]^T$ are developed

$$E_{ri} = \begin{cases} \alpha|E_i|^{\alpha-1}\dot{E}_i & E_i \neq 0 \\ 0 & E_i = 0 \end{cases} \tag{12}$$

where $\alpha = p_1/p_2$, p_1, p_2 are positive odd numbers and $0 < p_1 < p_2 < 2p_1$, $i = 1, \ldots, n$.

Define command vectors $\dot{r} \in \mathbb{R}^n$ and $\ddot{r} \in \mathbb{R}^n$ as the following

$$\begin{cases} \dot{r} = \dot{q}^d - (I + \gamma T)^{-1}\left[\gamma \dot{T} e + \Lambda_1 E + \Lambda_2 \mathrm{sig}(E)^\alpha\right] \\ \left\{ \begin{array}{l} \ddot{r} = \ddot{q}^d - \frac{\mathrm{d}(I+\gamma T)^{-1}}{\mathrm{d}t}\left[\gamma \dot{T} e + \Lambda_1 E + \Lambda_2 \mathrm{sig}(E)^\alpha\right] \\ \quad -(I + \gamma T)^{-1}\left[\gamma \ddot{T} \dot{e} + \Lambda_1 \dot{E} + \Lambda_2 E_r\right] \end{array} \right\} \end{cases} \tag{13}$$

where $\Lambda_1 \in \mathbb{R}^{6\times 6}$, $\Lambda_2 \in \mathbb{R}^{6\times 6}$ are positive diagonal control gain matrices.

In terms of expression (13), fast non-singular TSM is defined as

$$\begin{cases} s = \dot{q} - \dot{r} \\ \dot{s} = \ddot{q} - \ddot{r} \end{cases} \tag{14}$$

If system states reach the fast TSM, that is, $s = 0$, according to (14), the following equation holds.

$$\dot{e} + (I + \gamma T)^{-1}\left[\gamma \dot{T} e + \Lambda_1 E + \Lambda_2 \mathrm{sig}(E)^\alpha\right] = 0 \tag{15}$$

Since the matrix $(I + \gamma T)$ is non-singular, left multiplied the matrix $(I + \gamma T)$, expression (15) can be written as

$$\dot{E} + \Lambda_1 E + \Lambda_2 \mathrm{sig}(E)^\alpha = 0 \tag{16}$$

Considering i-th element of expression (16)

$$\dot{E}_i + \lambda_{ii}^1 E_i + \lambda_{ii}^2 |E_i|^\alpha \mathrm{sign}(E_i) = 0 \tag{17}$$

where $\lambda_{ii}^1 > 0$ and $\lambda_{ii}^2 > 0$ are the ii-th term of matrix Λ_1 and Λ_2, respectively, $i = 1, \ldots, n$.

If the initial value of E_i at time $t = 0$ is $E_i(0) \neq 0$, relaxation time t_j for the solution of system (17) is given as (Yu et al. 2005)

$$t_i = \frac{1}{\lambda_{ii}^1(1-\alpha)} \ln \frac{\lambda_{ii}^1 |E_i(0)|^{1-\gamma} + \lambda_{ii}^2}{\lambda_{ii}^2} \tag{18}$$

Remark 3: Expression (18) means that the coupling position error converges to zero in a finite time along the fast TSM. According to definition of coupling position error (11), as $E_i = 0$, $\varepsilon_i = 0$ and $e_i = 0$ at the same time. Note that the traditional TSM only includes position error (Yu et al. 2005), while the proposed TSM (14) considers convergence of both position error and synchronisation error.

Remark 4: $\Lambda_1 E$ guarantees system states to achieve fast convergence along TSM as they are far away from equilibrium point. $\Lambda_2 \mathrm{sig}(\mathrm{E})^{\alpha}$ guarantees system states to achieve fast convergence along TSM as they near the equilibrium point. Traditional TSMC for robot manipulators have not addressed the fast convergence along TSM when system states are far away from equilibrium (Man and Yu 1997). It should be noticed that Feng et al. (2002) has developed a novel non-singular TSM but this approach cannot achieve fast convergence of system states along TSM.

Remark 5: Due to $1/2 < \alpha < 1$, $\alpha - 1 < 0$. Before the errors dynamics reach the TSM, the cross-coupling error E_i may be zero ($E_i = 0$ and $\dot{E}_i \neq 0$) at some points, and therefore E_{ri} in expression (12) may tend to infinity at these points. To guarantee the bounded property, E_{ri} is defined as zero at these points. After the error dynamics reach the TSM in expression (16), E_{ri} can be written as (Man and Yu 1997)

$$E_{ri} = -\alpha\left(\lambda_{ii}^{1}|E_i|^{\alpha}\mathrm{sign}(E_i) + \lambda_{ii}^{2}|E_i|^{2\alpha-1}\mathrm{sign}(E_i)\right)$$

Since $1/2 < \gamma < 1$, E_{ri} is bounded in coupling position error space.

Remark 6: With the specified E_r in (12) the singularity problem can be avoided in the control law. It should be noticed that it is different from conventional TSMC approach. Control law u_0 of conventional TSMC approach (Man and Yu 1997) is set to be zero as arbitrary position error $e_i(t) = 0$, $i = 1, \ldots, n$. It can be seen in the next section that the control law of this article needs not to be zero at these points.

Substituting (13) and (14) into dynamic equation (3), yields

$$M(q)\dot{s} + C(q,\dot{q})s = -M(q)\ddot{r} - C(q,\dot{q})\dot{r} - G(q) + \tau \quad (19)$$

Several technical assumptions are employed for finite time position synchronised controller design.

(A1) The desired joint position trajectory q^d and its time derivatives $\dot{q}^d$ and $\ddot{q}^d$ are bounded signals. $q_i^d \neq 0$, $(i = 1, \ldots, n)$ all the time.
(A2) $||M(q)||$, $||C(q,\dot{q})||$ and $||G(q)||$ are bounded if q and $\dot{q}$ are bounded (Sun et al. 2006).
(A3) The matrix $(I + \gamma T)$ is non-singular.

Remark 7: In this article, $\|\cdot\|$ denotes L_2 norm for vector and induced norm for matrix, respectively. $\lambda_{\max}(\cdot)/\lambda_{\min}(\cdot)$ represents maximum/minimum eigenvalue of the matrix.

Remark 8: Assumption (A2) seems to be restrictive. If the actuated joints position q and velocity $\dot{q}$ are large or cannot be bounded, it means that q and $\dot{q}$ cannot converge to their desired values q^d and $\dot{q}^d$. This also means that the equilibrium of the closed-loop system is unstable and the controller is not designed well. If the controller is designed appropriately, q and $\dot{q}$ will be bounded. Later, in Section 4, simulation results will show that assumption (A2) is reasonable.

Remark 9: If the control gain matrix γ and the desired trajectory q^d are designed appropriately one can guarantee the matrix $(I + \gamma T)$ to be non-singular always.

The control objective of this article can be summarised as: under assumption (A1)–(A3), design a controller in coupling position error space to guarantee system states to reach fast TSM (14) in a finite time. According to the definition of fast TSM, the coupling position error will converge to equilibrium point along fast TSM in a finite time (18) with fast convergence rate. In light of the definition of coupling position error (10), both position error (6) and synchronisation error (8) will converge to zero in a finite time simultaneously.

3. Finite time position synchronised controller design and stability analysis

To make coupling position error converge to equilibrium point in a finite time, the following control law is developed.

$$\begin{aligned}\tau = &-K_1\mathrm{sig}(s)^{\beta} - K_2 s + K^M\ddot{r} + K^C\dot{r} + K^G \\ &- \mathrm{sign}(s)\left(\Delta^M\|\ddot{r}\| + \Delta^C\|\dot{r}\| + \Delta^G\right)\end{aligned} \quad (20)$$

where $K_1, K_2 \in \mathbb{R}^{n\times n}$ are positive definite diagonal feedback control gain matrices, $K^M, K^C \in \mathbb{R}^{n\times n}$, $K^G \in R^n$ are positive definite diagonal feedforward control gain matrices and vector, respectively, Δ^M, Δ^C and Δ^G are positive real numbers, $\mathrm{sign}(s) = [\mathrm{sign}(s_1), \ldots, \mathrm{sign}(s_n)]^T$.

Remark 10: $-K_1\mathrm{sig}(s)^{\beta} - K_2 s$ denotes fast TSM-type reaching law, which can make system states reach the TSM with fast rate in a finite time. By using this technique, the chattering can also be avoided in the operation (Yu et al. 2005).

Remark 11: $K^M\ddot{r} + K^C\dot{r} + K^G$ is feedforward part, which is used to compensate for the effect caused by $-M(q)\ddot{r} - C(q,\dot{q})\dot{r} - G(q)$ in dynamic equation (19).

Remark 12: $-\text{sign}(s)(\Delta^M\|\ddot{r}\| + \Delta^C\|\dot{r}\| + \Delta^G)$ is saturation control used to compensate for non-linear effect caused by the errors between the feedforward control gains and the modelling parameters, which was used in Utkin (1977) and Slotine and Sastry (1983) with stability analysis.

Remark 13: The proposed control approach (20) can achieve finite time stability in coupling position error space, while the existing parallel manipulator synchronised control approach (Lu 2005; Lu et al. 2006; Su et al. 2006; Sun et al. 2006) only can achieve asymptotic stability. The high gains are required in asymptotic stability control approach, which cannot be implemented in practice. It is obvious that the proposed controller does not explicitly use dynamic model parameters, while the existing TSMC approaches are all model-based without addressing the position synchronisation issue. In a word, the proposed control approach is simple and applicable, which can guarantee that position error and synchronisation error converge to zero in a finite time simultaneously.

A control gain tuning strategy is proposed as follows: first, select $\Delta^M = 0$, $\Delta^C = 0$ and $\Delta^G = 0$, tune the control gains K_1, K_2, K^M, K^C and K^G using a trial-and-error method. The controller at this time is a normal feedforward/feedback control. Second, gradually increase Δ^M, Δ^C and Δ^G from zero to introduce the saturation control. Finally, the previous tuned gains may need to be changed slightly, utilising trial-and-error method.

For stability analysis, several lemmas and definitions are presented in appendices A and B. In light of these lemmas and definitions, the following theorem is stated, which is the main result of this article.

Theorem 1: *Under assumptions* (A1)–(A3), *consider dynamic* equation (19) *of parallel manipulators subject to control law* (20). *If* $\Delta^M \geq ||K^M - M(q)||$, $\Delta^C \geq ||K^C - C(q,\dot{q})||$, $\Delta^G \geq ||K^G - G(q)||$, *both of position error e and synchronisation error ε converge to zero in a finite time at the same time.*

Proof: Consider the following Lyapunov function

$$V = \tfrac{1}{2}s^T M(q)s \tag{21}$$

Differentiating V with respect to time, yields

$$\dot{V} = s^T M(q)\dot{s} + \tfrac{1}{2}s^T\dot{M}(q)s \tag{22}$$

Considering the closed-loop dynamic Equations (19) and (22) can be written as

$$\begin{aligned}\dot{V} &= s^T\big(\tfrac{1}{2}\dot{M}(q) - C(q,\dot{q})\big)s + s^T\tau \\ &\quad - s^T M(q)\ddot{r} - s^T C(q,\dot{q})\dot{r} - s^T G(q)\end{aligned} \tag{23}$$

Applying property (P2), yields

$$\dot{V} = s^T\tau - s^T M(q)\ddot{r} - s^T C(q,\dot{q})\dot{r} - s^T G(q) \tag{24}$$

Substituting control law (20) into Equation (24), it can be given

$$\begin{aligned}\dot{V} &= -s^T K_1\text{sig}(s)^\beta - s^T K_2 s + s^T K^M\ddot{r} + s^T K^C\dot{r} + s^T K^G \\ &\quad - s^T\text{sign}(s)\big(\Delta^M\|\ddot{r}\| + \Delta^C\|\dot{r}\| + \Delta^G\big) \\ &\quad - s^T M(q)\ddot{r} - s^T C(q,\dot{q})\dot{r} - s^T G(q)\end{aligned} \tag{25}$$

Expression (25) also can be written as

$$\begin{aligned}\dot{V} &= -s^T K_1\text{sig}(s)^\beta - s^T K_2 s + s^T\big(K^M - M(q)\big)\ddot{r} \\ &\quad + s^T\big(K^C - C(q,\dot{q})\big)\dot{r} + s^T\big(K^G - G(q)\big) \\ &\quad - |s|\big(\Delta^M\|\ddot{r}\| + \Delta^C\|\dot{r}\| + \Delta^G\big)\end{aligned} \tag{26}$$

In terms of Lemma 2 (Appendix A), $\|s\| \leq |s|$, it yields

$$\begin{aligned}\dot{V} &\leq -s^T K_1\text{sig}(s)^\beta - s^T K_2 s + \|s^T\|\,\|K^M - M(q)\|\,\|\ddot{r}\| \\ &\quad + \|s^T\|\,\|K^C - C(q,\dot{q})\|\,\|\dot{r}\| + \|s^T\|\,\|K^G - G(q)\| \\ &\quad - \|s\|\big(\Delta^H\|\ddot{r}\| + \Delta^C\|\dot{r}\| + \Delta^G\big)\end{aligned} \tag{27}$$

Inequality (27) can be written as

$$\begin{aligned}\dot{V} &\leq -s^T K_1\text{sig}(s)^\beta - s^T K_2 s - \|s\|\,\|\ddot{r}\|\big(\Delta^M - \|K^M - M(q)\|\big) \\ &\quad - \|s\|\,\|\dot{r}\|\big(\Delta^C - \|K^C - C(q,\dot{q})\|\big) \\ &\quad - \|s\|\big(\Delta^G - \|K^C - G(q)\|\big)\end{aligned} \tag{28}$$

If control gains Δ^M, Δ^C, Δ^G are selected appropriately to make the conditions $\Delta^M \geq \|K^M - M(q)\|$, $\Delta^C \geq \|K^C - C(q,\dot{q})\|$, $\Delta^G \geq \|K^G - G(q)\|$ hold, it yields the following inequality

$$\dot{V} \leq -s^T K_1\text{sig}(s)^\beta - s^T K_2 s \tag{29}$$

Since K_2 is a positive definite matrix, $s^T K_2 s \geq 0$. The following inequality holds

$$\dot{V} \leq -s^T K_1\text{sig}(s)^\beta = -\sum_{i=1}^{n} k_i|s_i|^{1+\beta} \tag{30}$$

Since

$$\begin{aligned}\sum_{i=1}^{6} k_i|s_i|^{1+\beta} &\geq k_{\min}\sum_{i=1}^{6}|s_i|^{1+\beta} \\ &\geq k_{\min}\left(\frac{2}{\bar{m}}\right)^{1+\beta/2}\left(\sum_{i=1}^{6}\frac{\bar{m}}{2}s_i^2\right)^{1+\beta/2} \geq aV^\eta,\end{aligned}$$

where $a = k_{\min}(2/\bar{m})^{1+\beta/2} > 0$, $0 < \eta = 1 + \beta/2 < 1$. The following inequality holds

$$\dot{V} \leq -aV^\eta \tag{31}$$

Since V is clearly a positive definite function and $\dot{V}$ is negative semi-definite, the closed-loop systems (19)

is Lyapunov stable. Moreover, in light of results on differential inequalities (Definition 1, Lemmas 3 and 4, see Appendix B), $s = 0$ in a finite time $\tilde{t} = V^{1-\eta}(t_0)/k(1-\eta)$. This means that fast TSM is reached in a finite time. According to (18), E_i converges to zero along the fast TSM in a finite time $t_i = 1/\lambda_{ii}^1 \times (1-\alpha)\ln(\lambda_{ii}^1|E_i(0)|^{1-\gamma} + \lambda_{ii}^2/\lambda_{ii}^2)$. From $\tilde{t}$ and t_i, it follows that $E_i = 0$ in a finite time $t \geq t_i^T$ with

$$t_i^T = \tilde{t} + t_i \tag{32}$$

In terms of the definition of coupling position error E, both the position error e_i and the synchronisation error ε_i converge to zero in a finite time $t \geq t_i^T$ simultaneously. □

Remark 14: Note that $\Delta^M \geq \|K^M - M(q)\|$, $\Delta^C \geq \|K^C - C(q,\dot{q})\|$, $\Delta^G \geq \|K^G - G(q)\|$ are required to be held to guarantee inequalities (28) and (29) to be satisfied. One can use trial-and-error method to acquire Δ^M, Δ^C and Δ^G in controller implementation.

Remark 15: Note that in control law (20), signum function sign(·) may cause chattering. To avoid this problem, the function tanh(·) can be used to instead of sign(·) in practical controller implementation.

4. An illustrative example

Six degrees of freedom SPS-type Stewart Platform is a general parallel manipulator. Control approach developed for it can easily be applied to other types of industrial parallel manipulator.

To demonstrate the performance of the proposed approach, unexceptional simulations were considered as the first step in validation of theoretical design. Simulations were performed in Matlab SimMechnics toolbox; the model was downloaded from the website of Mathworks company: http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do? objectType=file&objectId=2334

Desired trajectories in work space were

$$X(t) = \begin{cases} 0.1 & 0 \leq t \leq 1 \\ 0.2t - 0.1 & 1 \leq t \leq 2 \\ 0.3 & 2 \leq t \leq 3 \\ -0.2t + 0.9 & 3 \leq t \leq 4 \\ 0.1 & 4 \leq t \end{cases},$$

$$Y(t) = \begin{cases} 0.1 & 0 \leq t \leq 2 \\ 0.2t - 0.3 & 2 \leq t \leq 3 \\ 0.3 & 3 \leq t \leq 4 \\ -0.2t + 1.1 & 4 \leq t \leq 5 \\ 0.1 & 5 \leq t \end{cases},$$

$$Z(t) = \begin{cases} 0.5t + 2.5 & 0 \leq t \leq 1 \\ 3 & 1 \leq t \end{cases},$$

$$\alpha(t) = \begin{cases} 0 & 0 \leq t \leq 1 \\ 0.025t - 0.025 & 1 \leq t \leq 5 \\ 0.1 & 5 \leq t \end{cases}, \quad \beta(t) = 0, \gamma(t) = 0.$$

The parameters of 6DOF Stewart Platform are given in Table 1.

For comparison purpose, three control algorithms including the proposed approach, asymptotic stability synchronous tracking control (ASSTC) developed by Sun et al. (2006) and traditional proportional-integral-derivative (PID) control were used to control the system respectively. The ASSTC controller is presented in Appendix C. It should be noticed that the proposed approach is different from ASSTC in synchronisation error, coupling position error and the controller itself. The proposed approach can achieve finite time stability, but ASSTC can only achieve asymptotic stability.

The selected control gains of the three control algorithms are listed in Table 2, where the control gains of PID control were chosen the values used in Matlab Demo directly. By using trial-and-error method, the control gains of the proposed approach and ASSTC are determined individually, so that the best trajectory tracking performance for each controller tested is achieved.

Note that the feedback control gains of ASSTC (K_{ri}, $K_{\varepsilon i}$) and PID (K_P, K_I, K_D) are much larger than the ones of the proposed control (K_1, K_2). The high gains are not desired in industrial applications.

Figures 2 and 3 illustrate the tracking performance in work space and joint space, respectively. The dotted lines are the desired trajectory, the solid lines are the performance with the proposed control, the dashed lines are the performance with ASSTC, and the dash dotted lines are the performance with PID. From these two figures, one can see that the proposed control can track the desired trajectory in a finite time. However, ASSTC and PID can only achieve asymptotic stability. Figure 2 shows that the PID control has overshoot during the response process, which is not expected in industry application. The corresponding joint tracking performances illustrated in Figure 3 shows the desired joint trajectory $q_i^d \neq 0$.

Figures 4–6 illustrate the attitude errors of upper platform, position errors of actuated joints and synchronisation errors, respectively. The solid lines are the errors with the proposed control, the dashed lines are the errors with ASSTC, and the dash dotted lines are the errors with PID. From comparing these figures, one can see that the tracking performances are improved by using the proposed control approach.

Table 1. Parameters of the Stewart Platform manipulator.

Variable	Description	Value	Unit
m	Mass of the moving platform	1216.9	kg
$I_X, I_Y(I_Z)$	Mass moment of inertia of moving platform about X, $Y(Z)$	304.48(608.46)	$\mathrm{kg\,m^2}$
$(m_d)_i$	Mass of lower part of i-th leg	92.11	kg
$(m_u)_i$	Mass of upper part of i-th leg	51.81	kg
$I_{\mathrm{d}X}, I_{\mathrm{d}Y}(I_{\mathrm{d}Z})$	Moment of inertia of lower of i-th leg (in local frame)	43.02(0.156)	$\mathrm{kg\,m^2}$
$I_{uX}, I_{uY}, (I_{uZ})$	Moment of inertia of upper of i-th leg (in local frame)	24.17(0.023)	$\mathrm{kg\,m^2}$
$T_1 \cdots T_6$	The joints position of the upper platform with respect to moving frame	$T_1[0.6428\ -0.7660\ 0]$ $T_2[0.6428\ 0.7660\ 0]$ $T_3[0.3420\ 0.9397\ 0]$ $T_4[-0.9848\ 0.1736\ 0]$ $T_5[-0.9848\ -0.1736\ 0]$ $T_6[0.3420\ -0.9397\ 0]$	m
$B_1 \cdots B_6$	The joints position of the lower platform with respect to inertia frame	$B_1[2.9971\ -0.1309\ 0]$ $B_2[2.9971\ 0.1309\ 0]$ $B_3[-1.3852\ 2.6610\ 0]$ $B_4[-1.6119\ 2.5302\ 0]$ $B_5[-1.6119\ -2.5302\ 0]$ $B_6[-1.3852\ -2.6610\ 0]$	m

Table 2. Control gains of the three controllers.

Controllers	Gains
The proposed control	$\gamma = \mathrm{diag}\{10\}$, $\Lambda_1 = \mathrm{diag}\{10\}$, $\Lambda_2 = \mathrm{diag}\{10\}$, $\alpha = \frac{3}{5}$, $K_1 = \mathrm{diag}\{50,000\}$, $K_2 = \mathrm{diag}\{50,000\}$, $\beta = \frac{3}{5}$, $K^M = \mathrm{diag}\{0.1\}$, $K^C = \mathrm{diag}\{0.1\}$, $K^G = \mathrm{diag}\{300\}$, $\Delta^M = 0.008$, $\Delta^C = 0.008$, $\Delta^G = 200$
ASSTC	$\mu = 10$, $\tilde{\Lambda} = \mathrm{diag}\{10\}$, $\tilde{K}_i^M = 1$, $\tilde{K}_i^C = 1$, $\tilde{K}_i^G = 300$, $D_i^M = 0.008$, $D_i^C = 0.008$, $D_i^G = 200$, $k_{ri} = 100,000$, $K_{\varepsilon i} = 100,000$
PID	$K_P = \mathrm{diag}\{2,000,000\}$, $K_I = \mathrm{diag}\{10,000\}$, $K_D = 4500$

It is important that the synchronisation errors with the proposed control and ASSTC are much smaller than those with PID control. Large synchronisation errors may damage the manipulator in practice. These three figures indicate that the system enters steady state after $t \geq 1$s. Table 3 lists the maximum absolute values of upper platform attitude errors, actuated joint position errors and synchronisation errors after $t \geq 1$s. Table 3 shows that the performance of the proposed and ASSTC are better than PID. It can also be seen from these data that the proposed approach is almost as good as in synchronised operation but is better than ASSTC in work space tracking and joint space tracking operation.

From Figures 4–6, one can see the attitude position error, joint position error and synchronisation error of the proposed approach converge to zero in finite time. These results also mean that the saturation control $-\mathrm{sign}(s)(\Delta^M\|\ddot{r}\| + \Delta^C\|\dot{r}\| + \Delta^G)$ have strong robustness, which can eliminate the non-linear effect caused by the errors between the feedforward control gains and the modelling parameters.

The torque output by each actuated joint is shown in Figure 7. From this figure one can see that the control input of three control algorithms are similar and bounded. This figure also means that the proposed control can achieve better performance than ASSTC and PID with them.

5. Conclusions

This article has studied the issues associated with the finite time position synchronised control for parallel manipulators with fast TSM. A novel synchronisation error and coupling position error are proposed. In light of them, a non-singular fast TSM is designed. The proposed approach can guarantee position error and synchronisation error to converge to zero without requiring the explicit use of a system dynamic model.

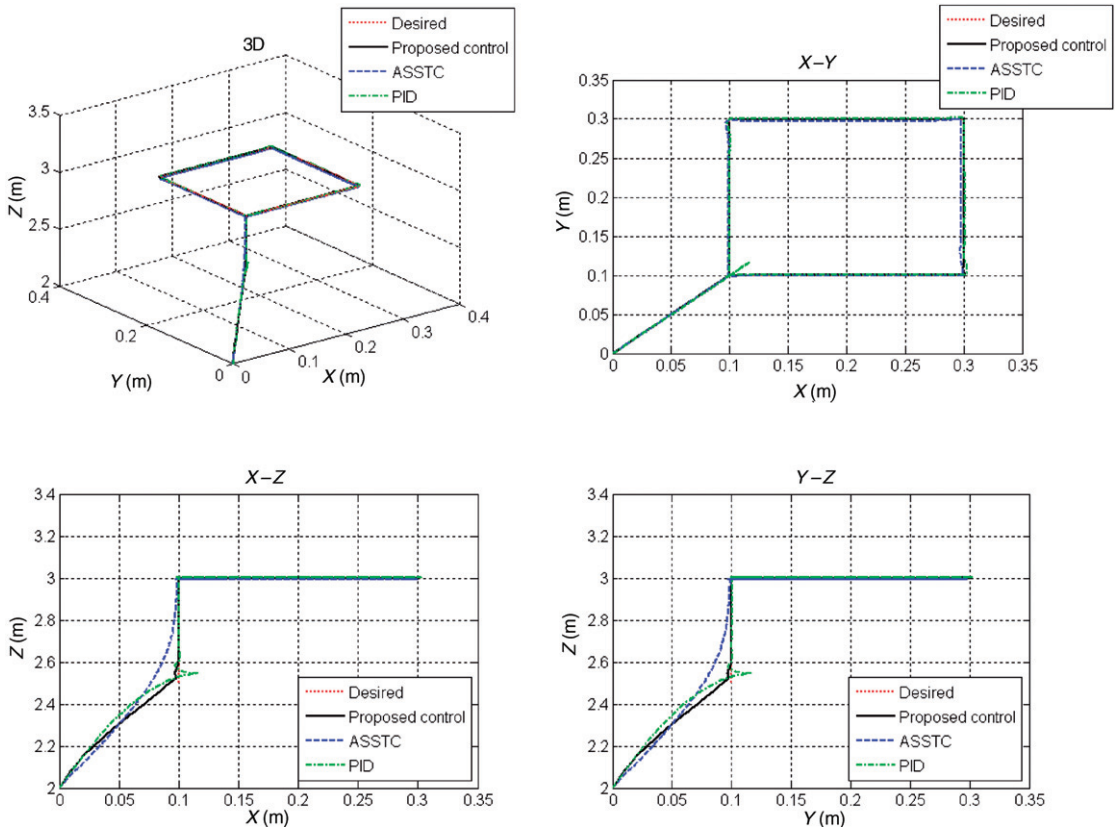

Figure 2. Tracking performance in workspace.

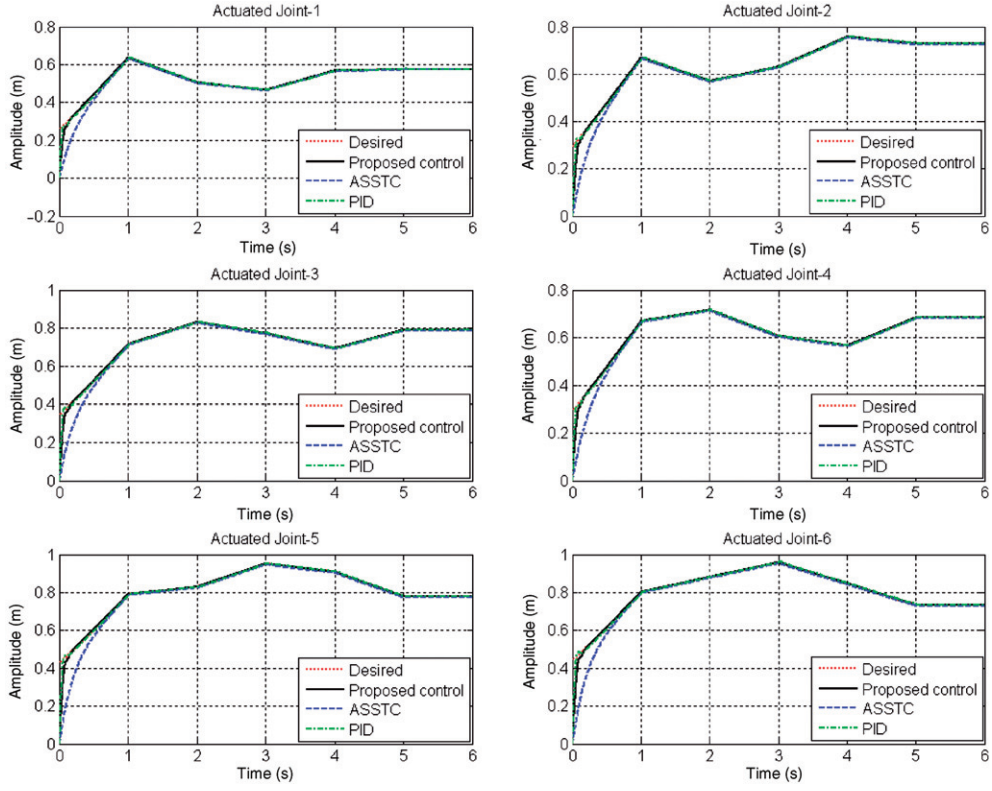

Figure 3. Tracking performance in joint space.

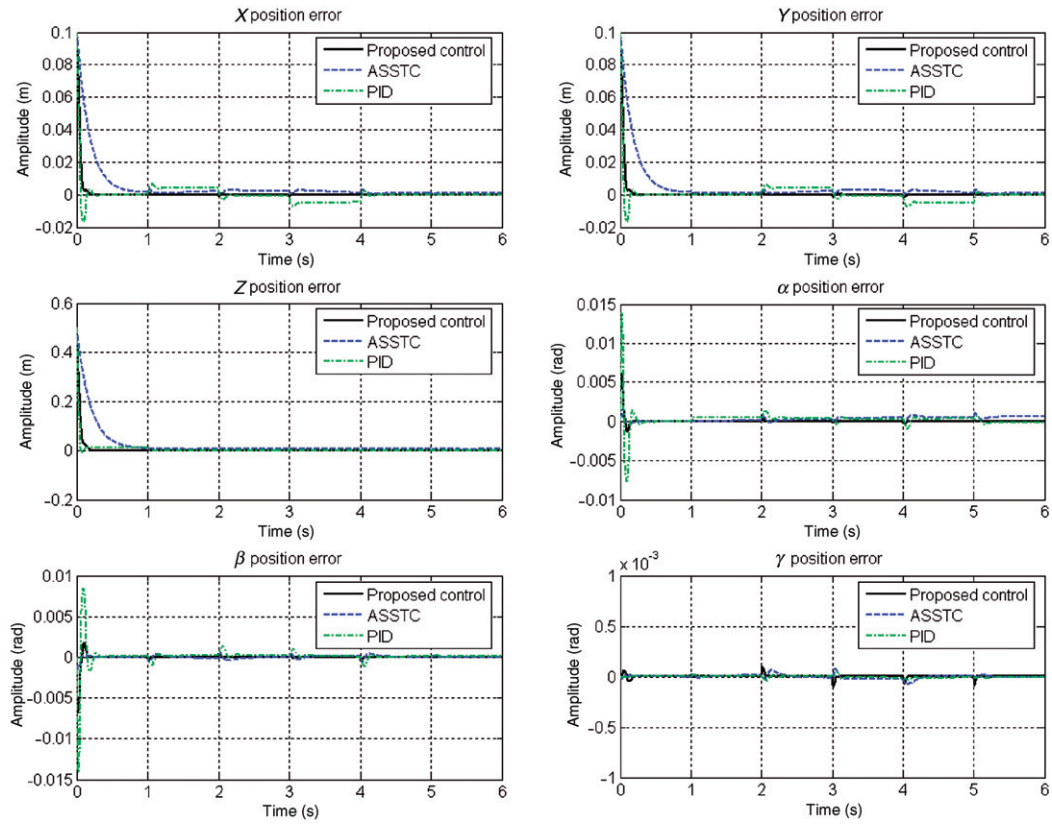

Figure 4. Attitude errors of the moving platform.

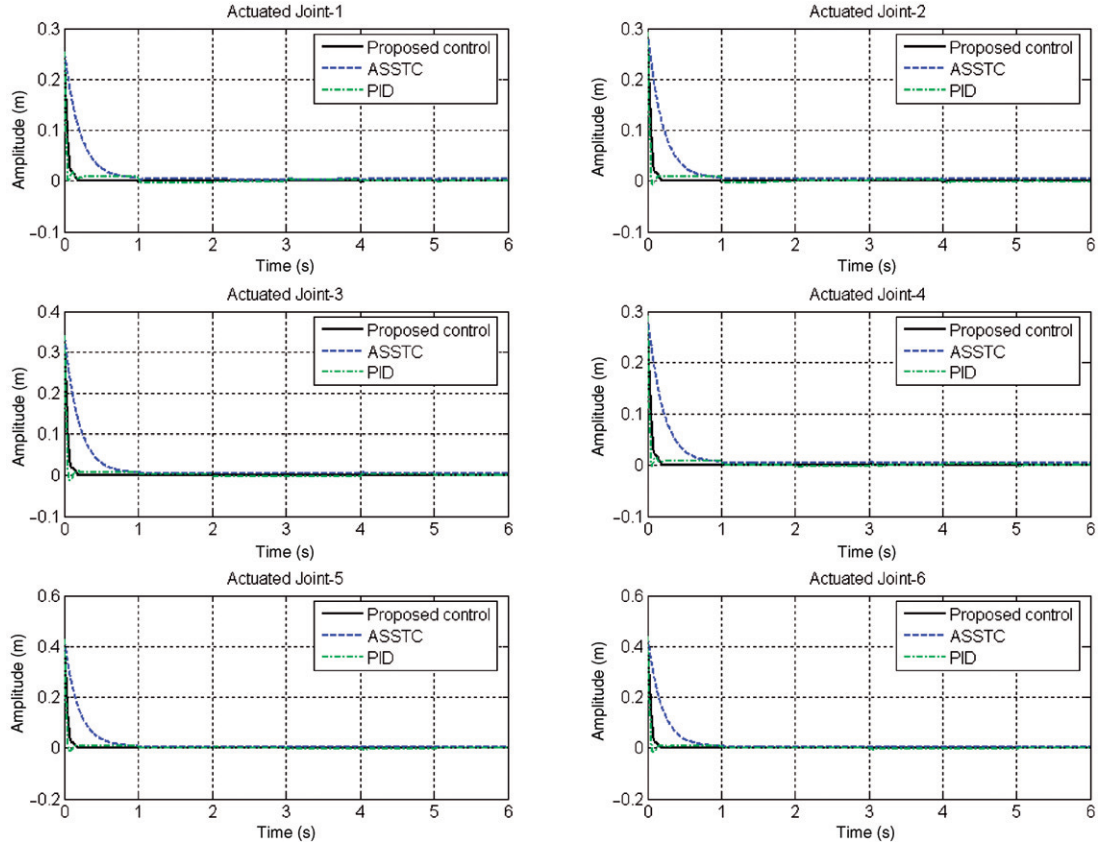

Figure 5. Position errors of actuated joints.

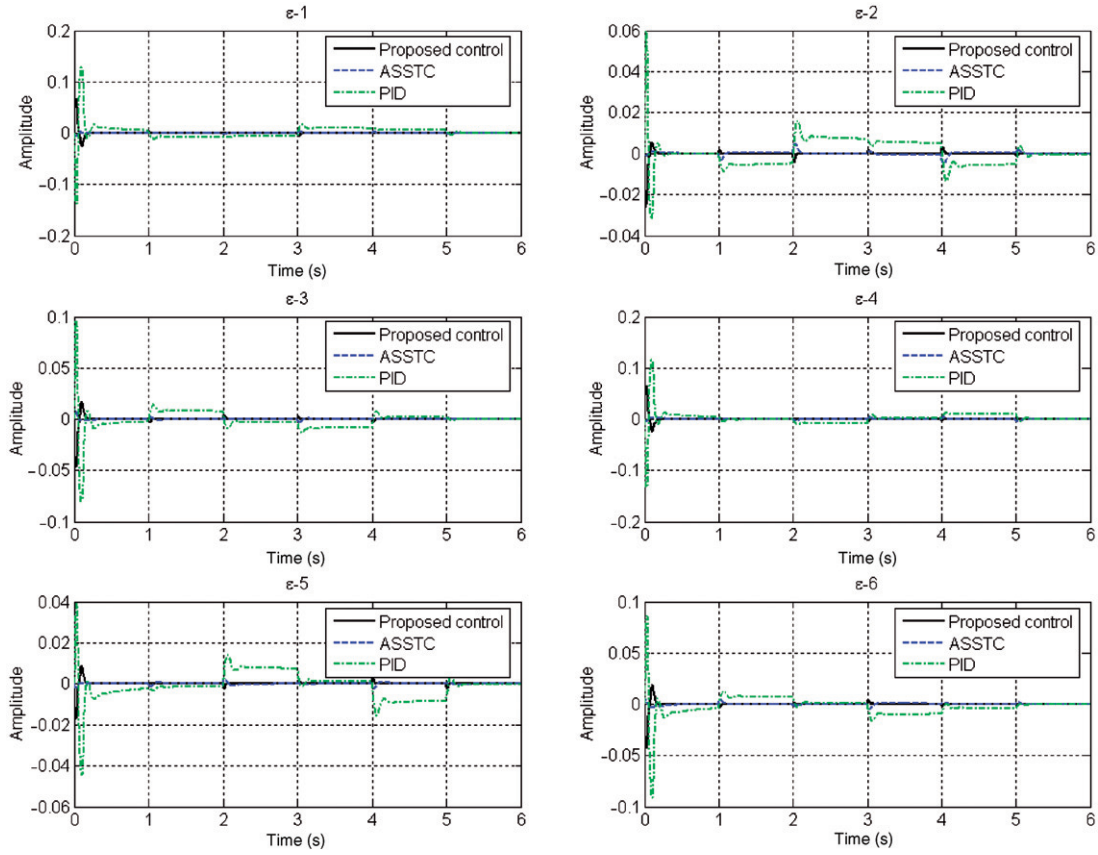

Figure 6. Synchronisation errors.

Table 3. Maximum absolute errors.

Maximum absolute errors	The proposed control	ASSTC	PID
X Position error of platform (m)	0.0014	0.0033	0.0074
Y Position error of platform (m)	0.0015	0.0031	0.0076
Z Position error of platform (m)	0.0037	0.0088	0.0095
α Orientation error of platform (rad)	0.0004	0.0010	0.0014
β Orientation error of platform (rad)	0.0005	0.0006	0.0014
γ Orientation error of platform (rad)	0.0001	0.0001	0.0001
Position error of actuated joint 1 (m)	0.0027	0.0060	0.0073
Position error of actuated joint 2 (m)	0.0027	0.0063	0.0072
Position error of actuated joint 3 (m)	0.0030	0.0069	0.0075
Position error of actuated joint 4 (m)	0.0029	0.0064	0.0077
Position error of actuated joint 5 (m)	0.0035	0.0078	0.0072
Position error of actuated joint 6 (m)	0.0033	0.0079	0.0072
Synchronisation error ε_1	0.0061	0.0029	0.0069
Synchronisation error ε_2	0.0044	0.0051	0.0099
Synchronisation error ε_3	0.0036	0.0028	0.0054
Synchronisation error ε_4	0.0039	0.0038	0.0118
Synchronisation error ε_5	0.0029	0.0022	0.0049
Synchronisation error ε_6	0.0044	0.0051	0.0139

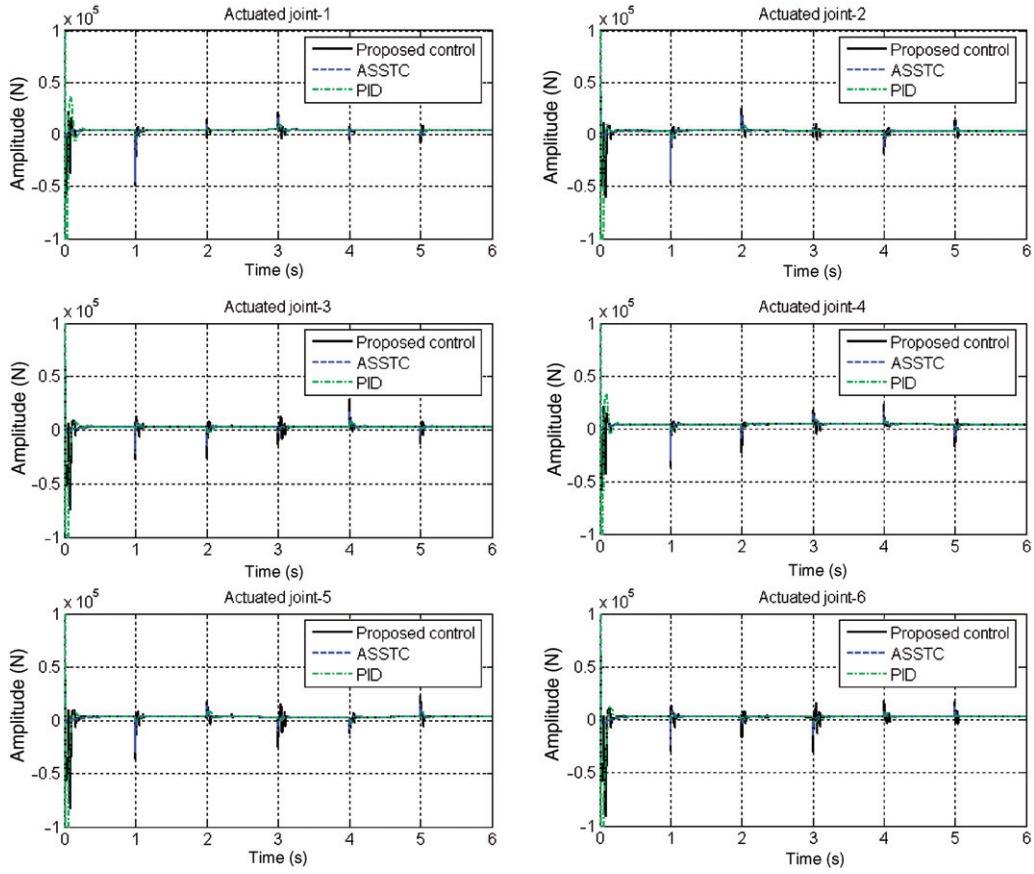

Figure 7. Torque of each actuated joint.

The corresponding stability analysis and an illustrative example are presented to validate the effectiveness of the proposed approach. In comparison with the existing synchronised control for parallel manipulator, the proposed control can achieve higher precision tracking performance. It is simple and can be applied to most of the industrial parallel manipulators easily. It should be mentioned that sound bench tests need to be conducted by simulations and lab demonstrations before applying the approach to control of real parallel manipulators. For stability analysis, it assumes the desired trajectory $q_i^d \neq 0$ and the matrix $(I + \gamma T)$ is non-singular. These two assumptions may be strict in some situation. The further studying for relaxing them is under the authors' work.

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Appendix A

Lemma 1: *Assume $a_1 > 0$, $a_2 > 0$ and $0 < c < 1$, the following inequality holds* (Mitrinovic 1970)

$$(a_1 + a_2)^c \leq a_1^c + a_2^c \tag{A1}$$

Lemma 2: *Suppose $a = [a_1, \ldots, a_n]^T$, $|a| = |a_1| + \cdots + |a_n|$, $\|a\| = (a_1^2 + \cdots + a_n^2)^{1/2}$ represent the Euclidean norm, then the following inequality holds*

$$\|a\| \leq |a| \tag{A2}$$

Proof: For $n = 1$, it is obvious that expression (A2) is satisfied. For $n = 2$, from Lemma 1, the follow inequality can be derived

$$\left(a_1^2 + a_2^2\right)^{1/2} \leq \left(a_1^2\right)^{1/2} + \left(a_2^2\right)^{1/2} \tag{A3}$$

Therefore

$$\left(a_1^2 + a_2^2\right)^{1/2} \leq |a_1| + |a_2| \tag{A4}$$

Assume that for $n = k$ the expression (A2) holds, i.e.

$$\left(a_1^2 + \cdots + a_k^2\right)^{1/2} \leq |a_1| + \cdots + |a_k| \tag{A5}$$

Then for $n = k + 1$

$$\left(a_1^2 + \cdots + a_k^2 + a_{k+1}^2\right)^{1/2} = \left[\left(a_1^2 + \cdots + a_k^2\right) + a_{k+1}^2\right]^{1/2} \tag{A6}$$

From Lemma 1, the right hand of Equation (A6) satisfies the following inequality

$$\left[\left(a_1^2 + \cdots + a_k^2\right) + a_{k+1}^2\right]^{1/2} \leq \left(a_1^2 + \cdots + a_k^2\right)^{1/2} + \left(a_{k+1}^2\right)^{1/2} \tag{A7}$$

According to the expression (A6) and (A7), the following inequality can be given

$$\left(a_1^2 + \cdots + a_k^2 + a_{k+1}^2\right)^{1/2} \leq |a_1| + \cdots + \left|a_{k+1}\right| \tag{A8}$$

By the principle of mathematical induction, the conclusion can be drawn that the expression (A2) is satisfied for any positive integer n. □

Appendix B

The following results on differential inequalities will be used for the stability analysis (Barambones and Etxebarria 2002; Yu 1998)

Definition 1: If $g(V, t)$ is a scalar function of scalars $V(t)$, t in some open connected set $D \in R^2$, then a function $V(t)$ on $[t_0, t_1)$ is a solution of the differential inequality

$$\dot{V}(t) \leq g(V(t), t) \tag{B1}$$

on $[t_0, t_1)$ if $V(t)$ is continuous on $[t_0, t_1)$ and its derivative on $[t_0, t_1)$ satisfies (B1).

Lemma 3: *Let $g(y(t), t)$ be continuous on an open connected set $D \in R^2$ and assume that the initial value problem for the scalar equation*

$$\dot{y}(t) = g(y(t), t), \quad g(t_0) = y_0 \tag{B2}$$

has a unique solution. If $y(t)$ is a solution of (B2) *on $[t_0, t_1)$ and $V(t)$ is a solution of* (B1) *on $[t_0, t_1)$ with $V(t_0) \leq y(t_0)$, then $V(t) \leq y(t)$* for $t_0 \leq t < t_1$.

Lemma 4: *Assume that a continuous positive definite function $V(t)$ satisfies the differential inequality*

$$\dot{V}(t) \leq -kV^{\eta}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0 \tag{B3}$$

where $k > 0$, $0 < \eta < 1$ are constants. Then, for any given t_0, $V(t)$ satisfies the inequality

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - k(1-\eta)(t - t_0), \quad t_0 \leq t \leq t_1 \tag{B4}$$

and $V(t) = 0$, $\forall t \geq t_1$, with t_1 given by $t_1 = t_0 + V^{1-\eta}(t_0)/k(1-\eta)$.

Appendix C

ASSTC control law of Sun et al. (2006) is given as follows

Synchronisation error

$$\begin{cases} \tilde{\varepsilon}_1 = c_1 e_1 - c_2 e_2 \\ \tilde{\varepsilon}_2 = c_2 e_2 - c_3 e_3 \\ \quad\vdots \\ \tilde{\varepsilon}_n = c_n e_n - c_1 e_1 \end{cases} \tag{C1}$$

where c_i is the coupling coefficient of i-th actuated joint.

Coupling position error

$$e_i^* = c_i e_i + \mu \int_0^t (\tilde{\varepsilon}_i(\omega) - \tilde{\varepsilon}_{i-1}(\omega))\mathrm{d}\omega \tag{C2}$$

where μ is a positive coupling parameter, ω is a variable from time zero to t, when $i = 1$, $i - 1 = n$.

Command vector u_i

$$u_i = c_i \dot{q}_i^d + \dot{c}_t \Delta q_i + \mu(\varepsilon_i - \varepsilon_{i-1}) + \Lambda e_i \tag{C3}$$

Linear sliding mode

$$\tilde{r}_i = u_i - c_i \dot{q}_i = \dot{e}_i + \tilde{\Lambda} e_i \tag{C4}$$

ASSTC control law

$$\begin{aligned} \tau_i &= \tilde{K}_i^M c_i^{-1}(\dot{u}_i - \dot{c}_i \dot{q}_i) + \tilde{K}_i^C c_i^{-1} u_i + \tilde{K}_i^G + \mathrm{sign}\left(c_i^{-1} r_i\right) K_i^N(t) \\ &\quad + K_{ri}(t) c_i^{-1} r_i + c_i^T K_{\varepsilon i}(\varepsilon_i - \varepsilon_{i-1}) \\ K_i^N(t) &= D_i^M \left\| c_i^{-1}(\dot{u}_i - \dot{c}_i \dot{q}_i) \right\| + D_i^C \left\| c_i^{-1} u_i \right\| + D_i^G \end{aligned} \tag{C5}$$

where $\tilde{K}_i^M, \tilde{K}_i^C, \tilde{K}^G > 0$ are positive feedforward control gains, $K_{ri}, K_{\varepsilon i} > 0$ are positive feedback gains, $D_i^M, D_i^C, D_i^G > 0$ are scalars.