Applied Soft Computing 14 (2014) 109–119

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

The Primitive Cognitive Network Process in healthcare and medical decision making: Comparisons with the Analytic Hierarchy Process

Kevin Kam Fung Yuen

Department of Computer Science and Software Engineering, Xi'an Jiaotong-Liverpool University, Suzhou Industrial Park, Suzhou, Jiangsu Province, China

ARTICLE INFO

Article history:
Received 31 March 2013
Received in revised form 30 May 2013
Accepted 25 June 2013
Available online 26 July 2013

Keywords:
Medical and healthcare decision making
Medical and healthcare measurement
Paired comparison
Pairwise comparison matrix
Psychometric
Cognitive computing

ABSTRACT

Analytic Hierarchy Process (AHP) is increasingly applied to healthcare and medical research and applications. However, knowledge representation of pairwise reciprocal matrix is still dubious. This research discusses the related drawbacks, and recommends pairwise opposite matrix as the ideal alternative. Pairwise opposite matrix is the key foundation of Primitive Cognitive Network Process (P-CNP), which revises the AHP approach with practical changes. A medical decision treatment evaluation using AHP is revised by P-CNP with a step-by-step tutorial. Comparisons with AHP have been discussed. The proposed method could be a promising decision tool to replace AHP to share information among patients or/and doctors, and to evaluate therapies, medical treatments, health care technologies, medical resources, and healthcare policies.

1. Introduction

Analytic Hierarchy Process (AHP) [1–3] and Analytic Network Process (ANP) [4] are ones of the popular decision making tools. [5–7] comprehensively reviewed and classified numerous AHP/ANP's applications in various domains. Regarding the AHP applications to medical and health care, [5] reviewed fifty articles from 1981 to 2006, and classified them in seven categories: diagnosis, patient participation, therapy/treatment, organ transplantation, project and technology evaluation and selection, human resource planning, and health care evaluation and policy.

AHP applications to health care and medical research are still growing. [8,9] demonstrated the tutorial review to promote applications of AHP in medical and healthcare decision making. [10] combined AHP and goal programming in strategic enterprise resource planning (ERP) in a health-care system. Many medical studies used plenty of questionnaires of AHP design to conduct empirical research to prioritize criteria. For example, [11,12] evaluated colorectal cancer. [13] prioritized a hierarchy of 12 user needs for computed tomography (CT) scanner. [14] elicited patient preferences for health technology assessment (HTA). [15] examined healthcare professionals' assessments of risk factors. [16] prioritized multiple outcome measures of antidepressant drug treatment. [17] assessed the expanded national immunization programs (ENIPs) and evaluated two alternative health care policies in Korea. [18] conducted a case comparison of the fuzzy logic and AHP methods in the development of medical diagnosis system involving basic symptoms elicitation and analysis. [19] compared the performance of AHP and Conjoint Analysis (CA) in eliciting patient preferences for treatment alternatives for stroke rehabilitation. [20] determined the most appropriate method for construction of a sequential decision tree in the management of rectal cancer, using various patient-specific criteria and treatments such as surgery, chemotherapy, and radiotherapy.

[21] reviewed methodological developments of AHP in views of problem modeling, pairwise comparisons, judgment scales, derivation methods, consistency indices, incomplete matrix, synthesis of the weights, sensitivity analysis and group decisions. Some debates of the AHP can be found in [22–30].

The core notion of AHP is the paired comparison using paired ratio scale and used in a pairwise reciprocal matrix. [31–33] indicated that the basic numerical definition of paired ratio scale $a_{ij} = w_i/w_j$ for paired comparison inappropriately represents the perception and cognition of paired difference and potentially producing misapplications, and suggested the paired interval (or differential) scale, $b_{ij} = v_i - v_j$, which replaces paired ratio scale, as the basis for Primitive Cognitive Network Process (P-CNP), which is to distinguish from AHP. Table 1 presents different terminologies to distinguish the Analytic Hierarchy/Network Process from proposed Cognitive Network Process, in order to avoid confusion of two methods. The Cognitive Hierarchy Process (or Primitive CNP) is the basic model of the CNP, whilst the Analytic Hierarchy Process is the basic model of the ANP.

E-mail addresses: kevinkf.yuen@gmail.com, kevin.yuen@xjtlu.edu.cn

1568-4946/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.asoc.2013.06.028

Table 1
Terminology of categories of pairwise comparisons.

	Analytic Hierarchy/Network Process	Cognitive Network Process
Rating scales		
Date type	Paired ratio scales	Paired interval scales
Model type	Analytical scales	Cognitive scales
Pairwise matrix		
Scale type	Pairwise ratio matrix	Pairwise interval matrix
Comparison type	Reciprocal comparison matrix	Opposite comparison matrix
Structure type	Pairwise reciprocal matrix	Pairwise opposite matrix
Model type	Analytical pairwise matrix	Cognitive pairwise matrix
Matrix validity		
Index	Accordance Index	Consistency index
Prioritization operator		
Scale type	Multiplicative prioritization operator	Differential prioritization operator
Model type	Analytic prioritization operator	Cognitive prioritization operator

The word "cognitive" implies that CNP uses the paired interval scale in a native way to represent our perception of paired difference as the expert judgment using paired ratio scale in AHP is questionable. For instance, we can more easily determine the answer or have less chance of errors for the difference of two than the ratio of two. The word "cognitive" could refer to cognitive sciences domain for the paired comparisons, as human cognitions of paired comparisons on the basis of measurement scale, number system and arithmetic operations are open to discuss.

The rest of this study is organized as follows. Section 2 discusses the flaws of paired ratio scale and the advantages of paired interval scale. Section 3 illustrates the concept of the Primitive Cognitive Network Process. Section 4 presents an application to Dogbite Treatment Decision using P-CNP. Section 5 discusses the comparisons with AHP, and the conclusion is in Section 6.

2. Drawbacks on paired ratio scale

The knowledge representation of pairwise comparison consists of two parts: syntactic form and semantic form. The syntactic form is a sentence using linguistic words for paired comparison. The semantic form is the mathematical expression or numerical representation for the syntactic form of the paired comparison. The semantic form for the syntactic form in AHP is open for discussion.

Definition of AHP's ratio scale is typically different from the ratio scale widely applied and defined in [34], as AHP's ratio scale does not have zero point. In order not to confuse with the conventional concept of ratio scale, the term "paired ratio scale" refers to the scale development using in the measurement of paired comparison of AHP, whilst paired interval/differential scale is used for CNP.

An example of the differences of paired comparisons in paired ratio scale and paired interval scale can be referred to in Fig. 1. The weights of two persons, e.g. Peter and Jason, are compared. If Peter is 60 kg and Jason is 61 kg, we can say that Jason is slightly heavier than Peter by our observation. However, if Saaty's ratio scale is applied, this is another story. The statement that Jason is slightly heavier than Peter will be interpreted as the statement that Jason is 2 times as heavy as Peter (viz. Jason is 1 time heavier than Peter). It is ridiculous to distort the fact to the exaggerated expression. If Peter is 60 kg and Jason is 70 kg by our perception, Jason is much taller than Peter. For numerical representation of Saaty's ratio scale, Jason is 5 times or 7 times as heavy as Peter.

There are various misleading examples. If Peter is 1.4 m, Jason will be 2.8 m if Jason is slightly higher than Peter (Jason is 1.5 m in fact). Providing that Peter is 15 years old, Jason will be 30 years old if Jason is slightly older than Peter (Jason is 16 in fact). Providing that Peter's IQ is 120, Jason's will be 240 if Jason is slightly more intelligent than Peter (Jason's IQ is 123 in fact). [35] indicated the flaws of paired ratio scales for time and temperature due to absence of the absolute zeros.

Whilst most applications use the paired ratio scale schema, 1, 2, . . ., 9, which appears to be the standard of the applications of the AHP, Saaty [3] mentioned that "When the elements being compared are closer together than indicated by the scale, one can use the scale 1.1, 1.2, . . ., 1.9. If still fine, one can use the appropriate percentage refinement". However, such scale 1.1, 1.2, . . ., 1.9 is much difficult (or impossible unless all entries of the matrix are 1) to get a perfectly consistent matrix, and induces much higher chance for higher inconsistency, due to the transitive property ($a_{ik} \cdot a_{kj} = a_{ij}$). For example, if $a_{12} \cdot a_{23} = 1.1 \cdot 1.2 = 1.32$, no scale point 1.32 (but only 1.3) is for a_{13}.

It appears that Saaty's ratio scale does not represent the reality of the cognition of paired difference, and usually produces exaggerated results beyond our common sense. The paired interval scale is more appropriate in above cases. For example, in the case of Peter and Jason who are 60 kg and 61 kg respectively, the statement "Jason is 1 kg heavier than Peter" matches with the statement "Jason is slightly heavier than Peter."

The perception of interval (or difference) of two objects is much more straightforward and native than the perception of the ratio of both. Operations of addition and subtraction are easier than the operations of multiplication and division, which are based on repeated additions and subtractions. In primary school, we learn addition and subtraction prior to multiplication and division. If we do not know addition and subtraction, it is impossible to understand multiplication and division. Usually if we do not memorize the simple multiplication table, we usually do not know how to perform multiplication and division quickly. Due to the invention of the calculator, many adults may use the calculator instead of the multiplication table in memory. Usually we do not call the multiplication table, linking with linguistic labels, for making comparison. Thus, addition and subtraction are straightforward and native for the comparison of two objects.

It may be suggested that transformation from $a_{ij} = w_i/w_j$ to $\ln a_{ij} = \ln w_i - \ln w_j$, which looks equivalent to $b_{ij} = v_i - v_j$. Such claim that the paired interval scale can be formed from paired ratio scale is misleading for two reasons. For one reason, the concepts of paired ratio scale and paired interval scale are completely different. When $a_{ij} = w_i/w_j$ is taken by a logarithm function, the scale $a_{ij} = 1, 2, \ldots, 9$ is distorted by the logarithm function as well, i.e. $\ln a_{ij} = \ln 1, \ldots, \ln 9$, which is definitely not equivalent to $b_{ij} = 0, \ldots, 8$. For example, "Object A is 8 units more than Object B" does not means "Object A is 2980 ($\ln^{-1}(8) = e^8$) times as many as Object B." Such suggestion completely dissociates mathematical form from the human cognition.

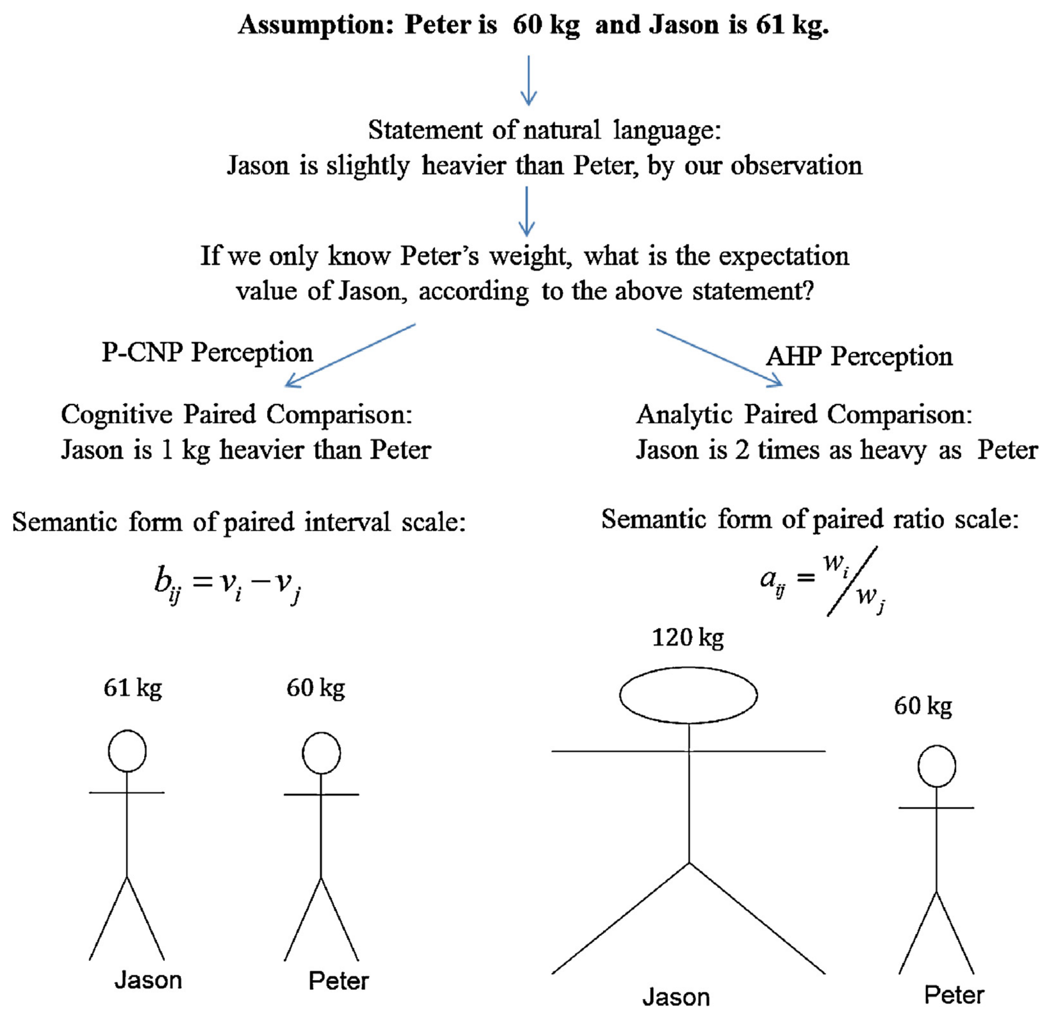


Fig. 1. Cognitive pairwise comparison and analytic pairwise comparison.

For another reason, the result is recovered after anti-logarithm operation. For example, if Peter is 1.4 m (known), and Jason is 1.5 m (unknown), then Jason is slightly taller than Peter by our observation. Regarding the transformation form, the semantic form means that "the logarithm value of the height of Jason is 0.6899 (ln 1.4 – ln 1.5) more than Peter". With judgment using the logarithm proposal, Jason is $w_i = \ln^{-1}(\ln a_{ij} + \ln w_j) = \ln^{-1}(\ln 2 + \ln 1.4) = 2.9$ m. This ridiculous result has no difference from the pure ratio one, but much more complicated to be understood and to be computed. Thus logarithm of paired ratio scale does not lead to paired interval scale. Section 5 discusses this issue further after Sections 4 and 5 introduce the related computational concepts.

3. Primitive Cognitive Network Processes

The Cognitive Network Process is the cognitive architecture which comprises five cognitive decision processes: the Problem Cognition Process (PCP), Cognitive Assessment Process (CAP), Cognitive Prioritization Process (CPP), Multiple Information Fusion Processes (MIP), and Decisional Volition Process (DVP).

3.1. Step 1: Problem Cognition Process

Problem Cognition Process is to formulate the Decision Problem as the measurable Structural Assessment Network (SAN) (Fig. 2), which comprises four units: an objective O, which is one sentence statement expressing the desired goal of the decision maker; a criteria structure (or structural criteria) C with a set of alternatives $T = [T_1, \ldots, T_i, \ldots, T_m]$ using a measurement scale schema $(\aleph, \bar{X})$. The questionnaire is developed according to the design of SAN.

Regarding measurement scale schema, let $\aleph$ be the space of linguistic labels of the paired interval scales such as {*Equally*, *Slightly*, . . ., *Absolutely*} and the opposite of the set. The numerical representation $\bar{X}$ of paired interval scales for differential comparison is in the following form:

$$\bar{X} = \left\{ \alpha_i = \frac{i\kappa}{\tau} | \forall i \in \{-\tau, \ldots, -1, 0, 1, \ldots, \tau\},\ \kappa > 0 \right\} \tag{1}$$

κ (read kappa) is the normal utility, which is the mean of the individual utility values of the comparison objects, and $\kappa > 0$; by default setting, $Max(\bar{X}) = \kappa$, especially if the mean of the individual utility values of the comparison objects or the scale sense is unknown. κ indicates how people perceive the difference value of paired objects in different scenarios. $2\tau + 1$ is the number of the intervals in the scale schema. $\tau + 1$ usually uses 7 ± 2 scale items in a rating process. An example of $(\aleph, \bar{X})$ is the default setting is shown in Table 2. Advantage of such definition associated with normalization function is discussed in step 3. The definition for rating scale could be disassociated from κ, especially when detailed information is provided.

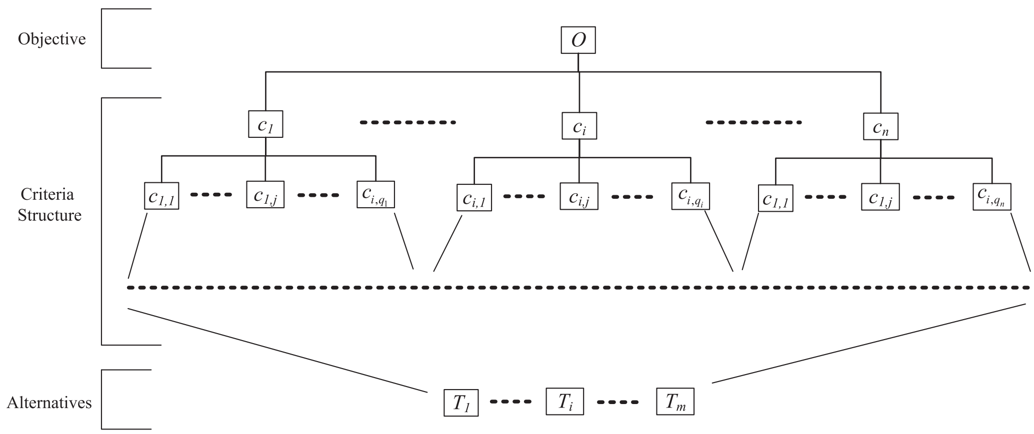


Fig. 2. A structured assessment network of P-CNP.

3.2. *Step 2: Cognitive Assessment Process*

In the Cognitive Assessment Process, the survey is conducted. A list of the Pairwise Opposite Matrices (POMs) is formed according to assessment result of the questionnaire yielded by the decision makers (or raters). The Accordance Index checks the validity of each POM.

The Pairwise Opposite Matrix (POM) is used to interpret the individual utilities of the candidates. Let an ideal utility set be $V=\{v_1,\ldots,v_n\}$, and the comparison score be $b_{ij}\cong v_i-v_j$. The ideal pairwise opposite matrix is $\tilde{B}=[v_i-v_j]$. A subjective judgmental pairwise opposite matrix using paired interval scales is $B=[b_{ij}]$. $\tilde{B}$ is determined by B as follows:

$$\tilde{B}=[\tilde{b}_{ij}]=\begin{bmatrix}0 & v_1-v_2 & \ldots & v_1-v_n\\ v_2-v_1 & 0 & \ldots & v_2-v_n\\ \vdots & \vdots & \ddots & \vdots\\ v_n-v_1 & v_n-v_2 & \cdots & 0\end{bmatrix}\cong\begin{bmatrix}0 & b_{12} & \ldots & b_{1n}\\ b_{21} & 0 & \ldots & b_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ b_{n1} & b_{n2} & \cdots & 0\end{bmatrix}=[b_{ij}]=B \quad (2)$$

When $i=j$, $b_{ij}=v_i-v_j=0$. $[b_{ij}]$ is given by an expert, and $b_{ij}\in\bar{X}$. The expert only fills an upper triangular matrix of the following form:

$$B^{+}=\begin{cases}b_{ij} & i<j\\ 0 & \text{otherwise}\end{cases},\quad \text{or written explicitly},\quad B^{+}=\begin{bmatrix}0 & b_{12} & \ldots & b_{1n}\\ 0 & 0 & \ldots & b_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0\end{bmatrix} \quad (3)$$

The lower triangular matrix is given by the opposite of an upper triangular matrix such that $b_{ij}=-b_{ji}$, and has the below form:

$$B^{-}=\begin{cases}b_{ij} & i>j\\ 0 & \text{otherwise}\end{cases},\quad \text{or written explicitly},\quad B^{-}=\begin{bmatrix}0 & 0 & \ldots & 0\\ b_{21} & 0 & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ b_{n1} & b_{n2} & \cdots & 0\end{bmatrix} \quad (4)$$

$[b_{ij}]$ is achieved by $B=B^{+}+B^{-}$. A complete POM needs $n(n-1)/2$ ratings. B is validated by the Accordance Index (AI) of the following form:

$$\text{AI}=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}d_{ij},\quad \text{where } d_{ij}=\sqrt{Mean\left(\left(\frac{1}{\kappa}(B_i+B_j^T-b_{ij})\right)^2\right)} \quad (5)$$

where $\text{AI}\geq 0$, and κ is the normal utility, B_i is the row vector of B, B_j^T is the row vector of B^T or column vector of B. If AI = 0, then B is perfectly accordant, i.e. $\tilde{B}\equiv B$; if $0<\text{AI}\leq 0.1$, then B is satisfactory. If AI > 0.1, then B is unsatisfactory.

A remark is that there was a careless typo for the presentation of AI form in [31–33] which was $\text{AI}=(1/n^2)\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}d_{ij}}$. Form (5) computes the average of root mean square errors. The square root in [31–33] should not be needed again. However, the AI results of examples in [31–33] corresponds to form (5), as the computational

Table 2
Paired Interval Scale schema for pairwise opposite comparison.

Level (i)	Numerical form ($\bar{X}$)	Verbal form (ℵ)	Definition (Object A vs. Object B)
0	0	Equally	Object A and Object B are equal
1	$\frac{\kappa}{8}$	Slightly	Object A is slightly over Object B
2	$\frac{2\kappa}{8}$	Moderately	Object A is moderately over Object B
3	$\frac{3\kappa}{8}$	Fairly	Object A is fairly over Object B
4	$\frac{4\kappa}{8}$	Highly	Object A is highly over Object B
5	$\frac{5\kappa}{8}$	Strongly	Object A is strongly over Object B
6	$\frac{6\kappa}{8}$	Significantly	Object A is Significantly over Object B
7	$\frac{7\kappa}{8}$	Outstandingly	Object A is outstandingly over Object B
8	κ	Absolutely	Object A is absolutely over Object B
$\{i\}$	Opposite values for reverse comparisons		Object B is over Object A in some degree

algorithm has been implemented in the computer program. Form (5) can explicitly be written in below format.

$$\mathrm{AI} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sqrt{\frac{1}{n}\sum_{p=1}^{n}\left(\frac{(b_{ip}+b_{pj}-b_{ij})}{\kappa}\right)^2} \quad (6)$$

During computation steps, the above form is not convenient to have clear view shown in Table 5 for any discordant location.

3.3. Step 3: Cognitive Prioritization Process

In the Cognitive Prioritization Process, each pairwise opposite matrix B is prioritized to the priority vector W, i.e. $\psi: B \to V$, where ψ is cognitive prioritization operator. Two methods are chosen in [31–33]: Primitive Least Squares (PLS) (or Row Average plus the normal Utility (RAU)) and Least Penalty Squares (LPS), specified as follows.

The vector of individual utilities can be derived by the Primitive Least Squares optimization model of B which is of the following form:

$$\mathrm{PLS}(B^+,\kappa) = Min\ \bar{\Delta} = \sum_{i=1}^{n}\sum_{j=i+1}^{n}(b_{ij}-v_i+v_j)^2 \quad (7)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} v_i = n\kappa,$$

where $n = |\{v_i\}|$, and κ is the normal utility.

The closed form solution can be solved manually and is the Row Average plus the normal Utility (RAU), given by:

$$\mathrm{RAU}(B,\kappa) = \left[v_i : v_i = \left(\frac{1}{n}\sum_{j=1}^{n} b_{ij}\right) + \kappa, \quad \forall i \in \{1,\ldots,n\}\right] \quad (8)$$

If $\exists v_i \in V$ is less than 0, κ can be increased such that $v_i \geq 0, \quad \forall v_i \in V$, or the Least Penalty Squares (LPS) operator is applied as an alternative, as follows:

$$\mathrm{LPS}(B^+,\kappa) = Min\ \widehat{\Delta} = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\beta_{ij}\cdot(b_{ij}-v_i+v_j)^2$$

$$\beta_{ij} = \begin{cases} \beta_1, & v_i > v_j \& b_{ij} > 0 \\ & \text{or } v_i < v_j \& b_{ij} < 0 \\ \beta_2, & v_i = v_j \& b_{ij} \neq 0 \\ & \text{or } v_i \neq v_j \& b_{ij} = 0 \\ \beta_3, & \text{otherwise} \end{cases}, \quad 1 = \beta_1 \leq \beta_2 \leq \beta_3 \quad (9)$$

$$\text{s.t.} \quad \begin{matrix}\sum_{i=1}^{n} v_i = n\kappa, \\ v_i \geq 0, i = 1, 2, \ldots, n\end{matrix}, \quad n = \left|\{v_i\}\right|, \text{ and } \kappa \text{ is the normal utility.}$$

For the most decision problems, summation of the priorities $W = \{w_1, \ldots, w_n\}$ is equal to one, i.e. $\sum_{i=1}^{n} w_i = 1$. W is said to be a normalized priority vector (or a priority vector in short). In order to use the proposed methods, the individual utility vector is rescaled (or normalized) as a normalized priority vector by the rescale function or the normalization function of the below form.

$$W = \left\{w_i : w_i = \frac{v_i}{n\kappa}, \forall i \in \{1,\ldots,n\}\right\}, \quad \text{which} \quad \sum_{i \in \{1,\ldots,n\}} v_i = n\kappa \quad (10)$$

W is the special case of V such that $n\kappa = 1$.

The advantage of normalization is that the normalized priority vectors and final normalized aggregation results in latter stage are

Table 3
Conversion between paired ratio scale and paired interval scale.

Label	Equally	Slightly	Moderately	Fairly	Highly	Strongly	Significantly	Outstandingly	Absolutely	
Paired ratio scale	1	2	3	4	5	6	7	8	9	Reciprocal values for reverse comparison
Paired interval scale ($\kappa = 8$)	0	1	2	3	4	5	6	7	8	Opposite values for reverse comparison

the same whatever positive κ is chosen once κ is associated with the paired interval scale in the format of Table 2. The disadvantage is that more compared objects produce less priority values, as all objects share the only one unit, and ultimately the differences among objects are very small. AHP also has such disadvantage.

3.4. Step 4: Multiple Information Fusion Processes

Let V' be a vector of criteria weights derived by a POM, and $\{V\}$ be a matrix of corresponding priorities derived by a list of POMs. Information Fusion Process is the function to aggregate a priority pair $(V', \{V\})$ to a result set Y by an aggregation operator, i.e. $AO : (V', \{V\}) \to Y$, such that the number of V' is the same as the number of the vectors of $\{V\}$. Multiple Information Fusion Process is the process to aggregate a list of hierarchical priority pairs $\{Y\}$ to a vector of the objective priorities (or global priorities) T according to the Structural Access Network by an aggregation operator, i.e. $AO : \{Y\} \to T$. Due to the popularity, simplicity, and efficiency, the weighted arithmetic mean is chosen as default AO.

$$wam(V', V) = \sum_{i=1}^{n} v'_i v_i \quad (11)$$

Whilst the evaluation involves a group of raters, aggregation of each rater is also performed in this stage. Aggregation of each individual priority vector of each rater is the recommended method.

3.5. Step 5: Decisional Volition Process

The Decisional Volition Process is the process to decide the final decision $t^* \in T = [T_1, \ldots, T_m]$ with respect to the aggregation results by a volition function $VL : T \to t^*$. The best alternative conventionally is determined by the highest score $T^* = Max(T)$, and its position γ is returned by the argument of the maximum function arg max.

$$t^* = VL(T) = T_\gamma, \quad \text{where } \gamma = \underset{i \in \{1,2,\ldots,m\}}{arg\max} (\{T_1, T_2, \ldots, T_i, \ldots, T_m\}) \quad (12)$$

The above form is used for the selection problem for one alternative. If more alternatives are needed, sorting algorithms are used to find a list the best candidates. The aggregation results or mere prioritization results could be used as ones of parameters for some specific functions.

4. Application to Dogbite Treatment Decision

According to the literature review in Section 1, many studies applying AHP to the medical and health care research and

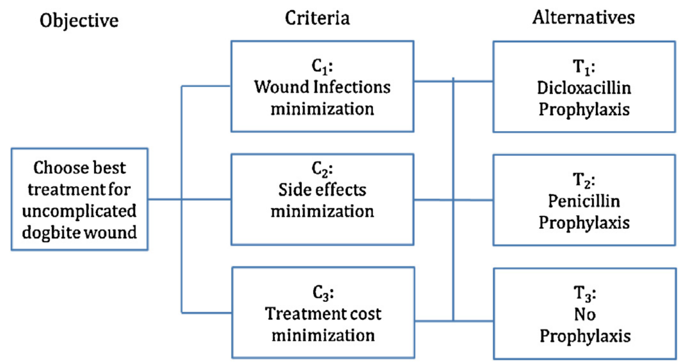


Fig. 3. Structural assessment network for Dogbite Treatment Decision.

applications. Most studies used pairwise reciprocal matrices of AHP as quantitative research to measure the weights of criteria, but a few studies presented the data of pairwise reciprocal matrices. The early study [8] illustrated a tutorial applying AHP to a clinical decision making problem regarding the use of prophylactic antibiotics in the treatment of a patient who has a dogbite wound. As [8] was the early introduction of AHP to medical decision and the expert judgment data were presented, this paper adopts the Dogbite Treatment Decision problem from [8] as background, revises it by using P-CNP, and presents the proposed approach step by step from laymen perspective.

4.1. Step 1: Problem Cognition Process

The measurable Structural Assessment Network (SAN) is reproduced from [8] and presented in Fig. 3. The objective is to choose the best treatment for uncomplicated dogbite wound. Three factors are used for the treatment selection: to minimize wound infections, to minimize side effects, and to minimize treatment cost. Three treatment alternatives are proposed: Dicloxacillin Prophylaxis, Penicillin Prophylaxis, and no Prophylaxis. The rating scale schema in Table 2 is applied, and κ is set to be 8. The formats of survey questions with judgment data are presented in the next step.

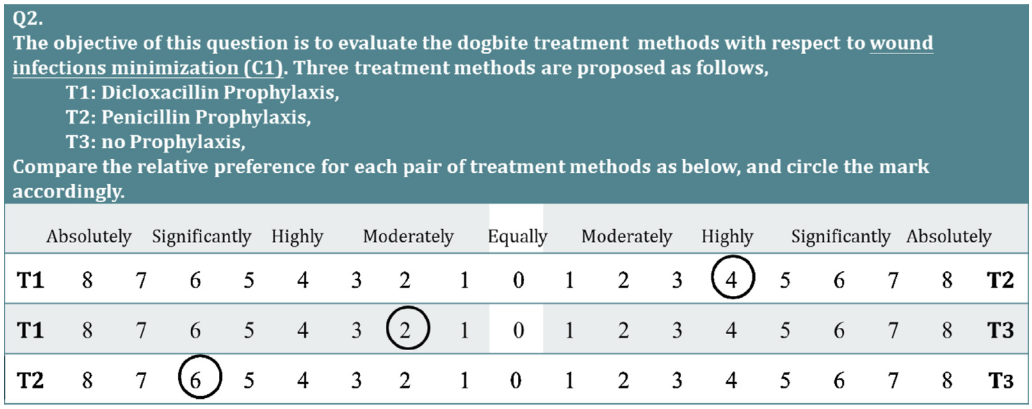
4.2. Step 2: Cognitive Assessment Process

A clinical expert fills the survey shown in Figs. 4–7. For tutorial purpose, Figs. 4–7 demonstrate different formats of the question designs. In real practice, a unified format is recommended in order to simplify raters' cognitions for the questions. The rating scores are converted from the pairwise reciprocal matrices presented in

Q1.
The objective of this question is to evaluate the weights of criteria for dogbite treatment decision. Three criteria are used as follows,
C1: Wound infections minimization,
C2: Side effects minimization,
C3: Treatment cost minimization
Compare the relative importance for each pair of criteria as below, and circle the mark accordingly.

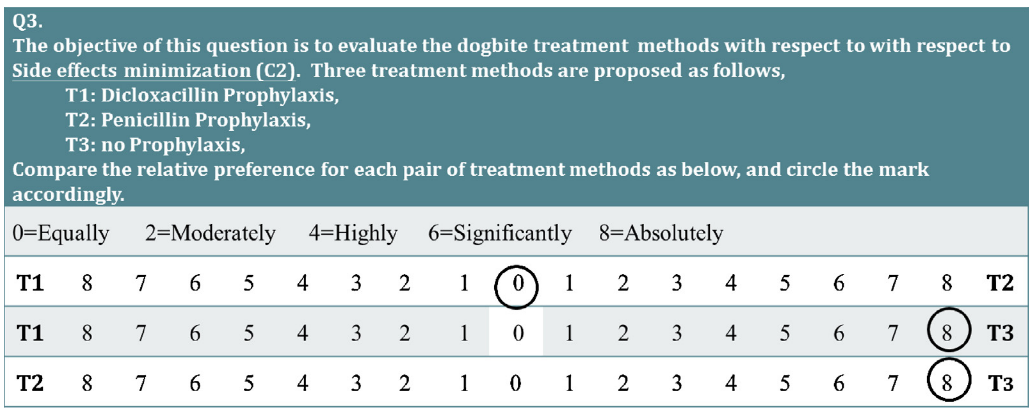
Criteria	Absolutely	Outstandingly	Significantly	Strongly	Highly	Fairly	Moderately	Slightly	Equally	Slightly	Moderately	Fairly	Highly	Strongly	Significantly	Outstandingly	Absolutely	Criteria
C1	8	7	6	5	4	3	②	1	0	1	2	3	4	5	6	7	8	**C2**
C1	8	7	6	5	④	3	2	1	0	1	2	3	4	5	6	7	8	**C3**
C2	8	7	6	5	4	3	②	1	0	1	2	3	4	5	6	7	8	**C3**

Fig. 4. Evaluation for criteria importance.


Q2.
The objective of this question is to evaluate the dogbite treatment methods with respect to wound infections minimization (C1). Three treatment methods are proposed as follows,
T1: Dicloxacillin Prophylaxis,
T2: Penicillin Prophylaxis,
T3: no Prophylaxis,
Compare the relative preference for each pair of treatment methods as below, and circle the mark accordingly.

	Absolutely		Significantly		Highly		Moderately		Equally		Moderately		Highly		Significantly		Absolutely	
T1	8	7	6	5	4	3	2	1	0	1	2	3	**4**	5	6	7	8	**T2**
T1	8	7	6	5	4	3	**2**	1	0	1	2	3	4	5	6	7	8	**T3**
T2	8	7	**6**	5	4	3	2	1	0	1	2	3	4	5	6	7	8	**T3**

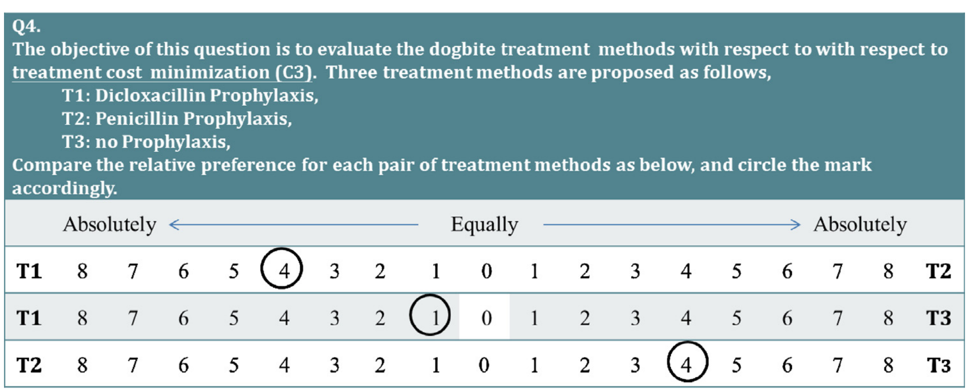
Fig. 5. Evaluation for treatment alternatives with respect to wound infections minimization.


Q3.
The objective of this question is to evaluate the dogbite treatment methods with respect to with respect to Side effects minimization (C2). Three treatment methods are proposed as follows,
T1: Dicloxacillin Prophylaxis,
T2: Penicillin Prophylaxis,
T3: no Prophylaxis,
Compare the relative preference for each pair of treatment methods as below, and circle the mark accordingly.

0=Equally 2=Moderately 4=Highly 6=Significantly 8=Absolutely

T1	8	7	6	5	4	3	2	1	**0**	1	2	3	4	5	6	7	8	**T2**
T1	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	**8**	**T3**
T2	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	**8**	**T3**

Fig. 6. Evaluation for treatment alternatives with respect to side effects minimization.


Q4.
The objective of this question is to evaluate the dogbite treatment methods with respect to with respect to treatment cost minimization (C3). Three treatment methods are proposed as follows,
T1: Dicloxacillin Prophylaxis,
T2: Penicillin Prophylaxis,
T3: no Prophylaxis,
Compare the relative preference for each pair of treatment methods as below, and circle the mark accordingly.

Absolutely ← Equally → Absolutely

T1	8	7	6	5	**4**	3	2	1	0	1	2	3	4	5	6	7	8	**T2**
T1	8	7	6	5	4	3	2	**1**	0	1	2	3	4	5	6	7	8	**T3**
T2	8	7	6	5	4	3	2	1	0	1	2	3	**4**	5	6	7	8	**T3**

Fig. 7. Evaluation for treatment alternatives with respect to cost minimization.

Table 4
Pairwise opposite matrices for Dogbite Treatment Decision.

B_0: weights

	C_1	C_2	C_3
C_1	0	2	4
C_2	−2	0	2
C_3	−4	−2	0

AI = 0

B_1: infections minimization

	T_1	T_2	T_3
T_1	0	−4	2
T_2	4	0	6
T_3	−2	−6	0

AI = 0

B_2: side effects minimization

	T_1	T_2	T_3
T_1	0	0	−8
T_2	0	0	−8
T_3	8	8	0

AI = 0

B_3: cost minimization

	T_1	T_2	T_3
T_1	0	4	1
T_2	−4	0	−4
T_3	−1	4	0

AI = 0.048

Table 5
Computational steps for Accordance Index of B_3 using form (5).

d_{ij}	b_{ij}	$B_i + B_j^T$	$B_i + B_j^T - b_{ij}$	$\left(\frac{1}{\kappa}(B_i + B_j^T - b_{ij})\right)^2$	$d_{ij} = \sqrt{Mean\left(\left(\frac{1}{\kappa}(B_i + B_j^T - b_{ij})\right)^2\right)}$
d_{11}	0	(0,0,0)	(0,0,0)	(0,0,0)	0
d_{12}	4	(4,4,5)	(0,0,1)	(0,0,0.016)	0.072
d_{13}	1	(1,0,1)	(0,−1,0)	(0,0.016,0)	0.072
d_{21}	−4	(−4,−4,−5)	(0,0,−1)	(0,0,0.016)	0.072
d_{22}	0	(0,0,0)	(0,0,0)	(0,0,0)	0
d_{23}	−4	(−3,−4,−4)	(1,0,0)	(0.016,0,0)	0.072
d_{31}	−1	(−1,0,−1)	(0,1,0)	(0,0,0)	0.072
d_{32}	4	(3,4,4)	(−1,0,0)	(0.016,0,0)	0.072
d_{33}	0	(0,0,0)	(0,0,0)	(0,0,0)	0
					$\text{AI} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} = 0.048$

Table 6
Computational steps for priority of B_0.

B_0	C_1	C_2	C_3	$\sum_{j=1}^{n} b_{ij}$	$\frac{1}{n}\sum_{j=1}^{n} b_{ij}$	$v_i = \left(\frac{1}{n}\sum_{j=1}^{n} b_{ij}\right) + \kappa$	$w_i = \frac{v_i}{n\kappa}$
C_1	0	2	4	6	2	10	0.42
C_2	−2	0	2	0	0	8	0.33
C_3	−4	−2	0	−6	−2	6	0.25
				$n\kappa = \sum v_i = 24$			$\sum w_i = 1$

Table A1 of appendix adopted from [8]. The numerical conversion among two rating scale schema is presented in Table 3. Although the linguistic labels proposed in this research are different from [8], understanding of the magnitudes of ordinal scales should have no difference, but the meanings of the numerical systems for rater's cognition of paired difference are completely different.

Once the survey is completed by the expert, POMs are formed and presented in Table 4. For example, the survey of Fig. 4 corresponds to the upper triangular matrix of B_0 in Table 4, and the lower triangular matrix in B_0 is given by the opposite of the upper triangular matrix such that $b_{ij} = -b_{ji}$. Similarly, Figs. 5–7 correspond to the upper triangular matrices of B_1, B_2, and B_3 in Table 4.

Accordance Index (AI) is used to check validity of a POM. In Table 4, B_0, B_1 and B_2 are perfectly accordant. B_3 is not perfectly accordant, but within the acceptable range. The demonstration to compute AI of B_3 using form (5) is presented in Table 5. In the column of $B_i + B_j^T - b_{ij}$, if AI of a POM is perfectly accordant, all elements in the vectors of this column are equal to zero. Thus B_3 is chosen as demonstration in Table 5. The number of non-zero elements means the number of contractions or discordance in the matrices. d_{ij} computes the root mean square error caused by a single entry b_{ij} of POM, and AI computes the average error of all entries. Although d_{ii} is always equal to zero, d_{ii} is regarded as an element in population and included for the average error calculation.

4.3. Step 3: Cognitive Prioritization Process

As AI values of all POMs are acceptable, RAU can be used next. The prioritization results of all POMs are shown in Table 7. Table 6 shows the steps to compute the priority set from B_0 using RAU of forms (8) and (10). The final result is the normalization of the row average plus κ. The sum of the row average values, viz. $(1/n)\sum_{j=1}^{n} b_{ij},\ i = 1, \ldots, n$, is zero-sum value, as the sum of all elements of POM is equal to zero, which the related proof can be referred to in [31,33]. Usually negative prioritization results are not used until the case is specified. The row arithmetic mean (or row average) of the POM could be regarded as a distribution method. κ is the normal utility that is the average value for each object. Such value can be called as priority, weight, utility, importance and so on. Object of more weight has more positive values getting from the object of less weight. The total value of all objects is $n\kappa$. Once κ is associated with the paired interval scale in the format of Table 2, whatever positive κ yields the same normalized prioritization result set $\{w_i\}$. If κ is disassociated with the paired interval scale, normalized prioritization result depends on the values of both κ and rating scales schema.

4.4. Steps 4 and 5: Multiple Information Fusion and Decisional Volition Processes

Once the prioritization results are derived, aggregation is performed, and leads to rank results. Table 7 summarizes the details. Regarding the computational details, the objective priority of T_1 is $0.42 \times 0.31 + 0.33 \times 0.22 + 0.25 \times 0.40 = 0.30$. The objective priorities of T_2 and T_3, which are computed in similar way, are 0.33 and 0.37. The closeness of preference results among three alternatives implies that the difficulty in making treatment decision, and thus the decision tool is needed. Although the aggregated preference

Table 7
Cognitive prioritization results, aggregation results, and ranks using P-CNP.

P-CNP	C_1	C_2	C_3	Objective priority	Rank
Weights	0.42	0.33	0.25		
T_1	0.31	0.22	0.40	0.30	1
T_2	0.47	0.22	0.22	0.33	2
T_3	0.22	0.56	0.38	0.37	3[a]

[a] The highest number means the highest preference.

Table 8
Cognitive prioritization results, aggregation results, and ranks using AHP.

AHP	C_1	C_2	C_3	Objective priority	Rank
W	0.64	0.26	0.11		
T_1	0.19	0.09	0.56	0.20	1
T_2	0.73	0.09	0.09	0.50	3[a]
T_3	0.08	0.082	0.35	0.30	2

[a] The highest number means the highest preference.

value of "no Prophylaxis" (T_3) is marginally higher than the others, T_3 is the best choice for the patient who has a dogbite wound.

5. Comparisons with AHP

The results from AHP are presented in Table 8. P-CNP recommends "no Prophylaxis" (T_3) (Table 7) with a slightly higher preference value (viz. 0.37) than the other treatments (viz. 0.30 and 0.33), but AHP suggests Penicillin Prophylaxis (T_2) treatment with distinct preference value (viz. 0.5) over the other alternatives (viz. 0.2 and 0.3).

The aggregation values among three alternative treatments by P-CNP are much closer than AHP (Tables 7 and 8) due to the critical reason that the paired ratio scales usually exaggerate the human perception of the paired difference in times, and result in much wider variance. Section 2 has discussed such exaggerated issues induced by paired ratio scale. That is the lowest one has much lower value whilst the highest one has much higher value (see Fig. 1). Thus the paired ratio scale of AHP to represent the perception of difference of two objects is dubious.

Whilst the objective priorities among three alternatives by P-CNP are much closer than AHP, P-CNP reflects the decision problem is difficult to make recommendation, but AHP reflects the decision problem is trivial. If the expert thinks C_1 is very important (0.64) and T_2 has distinct value (0.73) over the others with respect to this factor, T_2 can be chosen without any complex decision process. In fact, if the candidate is obviously very strong, selection process using any decision tool is redundant and just wastes time, as the expert can make decision immediately. The necessary selection process in SAN should be needed when it is difficult to make decision due to uncertainty, insufficient information and knowledge, or/and similar strengths among alternatives. In this case, the distinct priority values of AHP unlikely reflect the fact that the decision is difficult to make to select the best one from three alternatives, but reflect that using decision tool may be redundant.

Both pairwise reciprocal matrix (PRM) and pairwise opposite matrix (POM) use the same syntactic form, but different semantic forms, which lead to different Consistency/Accordance issues. Section 2 uses various examples to demonstrate flaws for using paired ratio scale as sematic representation. It could often be the case that the expert produces practical consistent/accordant rating; however, due to AHP's sematic misrepresentation, AHP tells the users the rating is inconsistent, and may require the decision maker to rate again. In the application of [8], A_0 and A_1 (Table A1 of appendix) were mistakenly interpreted as inconsistent evaluations, whilst their semantic forms (B_0 and B_1) are perfectly accordant in views of P-CNP.

Section 2 has discussed logarithm of paired ratio scale is not paired interval scale. To extend, logarithm of pairwise reciprocal matrix are definitely not pairwise opposite matrix, regarding the original rating sense and artificial prioritization results. The claim is that $a_{ij} = w_i/w_j$ is transformed to $\ln a_{ij} = \ln w_i - \ln w_j$, which looks equivalent to $b_{ij} = v_i - v_j$ such that $b_{ij} = \ln a_{ij}$, $v_i = \ln w_i$, and $v_j = \ln w_j$; or $a_{ij} = e^{b_{ij}}$, $w_i = e^{v_i}$, $w_j = e^{v_j}$; $A = (a_{ij}) = \exp(B) = (e^{b_{ij}})$. Such claim, however, completely make no sense. For example, according to such claim, B_0 (in Table 4) could be converted as the below form.

$$\exp(B_0) = \begin{pmatrix} e^0 & e^2 & e^4 \\ e^{-2} & e^0 & e^2 \\ e^{-4} & e^{-2} & e^0 \end{pmatrix} = \begin{pmatrix} 1 & 7.39 & 54.60 \\ 0.14 & 1 & 7.39 \\ 0.02 & 0.14 & 1 \end{pmatrix} = A'_0$$

The above PRM can typically be solved by eigenvector method. To easily understand the claim, it can be solved by Normalization of Geometric Means of Rows (NGMR) as below.

$$w'_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}, \quad i = 1, 2, \ldots, n \tag{13}$$

$$w_i = \frac{w'_i}{\sum_{i=1}^{n} w'_i}, \quad i = 1, 2, \ldots, n \tag{14}$$

However, the solution for the new PRM, $\exp(B_0)$ or A'_0 is (0.87, 0.12, 0.02), which is completely different from (0.42, 0.33, 0.25) derived from POM, B_0, in Table 6. To take a close look of the conversion,

$$w'_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n} = \left(\prod_{j=1}^{n} e^{b_{ij}}\right)^{1/n} = e^{(1/n)\sum_{j=1}^{n} b_{ij}}$$
$$= \exp\left(\frac{1}{n}\sum_{j=1}^{n} b_{ij}\right), \quad i = 1, 2, \ldots, n \tag{15}$$

The solution difference is mainly due to the reason that antilogarithm operation or exponential function (viz. exp(.)) is used for the average of paired interval scores in a row.

For the another reason of the difference, the normal utility κ is not included in $\exp((1/n)\sum_{j=1}^{n} b_{ij})$, but has no impact to the normalized prioritization result after form (14) is taken. Conversely, as demonstrated in Table 6, κ is needed for the RAU prioritization as the paired interval scale is associated with κ in this research (the benefit has been discussed earlier).

In addition, the rating scores in the survey interface and according to the PRM A'_0 are completely different from the original ones shown in Fig. 4. Especially, the entry of 54.6 of the PRM A'_0 is completely non-sense as there is no such rating option in the survey interface. POM and PRM, therefore, are two completely identical systems.

Finally, whilst there are a number of problems for AHP, why does AHP appear to be corrected? All potential alternatives are screened in advance to meet the basic requirements before they are included in the AHP tree for the further evaluation and selection processes. In other words, the unsatisfied or unrelated alternatives/factors are directly excluded for consideration in early stage. Although the best one may not be selected, at least the suitable one is selected to fulfill the tasks. No further valid measurement could be made to reflect the AHP decision.

6. Conclusions

Whilst there are increasing medical and healthcare research and applications using AHP, in addition to the other applications domains, this research discusses the defects of AHP, and proposed the better alternative, P-CNP. The AHP's Pairwise Reciprocal Matrix (PRM) for paired comparisons appears to be inappropriate or is open to discuss. PRM is based on paired ratio scales. It is questionable that the cognitive comparison of two objects can be represented by their ratio since human subjective judgment is unlikely to use multiplication or division for the difference. This

Table A1
Pairwise reciprocal matrices for Dogbite Treatment Decision [8].

A_0: weights

	C_1	C_2	C_3
C_1	1	3	5
C_2		1	3
C_3			1

Consistency ratio = 0.033

A_1: infections minimization

	T_1	T_2	T_3
T_1	1	1/5	3
T_2		1	7
T_3			1

Consistency ratio = 0.056

A_2: side effects minimization

	T_1	T_2	T_3
T_1	1	1	1/9
T_2		1	1/9
T_3			1

Consistency ratio = 0.00

A_3: cost minimization

	T_1	T_2	T_3
T_1	1	5	2
T_2		1	1/5
T_3			1

Consistency ratio = 0.046

research proposes a Pairwise Opposite Matrix (POM) where the cognitive comparison between two objects is represented by the native paired difference between both objects, rather than paired ratio. The perception of the linguistic terms should be paired interval scale rather than paired ratio scale with better matching our natural language. Development of P-CNP is based on the POM.

Dogbite Treatment Decision using AHP [8] is revised by P-CNP with a step-by-step tutorial demonstration. Shortcomings of AHP and the gap bridge are presented. The proposed method should be the promising alternative of AHP to medical and healthcare research and applications such as therapies, medical treatments, health care technologies, medical resources, and healthcare policies.

As the fuzzy number can be used in both paired ratio scale and paired interval scale, the Fuzzy Analytic Hierarchy Process (F-AHP) [36–40] and Fuzzy Cognitive Network Process (F-CNP) [41] are formed respectively. One of the future study directions is to apply the F-CNP to medical and healthcare research and applications, with comparing the F-AHP.

Appendix.

References

[1] T.L. Saaty, A scaling method for priorities in hierarchical structures, Journal of Mathematical Psychology 15 (1977) 234–281.
[2] T.L. Saaty, Analytic Hierarchy Process: Planning, Priority, Setting, Resource Allocation, McGraw-Hill, New York, 1980.
[3] T.L. Saaty, How to make a decision: the analytic hierarchy process, European Journal of Operational Research 48 (1990) 9–26.
[4] T.L. Saaty, Theory and Applications of the Analytic Network Process: Decision Making with Benefits, Opportunities, Costs and Risks, RWS Publications, Pittsburgh, USA, 2005.
[5] M.J. Liberatore, R.L. Nydick, The analytic hierarchy process in medical and health care decision making: a literature review, European Journal of Operational Research 189 (2008) 194–207.
[6] N. Subramanian, R. Ramanathan, A review of applications of Analytic Hierarchy Process in operations management, International Journal of Production Economics 138 (2012) 215–241.
[7] S. Sipahi, M. Timor, The analytic hierarchy process and analytic network process: an overview of applications, Management Decision 48 (2010) 775–808.
[8] J.G. Dolan, B.J. Isselhardt Jr., J.D. Cappuccio, The analytic hierarchy process in medical decision making: a tutorial, Medical Decision Making 9 (1989) 40–50.
[9] J.G. Dolan, Shared decision-making – transferring research into practice: the Analytic Hierarchy Process (AHP), Patient Education and Counseling 73 (2008) 418–425.
[10] C.W. Lee, N.K. Kwak, Strategic enterprise resource planning in a health-care system using a multicriteria decision-making model, Journal of Medical Systems 35 (2011) 265–275.
[11] K.-P. Chung, L.-J. Chen, Y.-J. Chang, Y.-J. Chang, M.-S. Lai, Application of the analytic hierarchy process in the performance measurement of colorectal cancer care for the design of a pay-for-performance program in Taiwan, International Journal for Quality in Health Care 25 (2013) 81–91.
[12] J.G. Dolan, E. Boohaker, J. Allison, T.F. Imperiale, Patients' preferences and priorities regarding colorectal cancer screening, Medical Decision Making 33 (2013) 59–70.
[13] L. Pecchia, J.L. Martin, A. Ragozzino, C. Vanzanella, A. Scognamiglio, L. Mirarchi, S.P. Morgan, User needs elicitation via analytic hierarchy process (AHP). A case study on a Computed Tomography (CT) scanner, BMC Medical Informatics and Decision Making 13 (2013).
[14] M. Danner, J.M. Hummel, F. Volz, J.G. Van Manen, B. Wiegard, C.M. Dintsios, H. Bastian, A. Gerber, M.J. Ijzerman, Integrating patients' views into health technology assessment: analytic hierarchy process (AHP) as a method to elicit patient preferences, International Journal of Technology Assessment in Health Care 27 (2011) 369–375.
[15] L. Pecchia, P.A. Bath, N. Pendleton, M. Bracale, Analytic Hierarchy Process (AHP) for examining healthcare professionals' assessments of risk factors: the relative importance of risk factors for falls in community-dwelling older people, Methods of Information in Medicine 50 (2011) 435–444.
[16] M.J.M. Hummel, F. Volz, J.G. Van Manen, M. Danner, C.M. Dintsios, M.J. Ijzerman, A. Gerber, Using the analytic hierarchy process to elicit patient preferences: prioritizing multiple outcome measures of antidepressant drug treatment, Patient 5 (2012) 225–237.
[17] T. Shin, C.B. Kim, Y.H. Ahn, H.Y. Kim, B.H. Cha, Y. Uh, J.H. Lee, S.J. Hyun, D.H. Lee, U.Y. Go, The comparative evaluation of expanded national immunization policies in Korea using an analytic hierarchy process, Vaccine 27 (2009) 792–802.
[18] F.-M.E. Uzoka, O. Obot, K. Barker, J. Osuji, An experimental comparison of fuzzy logic and analytic hierarchy process for medical decision support systems, Computer Methods and Programs in Biomedicine 103 (2011) 10–27.
[19] M.J. Ijzerman, J.A. Van Til, J.F.P. Bridges, A comparison of analytic hierarchy process and conjoint analysis methods in assessing treatment alternatives for stroke rehabilitation, Patient 5 (2012) 45–56.
[20] A. Suner, C.C. Çelikoğlu, O. Dicle, S. Sökmen, Sequential decision tree using the analytic hierarchy process for decision support in rectal cancer, Artificial Intelligence in Medicine 56 (2012) 59–68.
[21] A. Ishizaka, A. Labib, Review of the main developments in the analytic hierarchy process, Expert Systems with Applications 38 (2011) 14336–14345.
[22] V. Belton, T. Gear, On a short-coming of Saaty's method of analytic hierarchies, Omega 11 (1983) 228–230.
[23] J.S. Dyer, Remarks on the analytic hierarchy process, Management Science 36 (1990) 249–258.
[24] E.H. Forman, Facts and fictions about the analytic hierarchy process, Mathematical and Computer Modelling 17 (1993) 19–26.
[25] V. Belton, P. Goodwin, Remarks on the application of the analytic hierarchy process to judgmental forecasting, International Journal of Forecasting 12 (1996) 155–161.
[26] E.H. Forman, S.I. Gass, The analytic hierarchy process – an exposition, Operations Research 49 (2001) 469–486.
[27] S.I. Gass, Model world: the great debate – MAUT versus AHP, Interfaces 35 (2005) 308–312.
[28] R. Whitaker, Criticisms of the Analytic Hierarchy Process: why they often make no sense, Mathematical and Computer Modelling 46 (2007) 948–961.
[29] M. Bernasconi, C. Choirat, R. Seri, The analytic hierarchy process and the theory of measurement, Management Science 56 (2010) 699–711.
[30] M. Bernasconi, C. Choirat, R. Seri, A re-examination of the algebraic properties of the AHP as a ratio-scaling technique, Journal of Mathematical Psychology 55 (2011) 152–165.
[31] K.K.F. Yuen, Pairwise opposite matrix and its cognitive prioritization operators: comparisons with pairwise reciprocal matrix and analytic prioritization operators, Journal of the Operational Research Society 63 (2012) 322–338.
[32] K.K.F. Yuen, The primitive cognitive network process: comparisons with the analytic hierarchy process, International Journal of Information Technology and Decision Making 10 (2011) 659–680.
[33] K.K.F. Yuen, Cognitive network process with fuzzy soft computing technique for collective decision aiding, The Hong Kong Polytechnic University, 2009 (Ph.D. thesis).
[34] S.S. Stevens, On the theory of scales of measurement, Science 103 (1946) 677–680.

[35] J. Barzilai, Measurement and preference function modelling, International Transactions in Operational Research 12 (2005) 173–183.
[36] D.-Y. Chang, Applications of the extent analysis method on fuzzy AHP, European Journal of Operational Research 95 (1996) 649–655.
[37] Y.-M. Wang, Y. Luo, Z. Hua, On the extent analysis method for fuzzy AHP and its applications, European Journal of Operational Research 186 (2008) 735–747.
[38] K.Y. Chan, C.K. Kwong, T. Dillon, Determination of importance of customer requirements using the fuzzy AHP method, in: Computational Intelligence Techniques for New Product Design, Springer, Berlin Heidelberg, 2012, pp. 59–77.
[39] K.Y. Chan, C.K. Kwong, T. Dillon, An enhanced fuzzy AHP method with extent analysis for determining importance of customer requirements, in: Computational Intelligence Techniques for New Product Design, Springer, Berlin Heidelberg, 2012, pp. 79–93.
[40] K.K.F. Yuen, Membership maximization prioritization methods for fuzzy analytic hierarchy process, Fuzzy Optimization and Decision Making 11 (2012) 113–133.
[41] K.K.F. Yuen, Fuzzy cognitive network process: comparisons with fuzzy analytic hierarchy process in new product development strategy, IEEE Transactions on Fuzzy Systems (2013), http://dx.doi.org/10.1109/TFUZZ.2013.2269150.