A Non-linear Interior point Based Optimal Power Flow Algorithm Incorporating Series FACTS Devices

Ye Peng, Song Jiahua, Ye Yumin, Jia Qingquan, Li Jing

***Abstract*-- This paper presents a new method to incorporate flexible AC transmission system (FACTS) devices in optimal power flow (OPF) problem. Through power injection model of FACTS devices, their control to power system is expressed as the additional power equations at the nodes and the branches where FACTS devices are located. These additional power equations are convenient in combination with OPF algorithm based on non-linear interior point (IP) programming. A two-part calculation structure is introduced in order to make full use of the existing OPF algorithm and related software in EMS. Digital simulations of the modified IEEE 30-node system located with multiple FACTS devices are present to test the effectiveness and efficiency of this work. The study also shows that FACTS devices are capable of providing an economically and technically attractive solution to power systems congestion problems.**

***Index Terms*-- FACTS, optimal power flow, non-linear interior point algorithm, and congestion**

I. INTRODUCTION

The capacity of transmission lines is becoming the main bottleneck of electricity transmission in the deregulated power industry. The competition of electricity may aggravate loadability of some transmission lines. An even worse case congestion may happen to some lines [1], while other lines still have a wide capacity margin. To meet the load demands in a power system and satisfy the stability and reliability criteria, the existing transmission lines must be utilized more efficiently. A technically attractive solution to above problems is to use some efficient controls with the help of FACTS (flexible AC transmission system) devices. Hingorani.N.G first defined the concept of FACTS in 1988. Up to now, many advanced FACTS devices have been put forward such as TCSC, TCPS and UPFC. These FACTS devices have a large potential ability to adjust power flows, thus to assure power systems operate in a more flexible, secure and economic way. With the technical development of several FACTS devices and the ongoing of the power market, there is an urgent need for incorporating FACTS devices into the analysis and simulation of power system. Unfortunately, most of the existing optimal power flow program and related software algorithms in modern Energy Management System (EMS) cannot take the optimal FACTS control into account.

Ye Peng is with North China electric university, Baoding China 071003 (e-mail: yepeng_126@sina.com).

Reference [2-10] made some research work about the optimal calculation incorporating FACTS devices. In reference [2-3], the optimal method incorporating series FACTS based on DC model and linear programming [LP] is researched. In order to solve the non-linearity caused by the introduction of FACTS control parameter, a Benders Decomposition method is used in reference [2] and excepted power flow control values are introduced in reference [3]. Either method makes the main optimal problem solved by two sub-optimal stages. In reference [4], The FACTS control is divided into two sub-problems with specified power flower control values, namely active power control and reactive power control, LP based method and sequence quadratic programming [SQP] is used to solve the two sub-problems respectively. Further researches in reference [5-7] SQP is used to bridge this optimal problem based on the full AC power system model. To make full use of FACTS controllability, no specified control values are introduced. Some heuristic methods such as genetic algorithm are also used to solve this optimal problem [8,9], but they are time consuming in some extent.

IP methods have been thoroughly studied and successfully applied to the solution of large-scale linear optimization problem [10]. Although, these methods were first introduced into non-linear programming by Fiacco and McCormick more than twenty years ago, not until recently they have been applied to power system and other non-linear problems. Non-linear IP algorithm is brought to much attention due to its outstanding performance, experience with these methods has been quite positive. In 1992, Mehrotra's predictor-corrector interior point method was regard as an important improvement. Some research about this algorithm is still going on. In this paper, a direct non-linear interior point algorithm based method to OPF problem considering FACTS control is present. The OPF problem is modeled as an optimal congestion dispatch with FACTS devices. Through injection model of FACTS device, the FACTS control to power systems is described as the additional power equations at the nodes and the branches where FACTS devices are located. These additional equations are convenient to be combined into non-linear IP algorithm. The main calculation of this optimal problem can be decomposed into two sub-parts: One is the main part, which is same with the non-linear IP OPF programming only minor changes needed to include the FACTS control parameters. The other is the assistant part. It is used to process the FACTS addition power equations, which only need less calculation and make no change to the sparsity of the related Jacobian and Hessian matrix. Digital simulations are present to show the efficiency and effectiveness of this work.

II. Injection Model of FACTS Devices

There are several FACTS devices powerful for power flow control, such as TCSC, TCPS and UPFC. In this paper, only the first two devices are considered. Supposing that there is a FACTS device located in branch *i-j*, Let the complex voltages at bus-*i* and bus-*j* be denoted by $v_i \angle \theta_i$ and $v_j \angle \theta_j$, z_{ij} is the impedance of the branch, $z_{ij}=r_{ij}+jx_{ij}$; y_{ij} is the admittance of the branch, $y_{ij}=g_{ij}+jb_{ij}$; B_c is the charging capacitance of the branch.

Generally speaking, there are two type models of FACTS devices for the stead-state power flow control and calculation. The first model is Controllable Source Model (CSM), which is formed in light with the physical operating principles of FACTS devices. CSM is straightforward but will destroy the symmetric of the network admittance matrix. The second FACTS model is called as Power Injection Model (PIM), which substitute the CSM for the equivalent power injection to specified nodes according to the circuit theory. With PIM, FACTS devices can be embedded into optimal power flow equations without any modification of network admittance matrix. Here we use PIM.

In steady-state analysis, TCSC can be represented by a controllable voltage source, which phase is vertical with the current vector of the branch. Its equivalent voltage source model is shown in Fig.1. TCPS can be modeled as a controllable voltage source series connected with the branch and a current source parallel connected with the branch as shown in Fig.2. TCPS regulates power flows by the injected voltage source, which multitude is adjustable and phase is vertical to the voltage phase of the connected node. The current source is used to compensate the power that the controllable voltage source injected into the branch.

By equivalent transformation, The voltage source and current source in Fig.1 and Fig.2 can be replaced by the additional injection power at node *i* and node *j* as shown in Fig.3. More details can be found in reference [7]

The additional injection power of TCSC can be written as:

$$\begin{cases} \Delta P_i = V_i^2 \Delta G_{ij} - V_i V_j (\cos\theta_{ij} \Delta G_{ij} + \sin\theta_{ij} \Delta B_{ij}) \\ \Delta Q_i = -V_i^2 \Delta B_{ij} + V_i V_j (\cos\theta_{ij} \Delta B_{ij} - \sin\theta_{ij} \Delta G_{ij}) \\ \Delta P_j = V_j^2 \Delta G_{ij} - V_i V_j (\cos\theta_{ij} \Delta G_{ij} - \sin\theta_{ij} \Delta B_{ij}) \\ \Delta Q_j = -V_j^2 \Delta B_{ij} + V_i V_j (\cos\theta_{ij} \Delta B_{ij} + \sin\theta_{ij} \Delta G_{ij}) \end{cases} \quad (1)$$

Where

$$\Delta G_{ij} = \frac{K_C x_{ij}^2 r_{ij} (K_C - 2)}{(r_{ij}^2 + x_{ij}^2)[r_{ij}^2 + x_{ij}^2 (1 - K_C)^2]}$$

$$\Delta B_{ij} = \frac{K_C x_{ij} (x_{ij}^2 (1 - K_C) - r_{ij}^2)}{(r_{ij}^2 + x_{ij}^2)[r_{ij}^2 + x_{ij}^2 (1 - K_C)^2]}$$

In which K_C is the compensate percentage of TCSC. When capacitance compensation is used, K_C has a positive value. While when inductance compensation is used, it has a negative value. K_C is chosen to be the control parameter of TCSC. We defined that:

$$-K_I^{\max} \le K_C \le K_C^{\max} \quad (2)$$

Where $K_C^{\max}$ and $K_I^{\max}$ is the maximum compensation percentage for capacitate and inductive compensation each.

The additional injection power of TCPS can be written as:

$$\begin{cases} \Delta P_i = -K_P^2 V_i^2 g_{ij} + K_P V_i V_j (\cos\theta_{ij} b_{ij} - \sin\theta_{ij} g_{ij}) \\ \Delta Q_i = K_P^2 V_i^2 b_{ij} + K_P V_i V_j (\cos\theta_{ij} g_{ij} + \sin\theta_{ij} b_{ij}) + K_P^2 V_i^2 \frac{B_C}{2} \\ \Delta P_j = -K_P V_i V_j (\cos\theta_{ij} b_{ij} + \sin\theta_{ij} g_{ij}) \\ \Delta Q_j = -K_P V_i V_j (\cos\theta_{ij} g_{ij} - \sin\theta_{ij} b_{ij}) \end{cases} \quad (3)$$

In which $K_P = tan(\Psi)$, Ψ is the shift angle of TCPS. K_P is chosen to be the control parameter of TCPS. We defined that:

$$tan(\psi^{min}) \le K_P \le tan(\psi^{max}) \quad (4)$$

Where $\psi^{\max}$ and $\psi^{\min}$ is the maximum and minimum shift angle of TCPS.

Notice that the expression of the transmission power in the branch where FACTS located is also changed. The active transmission power of FACTS branch from *i* to *j* is:

$$P_{ij}^F = P_{ij} + \Delta P_{ij} \quad (5)$$

Where P_{ij}^F is the active transmission power expression of the branch located with FACTS, P_{ij} is the active branch power expression without FACTS, ΔP_{ij} here is defined as the additional active transmission power caused by FACTS. And we have:

$$\Delta P_{ij} = -\Delta P_i \quad (6)$$

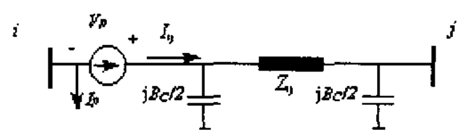

Fig.1 The equivalent voltage source model of TCSC

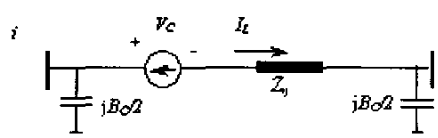

Fig.2 The equivalent source model of TCPS

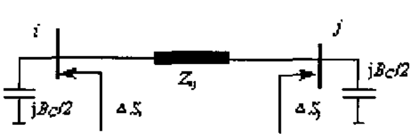

Fig.3 The equivalent injection model of FACTS

Through above, the control of the two FACTS devices to power system is expressed as the additional power at the nodes and the branches where FACTS devices are located, while without any changing of the original network parameters.

III. OPF ALGORITHM WITH FACTS DEVICES

A. Optimal model

Using FACTS models mentioned above, The optimal congestion dispatch incorporating FACTS devices can be formulated as followed OPF problem:

To minimize:

$$minF = \sum_{i=1}^{mp} f(p_{gi}) \quad (7)$$

Subject to:

$$\begin{cases} P_i = P_{gi} - P_{di} + \Delta P_i \\ Q_i = Q_{gi} - Q_{di} + \Delta Q_i \quad i \in n \\ P_{gi}^{min} \le P_{gi} \le P_{gi}^{max} \quad i \in mp \\ Q_{gi}^{min} \le Q_{gi} \le Q_{gi}^{max} \quad i \in mq \\ V_i^{min} \le V_i \le V_i^{max} \quad i \in n \\ Pl_{ij}^{min} \le Pl_{ij} + \Delta Pl_{ij} \le Pl_{ij}^{max} \quad i,j \in n, i \ne j \\ K_i^{min} \le K_i \le K_i^{max} \quad i \in nk \end{cases} \quad (8)$$

With

$$\begin{cases} P_i = -V_i V_j \sum_{i=1}^{n} (\cos\theta_{ij} G_{ij} + \sin\theta_{ij} B_{ij}) \\ Q_i = -V_i V_j \sum_{i=1}^{n} (\cos\theta_j B_{ij} - \sin\theta_{ij} G_{ij}) \quad i, j \in n \end{cases} \quad (9)$$

Where in formula (7-9)

n, mp, mq: number of node, real and reactive power source

i, j: bus indices

$f()$: bid function for generators

p_{gi}, Q_{gi}: real and reactive power output at bus i

p_{di}, Q_{di}: real and reactive power loads at bus i

ΔP_i, ΔQ_i: real and reactive power injection of FACTS devices at bus i

v_i, θ_i: voltage magnitude and angle at bus i

G_{ij}, B_{ij}: the i-jth element of admittance matrix

pl_{ij}: the active transmission power of branch i-j

Δpl_{ij}: the additional active transmission power of FACTS devices at branch i-j

K_i: control parameter of the ith FACTS device

B. Formulation of nonlinear IP algorithm

To solve above problem, a non-linear IP algorithm is introduced. The non-linear IP OPF algorithm is essentially the combination of Lagrangian function, Logarithm barrier function and Newton direction. It can inherit the advantages of original Newton OPF and be further developed to combine the handling of inequality constraints into Newton iteration directly instead of finding the active binding set of inequality constrains. The optimal flow model mentioned above can be simply expressed as:

$$Min(F(z)) \quad s.t.\, g(z) = 0, \quad \boldsymbol{h}_l \le \boldsymbol{h}(z) \le \boldsymbol{h}_u \quad (10)$$

By introducing slack variable vectors, the inequality constrains are transformed to the following equality constrains:

$$\boldsymbol{h}(z) - \boldsymbol{s}_l = \boldsymbol{h}_l \qquad \boldsymbol{h}(z) + \boldsymbol{s}_u = \boldsymbol{h}_u \quad (11)$$

To eliminate the non-negativity constrains of slack variables, logarithmic barrier functions are introduced. Hence, Lagrange function is formulated as:

$$L(z,\lambda,\boldsymbol{w}_l,\boldsymbol{w}_u,\boldsymbol{s}_l,\boldsymbol{s}_u) = f(z) - \lambda^T g(z) + \boldsymbol{w}_l^T(\boldsymbol{h}(z) - \boldsymbol{s}_l - \boldsymbol{h}_l)$$
$$+ \boldsymbol{w}_u^T(\boldsymbol{h}(z) + \boldsymbol{s}_u - \boldsymbol{h}_u) - \mu(\ln \boldsymbol{s}_l + \ln \boldsymbol{s}_u) \quad (12)$$

According to the Kuhn-Tucker stationary condition, the following formula can be derived:

$$\begin{cases} \nabla_z L = \Delta_z f(z) - \boldsymbol{J}^T(z)\lambda + \Delta_z \boldsymbol{h}^T(\boldsymbol{w}_l + \boldsymbol{w}_u) = 0 \\ \nabla_\lambda L = \boldsymbol{g}(z) = 0 \\ \nabla_{sl} L = \boldsymbol{w}_l + [\boldsymbol{s}_l]^{-1}\mu = 0 \\ \nabla_{su} L = \boldsymbol{w}_u - [\boldsymbol{s}_u]^{-1}\mu = 0 \\ \nabla_{wl} L = \boldsymbol{h}(z) - \boldsymbol{s}_l - \boldsymbol{h}_l = 0 \\ \nabla_{wu} L = \boldsymbol{h}(z) + \boldsymbol{s}_u - \boldsymbol{h}_u = 0 \end{cases} \quad (13)$$

Where in formulator (11-13)

$F(\,)$: object function

$g(\,)$ $h(\,)$: equality and inequality constrains

$L(\,)$: Lagrangian function

z: variable vector. $z = [P_g^T \; Q_g^T \; \theta^T \; V^T \; K^T]^T$

h_l, h_u: lower and upper limits of inequality constrains

s_l, s_u: slack variables for inequality constrains. $s_l, s_u > 0$

λ: the multipliers of equality constrains. $\lambda > 0$;

w_l, w_u: the multipliers of inequality constrains, $w_l < 0, w_u > 0$.

μ: the barrier parameter, $\mu > 0$.

Δ: The change in variables.

∇: Differentiation operation.

J: Jacobian matrix.

$[s_l],[s_u],[w_l],[w_u]$: diagonal matrix of s_l, s_u, w_l, w_u

By applying Newton method to the equation (13), we obtain:

$$\begin{cases} \boldsymbol{h}(z,\lambda)\Delta_z - \boldsymbol{J}^T(z)\Delta\lambda + \nabla_z \boldsymbol{h}^T \Delta \boldsymbol{w}_l + \nabla_z \boldsymbol{h}^T \Delta \boldsymbol{w}_l = -\nabla_z L \\ \boldsymbol{J}(z)\Delta z = -\nabla_\lambda L \\ [\boldsymbol{s}_l]\Delta \boldsymbol{w}_l + [\boldsymbol{w}_l]\Delta \boldsymbol{s}_l = -\nabla_{sl} L \\ [\boldsymbol{s}_u]\Delta \boldsymbol{w}_u + [\boldsymbol{w}_u]\Delta \boldsymbol{s}_u = -\nabla_{su} L \\ \nabla_z \boldsymbol{h}\Delta z - \Delta \boldsymbol{s}_l = -\nabla_{wl} L \\ \nabla_z \boldsymbol{h}\Delta z + \Delta \boldsymbol{s}_u = -\nabla_{wu} L \end{cases} \quad (14)$$

Where $h(z,\lambda) = \nabla_z^2 f(z) - \sum_{i=1}^{2n} \lambda_i \nabla_z^2 g_i(z) + \sum_{i=1}^{N} (w_{li} + w_{ui}) \nabla_z^2 h_i(z)$,

N is the inequality constrains number.

Finally solving the above (14), we have:

$$\begin{cases}\Delta s_l = \nabla_z h \Delta z \\ \Delta s_u = -\nabla_z h \Delta z \\ \Delta w_l = -[s_l]^{-1}([s_l]w_l + \mu) - [s_l]^{-1}[w_l]\nabla_z h \Delta z \\ \Delta w_u = -[s_u]^{-1}([s_u]w_u - \mu) + [s_u]^{-1}[w_u]\nabla_z h \Delta z \\ \begin{bmatrix}\Delta z \\ \Delta \lambda\end{bmatrix} = -A^{-1}\begin{bmatrix}\tilde{l} \\ -g(z)\end{bmatrix}\end{cases} \tag{15}$$

In which

$$\tilde{l} = \nabla_z f(z) - J^T(z)\lambda + \nabla_z h^T(w_l + w_u - [s_l]^{-1}([s_l]w_l + \mu) - [s_u]^{-1}([s_u]w_u - \mu))$$

$$A = \begin{bmatrix}\tilde{H}(z,\lambda) & -J^T(z) \\ -J(z) & 0\end{bmatrix}$$

And $\tilde{H}(z,\lambda) = H(z,\lambda) + \nabla_z h^T(-[s_l]^{-1}[w_l] + [s_u]^{-1}[w_u])\nabla_z h$

A is a large sparse matrix. Most of the calculation of this algorithm is to format and factorize this symmetric matrix.

C. Solution procedure

In order to keep the feasibility of the solution proper step for each iteration is necessary. During iterations, the barrier parameter should be adjusted automatically according to the complimentary gap. In our implementation, we define the complementary gap (G_{gap}) as:

$$G_{gap} = \sum_{i=1}^{N}(S_{li}W_{li} - S_{ui}W_{ui}) \tag{16}$$

And μ is determined by:

$$\mu = \frac{G_{gap}}{2N} \cdot \frac{1}{\beta} \tag{17}$$

Where β is a parameter specified by the user which is called accelerate parameter in this paper and $\beta > 1$.

Generally, the IP non-linear method can converge to an optimal solution as $\mu \to 0$. The values of the complementary gap reflect the extent to which the inequality constraints are satisfied. The extent to which the equality constraints are satisfied can be expressed by the maximum mismatch (M_{max}) of equality constraints. Because of the possible difference in the change rate of G_{gap} and M_{max} with iteration cycle, β here is used to balance their convergence rates.

Based on the above discussion, G_{gap} and M_{max} are jointly selected as the convergence stop criterion. The solution procedure of the prime-dual IP method to above optimal problem can be concluded as follows:

Step1: Give the initial values of all the variables and make sure that $\lambda > 0$, $w_l < 0$, $w_u > 0$, $s_l < 0$, $s_u > 0$. Let $\varepsilon_1 = 10^{-6}$, $\varepsilon_2 = 10^{-4}$, Giving a proper value for β ($\beta > 1$).

Step2: Calculate G_{gap} and M_{max}. If $G_{gap} < \varepsilon_1$ and $M_{max} < \varepsilon_2$ then go step5.

Step3: calculate the barrier parameter and solve equation (13) and (14), then get $\Delta z, \Delta\lambda, \Delta s_l, \Delta s_u, \Delta w_l, \Delta w_u$

Step4: Calculate iteration steps of variables and revise the variables. Go to step2.

Step5: Output the optimal solution.

As to the prime variable:

$$step^p = 0.9995 \min\left\{\min_{\Delta S_{li} < 0} \frac{-s_{li}}{\Delta s_{li}}, \min_{\Delta S_{ui} < 0} \frac{-s_u}{\Delta s_{ui}}, 1, i \in N\right\}$$

As to the dual variable:

$$step^d = 0.9995 \min\left\{\min_{\Delta W_{li} > 0} \frac{w_{li}}{\Delta w_{li}}, \min_{\Delta W_{ui} < 0} \frac{-w_u}{\Delta w_u}, 1, i \in N\right\}$$

IV. IMPLEMENTATION ISSUES

Contrast with the implementation without FACTS devices, several main changes are concluded as follows:

(1) Due to the introduction of FACTS control parameters, the dimension of variables is increased.

(2) The inequality constrains of FACTS control parameter is added. It only influence the Jacob matrix of inequality constrains equations.

(3) Additional power is added in the equality power-flow constrains of the node connected with FACTS branch and in the inequality constrains of the branch located with FACTS.

The above two is easy to deal with. It only needs a little modification to the OPF non-linear IP algorithm with the situation without FACTS devices. The third is the main issue. There are three-part calculations that are mainly concerned. That is the calculation of initial values (in formula (13)), Jacobian matrix and Hessian matrix (in formula (15)) associated with the equations embedded with FACTS additional power. To make full use of the existing algorithm and program, we decompose these equations into two parts: One is the same as the traditional equations without FACTS; the other is FACTS additional power equation. So the main calculations can be decomposed into two parts: One is the main part, which is similar to the non-linear IP OPF except some small change due to the introduction of FACTS control parameters. The other is assistant part, which is used to process the FACTS addition power equations and is calculated separately. Notice that the additional power only relates with the complex voltage of nodes that connect with the FACTS branches and the network parameters of FACTS branches, so the calculation caused by the FACTS additional power equation is very limit. And finally we get the matrix we need in formula (13) and (15) by adding this two kind of results together at proper place.

Matrix A is a large sparse matrix. We take the variable order in z as follows:

$$[p_1 \ldots p_i \ldots p_n, q_1 \ldots q_i \ldots q_n, k_1 \ldots k_i \ldots k_{nk}, \varphi_1 v_1 \lambda_{P1} \lambda_{Q1}, \ldots, \varphi_i v_i \lambda_{Pi} \lambda_{Qi}, \ldots, \varphi_n v_n \lambda_{Pn} \lambda_{Qn}]$$

Matrix A can be written as: $A = \begin{bmatrix} G & -I^T \\ -I & H \end{bmatrix}$

Where G is a diagonal matrix; I is make up of "0" and "1". Notice the facts those FACTS additional power equations make no change to the variables sparse relationship (FACTS control parameters themselves not included). So we can use the blocking technique of Newton OPF in matrix H, which has essentially become a standard in OPF problem [11].

Because the small numbers of FACTS control parameters and the variables related in the FACTS additional power equation, the fill-in terms caused by FACTS control parameters in the course of formatting and factorising this Matrix is very limit.

Based on the analysis above, the calculation of this optimal problem can be conveniently solved by the two parts structure: Main part is almost the same to the original OPF algorithm, only need a little modification to include the FACTS control parameters. Assistant part aid the main part to complete the calculation associated with FACTS additional power equations when needed. This structure is available to make full use of the existing OPF algorithm and related software in EMS.

V. CASES STUDY

The proposed methodology is applied to a simple 3-node system and a modified IEEE 30-node system with competitive supply conditions to meet the demand of electricity. In the calculation, β is appointed at 10, The maximum compensation percentage of k_c is appointed at 50%. The maximum control range of k_p is appointed at 0.2. Their initial values are set at middle point of their range. The diagram of 3-node system is show in Fig.4. In the 3-node system, the reactance of each branch is set at 0.03+j0.12pu. The bid price of generator and load level is also shown in Fig.4. In the modified IEEE 30-node system, the bid price of each generator is shown in Table.1. The simulation cases were designed as a congestion dispatch problem. The calculation results of 3-node system were shown at table.2. The location of FACTS devices and calculation results of modified IEEE 30-node system was shown at table.3.

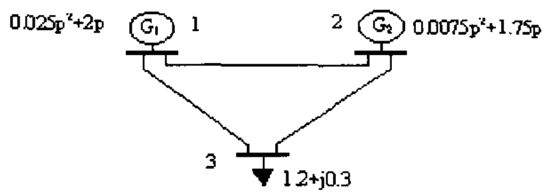

Fig.4. diagrom of 3-node system

Table.1 Bid function of 30-node system

Generator	Bid function($/MW)	Generator	Bid function($/MW)
1	$0.02p^2+2p$	22	$0.0625p^2+2p$
2	$0.0175p^2+1.75p$	23	$0.025p^2+3p$
13	$0.035p^2+4p$	27	$0.0083p^2+3.25p$

First, in order to understand the FACTS control to power system, we see the simple 3-node system. In case 1, the transmission lines have ample capacity, the production cost is only related with the bid price of generators. When the active transmission limit of branch 2-3 is redefined at 0.7 and 0.5 in case 2 and case 3 respectively, transmission congestion happened in branch 2-3. Now system operator can not but make a more purchase of the expensive electricity to meet the load demands. More production cost is produced. In case 4-7, series FACTS was appointed at branch 2-3 to relieve the line congestion. By the optimal calculation, FACTS got their optimal control parameter. When the line congestion is fully relieved by FACTS, the cost reduced approximately equal to the case without transmission congestion happened. When the line congestion is partly relieved, the production cost is reduced in a large extent. In the calculation, the FACTS adjustment is limited by two factors: One is the control parameter's control range as shown in case 5. TCSC reach the maximum compensation percentage. The other is the adjustable network transmission capacity. For example, in case 7, FACTS control parameter did not reach their limit, but the congestion was not fully relieved by FACTS control. Because two branches in the loop congested, there has not transmission capacity left for FACTS to adjust.

In the 30-node system, we appointed the active transmission limit of line 2-4, 27-28 and 21-22 at 0.18pu,

Table 2. Calculation results of 3-node system

Case	Cost($)	Generation mode		FACTS		Line flow limit			Line flow			Iteration times
		G_1	G_2	type	Parameter	1-2	1-3	2-3	1-2	1-3	2-3	
1	3.1132	0.2559	0.9808	\	\	1	1	1	-0.2391	0.4950	0.7393	8
2	3.1547	0.3633	0.8726	\	\	1	1	0.7	-0.1711	0.5345	**0.7000**	8
3	4.8286	0.9732	0.2642	\	\	1	1	0.5	0.2381	0.7350	**0.5000**	8
4	3.1141	0.2579	0.9790	TCSC	-0.1647	1	1	0.7	-0.2759	0.5338	**0.7000**	11
5	3.8253	0.7163	0.5188	TCSC	-0.5	1	1	0.5	-0.0187	0.7350	**0.5000**	8
6	3.1388	0.2686	0.9757	TCPS	-0.0972	1	1	0.5	-0.4667	0.7353	**0.5000**	9
7	3.2342	0.4350	0.8037	TCPS	-0.0738	0.3	1	0.5	**-0.3000**	0.7350	**0.5000**	10

Table 3. Calculation results of modified IEEE-30 node system with multiple FACTS devices

Case	Cost($)	FACTS-1			FACTS-2			FACTS-3			Iteration times
		type	location	parameter	type	location	parameter	type	location	parameter	
1*	601.71	\	\	\	\	\	\	\	\	\	14
2*	587.43	TCSC	2-6	0.3306	TCSC	21-22	-0.5000	\	\	\	16
3*	584.85	TCSC	2-6	0.3429	TCPS	21-22	0.0113	\	\	\	15
4*	583.40	TCPS	4-6	-0.0216	TCPS	10-21	0.0076	TCPS	27-28	0.0435	14
5*	584.25	TCSC	2-6	0.4340	TCPS	21-22	0.0097	TCSC	27-28	0.0309	15

0.15pu and 0.18pu each, other branches at 0.3pu. Now, a serious congestion accrued with multiple lines reaching their transmission limit. Multiple FACTS devices were introduced in the optimal congestion dispatch. When multiple FACTS coexist, according to the optimal algorithm, each FACTS get its optimal control parameter, thus have a co-ordinate control to power system. The congestion problem is relieved by different extent due to the powerful regulation of FACTS to power flow. Thus better social benefits achieved. The location of FACTS has a much influence on the results. Proper location of FACTS is the key to fully exert its function.

To further understand the FACTS control to congestion problem, active spot price of each node in case 1*, 3* and 5* is drawn in Fig.5. They can be easily got from the Lagrange multipliers of active equality constrains [12]. When no FACTS aided (case 1*), the transmission line congestion lead to a wide variation of spot price. By the optimal control of multiple FACTS devices (case 5*), the congestion is relieved completely, the difference of spot price only relates with the contribution to network losses of each node. In case 3, constrains of transmission lines are not fully relieved by FACTS control. The spot price still has some vibration but is alleviated much compared with the case without FACTS devices.

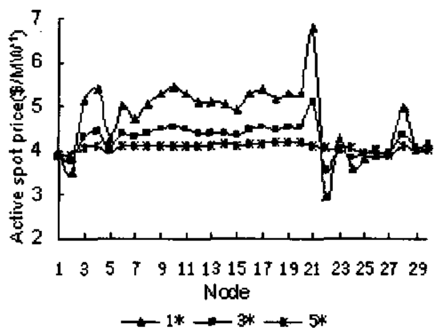

Fig.5 the active spot price of case 1*, 3* and 5*

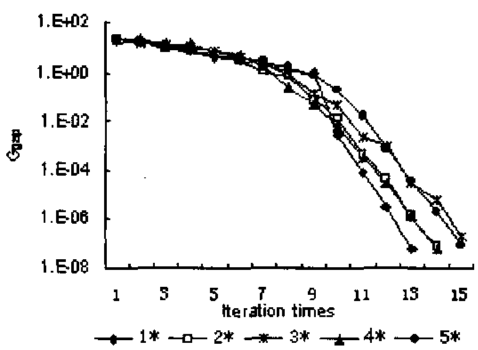

Fig.6 the convergence of case 1*—5*

The iteration times of this algorithm are shown in table.2 and table.3. Fig.6 shows the compensation gap value at each iteration time of different cases. It reflects the convergence characteristic in this method. We can see that the optimal method incorporating FACTS device proposed by this paper has a consistent and robust convergence. Compared with the case without FACTS, the OPF with FACTS devices will only need a little more iteration steps. Some more examples are studied in different systems. The simulation results also show this algorithm has a good convergence performance. As space limited, they are not displayed.

VI. Conclusion

A non-linear interior point based OPF algorithm for incorporating FACTS devices is present in this paper. Compared with previous work, some of its advantages are:

• The introduction of FACTS control is described as the additional power equations, which are convenient to be combined into non-linear IP algorithm.

• It can make full use of the existing OPF algorithm and related software in EMS.

• The method has a good convergence performance.

The study shows that non-linear IP algorithm is an attractive method for OPF problems incorporating multiple FACTS devices. The simulation result also shows that FACTS devices are capable of providing an economically and technically attractive solution to power systems congestion problems.

VII. Reference

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