Applied Mathematical Modelling 000 (2015) 1–20

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Multi-objective hub network design under uncertainty considering congestion: An M/M/c/K queue system

Yaser Rahimi [a], Reza Tavakkoli-Moghaddam [a,b], Mehrdad Mohammadi [a,b,*], Marjan Sadeghi [a]

[a] School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran
[b] LCFC, Arts et Métier Paris Tech, Metz, France

ARTICLE INFO

*Article history:*
Received 2 January 2014
Revised 30 October 2015
Accepted 9 November 2015
Available online xxx

*Keywords:*
Hub location problem
Congestion
Queue system
Uncertainty
Meta-heuristics

ABSTRACT

Hub location problems have applications in a variety of fields including cargo delivery systems and telecommunication network design. Hub location problems deal with locating a set of hub nodes and allocating non-hub nodes to the located hubs. This paper presents a new bi-objective model for a multi-modal hub location problem under uncertainty considering congestion in the hubs. The objective functions attempt to minimize the total transportation cost as well as minimize the maximum transportation time between each pair of Origin-Destination (O-D) nodes in the network. To cope with the computational complexity of the problem, a well-known meta-heuristic algorithm, namely differential evolution (DE), is developed to obtain near-optimal Pareto solutions. Furthermore, several computational experiments and sensitivity analyses are provided to demonstrate the efficiency and applicability of the presented model and solution algorithm. Finally, the conclusion is presented.



## 1. Introduction

Hub Location Problems (HLPs) consist of locating the hub nodes, allocating the non-hubs to the located hubs, and designing a network. At the hub networks, a large set of origin nodes is connected by some intermediate nodes known as hubs to a set of destination nodes. Therefore, hub networks provide a situation to use less connection links between the nodes. Using fewer number of transportation links, this approach concentrates on flows and reduces the total transportation cost in the network since it employs the economies of scale in the links between hubs while hub nodes perform the switching and flow distribution. It can be then concluded that the hubs are critical elements of transportation networks. Recently, hub location topology is commonly used in transportation and telecommunication fields, such as air transport services; express shipments; postal operations and rapid transit systems.

An HLP is called a *p*-hub location problem when the number of hub nodes is pre-determined. There are two primary assumptions in most of HLPs as: (1) flows have to be routed via at least one hub node and (2) the network between hubs is a complete graph with more effective and higher volume pathways that enable a discount factor denoted by $\alpha$ $(0 < \alpha < 1)$ to be applied to all transportation costs associated with the flows routed between each pair of hub nodes [1–7]. Besides, two types of allocation strategies are applicable for HLP that are single and multiple allocations. In single-allocation, each non-hub node is allocated to exactly one hub. Hence, all flows originating from an origin node must be transferred by this specific hub. On the

* Corresponding author at: School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran. Tel.: +989188729120.
*E-mail address:* mehrdadmohamadi@ut.ac.ir, mehrdad.mohammadi@ensam.eu (M. Mohammadi).

http://dx.doi.org/10.1016/j.apm.2015.11.019
S0307-904X(15)00733-7/

other hand, in multiple-allocation each non-hub node is allocated to at least one hub. The multiple-allocation strategy causes more flexibility in the hub network comparing to single-allocation, since the flows between each pair of O-D nodes can pass through a variety of hub links. In addition, classical HLPs have been developed to minimize the total transportation cost and the fixed cost of locating hubs in the network. A HLP is a form of a capacitated fixed-charge location problem [8]. For more information, enthusiastic readers are referred to surveys on HLPs present by Alumur and Kara [9], Campbell et al. [10] and more recently Zanjirani Farahani et al. [11].

One of the most important applications of HLP is freight transportation. Freight transportation has become a vital part of each country's economy because the trade rate across the globe has highly increased. In order to successfully manage this part of the economy, the underlying transportation network plays a key role. In this particular application of HLP, which is the main issue that our model deals with, the structure of the hub network and the effective utilization of resources play major roles in the level of service offered to the customers. These are also vital for the sustainability of businesses in such highly competitive environments. Hubs are responsible for consolidating, processing, and distributing shipments across the network. In order to route the flows through the hub network, some special operations, such as unloading, sorting, weighting, packaging, and loading are performed at the hubs every day. For instance, companies such as FedEx, UPS, DHL, and the United States Postal Service (USPS) receive and deliver millions of shipments each day. USPS handles 2.3 million shipments a day, accounting for 22% of its domestic revenues. Moreover, at the FedEx Memphis hub, 2.2 million shipments are unloaded, sorted, weighted, and transferred every night [12]. Considering the fact that the shipments are not homogenous and have different service delivery requirements, handling large volumes of arrival shipments within the specified time horizon is challenging. With such large volumes of shipments delivered between many different O-D nodes, many sorting hub nodes are involved.

In a hub network design, decisions regarding the location of hubs and the allocation of the non-hub nodes to the located hubs directly affect the operational performance of the transportation networks [13,14]. As one of the operational performance metrics is service time level in the network, a mismatch between the arrival rate of flows and the processing rate of hub resources may cause congestion in the hubs. Neglecting this situation may result in a queue and long waiting times for flows which has a direct effect on the delivery service performance. Variability of transit time in different transportation modes can be stated as another source of delays in hub networks [15–20]. Two common transportation modes used in the logistics networks are road and rail transportation modes. Both of these transportation modes differ in travel times when they act between the same origins and destinations. Major variations in the flow transit times increase the uncertainty in the volume of flows that need to be processed in a hub over a limited period of time. On the other hand, the limited capacity of the hubs leads to a level of congestion. This congestion forms a queue and results in extra waiting time for shipping flows in the hubs. This extra waiting time and delay of deliveries have become significant concerns for transportation companies [21,22].

In real settings, the system works in a dynamic and chaotic environment. Due to uncertain circumstances that environment may bring, the respective costs, demands, distances, times, and other relevant parameters might change after the design of the hub network. For instance, in HLPs, with variability in the transportation time from an origin to a destination, there is a possibility of not delivering a flow on time. A failure in on-time delivery may result in huge costs, such as lost opportunity and lost sales, due to unsatisfied customers. Typically, there are three types of modeling techniques to cope with uncertain data, namely stochastic programming, robust optimization, and fuzzy programming. Several papers have been published using these techniques and the interested readers are referred to a rich body of literature regarding facility location under uncertainty [23].

Accordingly, the main purpose of this paper is to investigate the effects of delivery requirements for different O-D nodes in the configuration of the hub network and to analyze the tradeoff between network's timely responsiveness and efficiency. The timely responsive networks are those networks that are able to meet delivery requirements and the efficient networks are the networks with the minimum transportation cost. An important factor to maintain in the competition is to maximize responsiveness of the network. On the other hand, customers are more sensitive to delivery requirement and choose a company with the lowest delivery time at a reasonable cost. Therefore, this paper extends a bi-objective mixed-integer nonlinear mathematical programming (BOMINLP) to minimize the total transportation cost along with the maximum travel time between each pair of O-D nodes in order to meet delivery requirement. This time includes both travel time at connection links and processing time at hubs.

In general, whenever randomness is the main source of uncertainty in input data for random variables with known probability distributions, stochastic programming methods can be utilized. In the presence of randomness, robust optimization is a suitable approach if no distributional information is available. However, in many practical situations, due to lack of historical data for some parameters, such as the demand of non-hub nodes, or fully subjectivity of judgmental data extracted from expert(s); for example, about the capacity parameter at one hub, it is hard or even impossible to fit a probability distribution. In these cases, it is more reasonable to adopt a suitable possibility distribution (i.e., distribution of fuzzy numbers) [24] for each parameter based upon the available (but often insufficient) objective data as well as subjective opinions of decision makers, or a fully subjective (preference-based) fuzzy set for each judgmental data based upon expert's subjective knowledge, experience and professional feelings. However, in both cases, fuzzy programming approaches should be used to cope with such vague/epistemic uncertainties. Bashiri et al. [25] represented a combined approach to $p$-hub center that locating of hub facilities by two parameters simultaneously and applied Fuzzy Vikor method to solving this combined model which results gain by genetic algorithm and also Yang et al. [26] represented a new Robust parametric optimization approach for p-hub center problem with considering value at risk to solve model. Accordingly, in this paper, we will consider different sorts of uncertainties for input data and develop fuzzy programming approach to cope with these uncertainties.

The main contributions of this paper, which differentiate our study from those already published on the subject, are as follows:

- Designing a bi-objective hub location model to minimize the total transportation cost and the maximum travel time between each pair of O-D nodes;
- Considering multi-capacity levels for hub establishment;
- Considering different transportation modes to make the hub network more reliable and flexible;
- Modeling the congestion at hubs and calculating the waiting time of shipments using an M/M/c/k queue system;
- Proposing a model that encompasses uncertain parameters in order to make the model more realistic;
- Developing efficient meta-heuristic algorithms, namely simulated annealing (SA) and differential evolution (DE) to obtain near-optimal Pareto solutions.

The rest of this paper is organized as follows. Section 2 presents a brief review of the relevant literature with a focus on application of queuing systems in HLPs. Section 3 provides a formal description of the problem and its multi-objective mathematical model. The proposed fuzzy programming to convert the original fuzzy model into its crisp counterpart is elaborated in Section 4. In Section 5, the developed meta-heuristic solution algorithms are discussed. The computational experiments and sensitivity analyses are presented in Section 6. Finally, the paper concludes in Section 7.

## 2. Literature review

This section presents a review of the literature on HLPs with service considerations due to delays at hubs. Prior studies in the area of HLPs are reviewed followed by a review of the previous studies on the use of queuing models in HLPs.

The first HLP as a quadratic mathematical model is presented by O'Kelly [27]. Campbell [28] simplified this model and converted it to a linear mathematical model. A mixed-integer linear problem (MILP) is introduced by Ernst and Krishnamoorthy [29] with fewer variables and constraints than earlier proposed models. By performing a tight linear relaxation on Campbell [28] model, Skorin-Kapov et al. [30] could reach an exact solution for the *p*-hub median problem.

A queuing theory has been widely used for managing the congestion at hub centers. Marianov and Serra [31] presented several probabilistic maximal covering location-allocation models with constrained waiting time for queue length to consider service congestion. In another study, Marianov and Serra [32] formulated new models for the location of congested facilities to maximize population that are covered by the system with short queues or waiting time. They also presented an extension of location-allocation models which seeks to cover all the population and includes server allocation to the facilities. Their new model was intended for the design of service networks including health and EMS services, and banking or distributed ticket-selling services. Marianov and Serra [33] presented a one-stage single-allocation model by considering the M/D/c queuing system at the HLP for an airport system. They used a probabilistic constraint for limiting the waiting time at hubs. Effects of congestion at hubs were shown in a single-allocation *p*-hub model by Elhedhli and Hu [34]. In addition, the objective function of their model was improved by adding a convex (cost) function. As a consequence, if more flow units are allocated to hubs, objective function will be increased with a non-linear rate.

Berman et al. [35] suggested an M/M/1/K queue model for a location-allocation problem with respect to two objectives as: (1) the number of facilities that are treated by limiting the system capacity and (2) the percentage of demand that may be lost due to the model's limitations. Besides, Rahmati et al. [36] investigated a practical bi-objective model for the facility location–allocation problem with immobile servers and stochastic demand within the M/M/1/K queue system. The first goal of their research was to develop a mathematical model, in which customers and service providers are considered as perspectives. The objectives of the developed model were to minimize the total cost of server provider and minimize the total time of customers.

Rodriguez et al. [21] designed a hub network in a way that arriving trucks to hubs should wait in a queue if unloading services are busy while the arrival rate is related to the number of trucks assigned to each hub. Mohammadi et al. [37] presented an M/M/c model for a hub covering location problem by limiting the length of the queue at each hub to a predefined value. Ishfaq and Sox [15] modeled the hub operations as a GI/G/1 queuing network and the shipments as multiple job classes with deterministic routing.

Vidyarthi et al. [12] presented a model for designing a capacitated single allocation *p*HLP with stochastic demand and time-based service level constraints at the hubs. They used a matrix geometric approach to deal with the service level constraint associated with priority flow classes. Through a numerical example, it was shown that substantial improvement in service levels could be achieved with a small increase in the total costs of the HLP design. Mohammadi et al. [17] developed a novel sustainable hub location problem (SHLP) in which two new environmental-based cost functions, accounting for air and noise pollution of vehicles, are incorporated. In order to cope with the uncertain data incorporated in the model, they proposed a mixed possibilistic–stochastic programming approach to construct a deterministic model. They also applied two meta-heuristic algorithms, namely Simulated Annealing (SA) and Imperialist Competitive Algorithm (ICA), with a new solution representation to solve real-sized instances, and compared the performance of the algorithms with a lower bound.

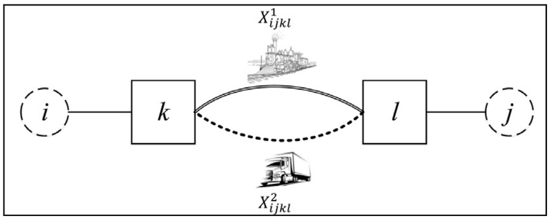


**Fig. 1.** Path between a pair of O-D nodes.

## 3. Problem description and proposed model

### 3.1. Model framework

This paper presents a bi-objective single allocation, multi-mode hub location problem under congestion. One of the objectives of this problem is to minimize the transportation time between each pair of O-D nodes. The transportation time of a path between each O-D node is the sum of the travel time at the *node- hub, hub-hub* and *hub-node* connection links plus the process time (i.e., service time + waiting time) at hubs, while *hub-hub* travel time depends on the utilized transportation mode on it. The structure of a path between an O-D pair has been depicted in Fig. 1. The choice of a specific transportation mode for *hub-hub* travel, represented by the binary variable $X^t_{ijkl}$, refers to the use of the corresponding transportation mode $t$. For example, in the case of a road-rail intermodal network (see Fig. 1), $t = 1$ and $t = 2$ represent road and rail modes, respectively. In such a network, $X^1_{ijkl} = 1$ for an O-D pair $(i, j)$ and a hub pair $(k, l)$ implies road mode and $X^2_{ijkl} = 1$ implies rail mode for the *hub-hub* travel.

As explained in Section 1, due to the limited capacity of the hubs, the entering flow unit should wait in a queue to receive services (e.g., unloading, sorting and loading). Since the amount of flow unit between nodes is stochastic, calculating the exact process time at a hub is challenging. Since the process time has been considered to be the sum of processing time and waiting time, calculating the waiting time with stochastic flow units requires developing a queuing model for the system.

Several papers have justified using queuing approach by both empirical data and simulation [38–40] in order to represent the waiting time in practice. Justification results proved that the queuing models can be adequately used for calculating the waiting time of the flow units. In an application of queuing theory, the transient behavior of a queuing system at a hub was modeled by Peterson et al. [41]. They concluded that there was always a variation around scheduled arrival times, so a Poisson process was a good representation for the arrival rates. They also declared this Poisson assumption is generalizable to other service systems where arrival rates and capacity levels vary significantly over time.

In this paper, since it has been considered that the flows are transferred in batches by different transportation modes, the number and frequency of entering flow units to the hubs are low. Moreover, there are several events such as environmental conditions, traffic jams, link disruptions, etc. that always make variations in the arrival times of transporters. Hence, a Poisson distribution is considered for modeling the waiting times of entering flow units to the hub network. In this regard, Peterson et al. [41] has already confirmed this deduction using the real data and similarly, Marianov and Serra [31–33], Mohammadi et al. [5,17,18] have also modeled entering flow units to the hub nodes as a Poisson process. In this way, under specific assumptions, the analysis of the queue formed by truck waiting for receiving services is applicable for their unloading, loading, and getting stuck in traffic. We use the peak hours analysis, assuming that during the peak hours the average arrival rate and the service rate are both constant and therefore the arrivals of flow units to hub nodes follow a Poisson distribution during peak hours. Additionally, a finite capacity ($K$) has been considered for the queue which limits the number of entering flow units and can control the congestion at hubs. This allows us to model the queue formed by trucks as an M/M/c/K queuing system.

In this queuing system, the maximum number of flow units in the hubs is $K$, which implies a maximum queue length of $K - c$. An arriving flow unit enters the hub if it finds fewer than flow unit in the system and is lost otherwise. By this approach, the congestion at hubs could be better controlled than considering an infinite queuing model for the system for the hub nodes [37].

Another approach to control congestion at the hubs is adding a capacity level constraint at the network design stage that has been used by a number of studies [42–45]. Therefore, in this paper, different capacity levels are considered for the hubs. Although the hubs with higher capacity levels need higher cost to be located; these hubs can absorb a higher volume of flow units and have better control on congestion. Accordingly, the effect of a capacity level constraint on the structure of the hub network as well as delivery requirements are also investigated in this paper.

### 3.2. M/M/c/K queue system

In this subsection, an M/M/c/K queuing system is developed for the single allocation, multi-mode hub location problem. The hub nodes in this queuing system are multiple servers with finite capacities. The following notations are used for the finite capacity queuing system.

***Sets***

| | |
|---|---|
| $i, j, k, l \in I$ | Set of nodes (i.e., $i$ and $j$ denote non-hub nodes and $k$ and $l$ denote hub nodes). |
| $t \in M$ | Set of transportation modes. |
| $s \in S$ | Set of hub capacity levels. |

***Parameters***

| | |
|---|---|
| $Wq_k^s$ | Total queue waiting time at hub $k$ with level $s$. |
| $W_k^s$ | Total service time at hub $k$ with level $s$ (i.e., $W_k^s = Wq_k^s +$ processing time). |
| $\lambda_k$ | Arrival rate of flow unit to hub $k$. |
| $\mu_k^s$ | Service rate of hub $k$ with level $s$. |
| $c_k^s$ | Numbers of service providers at hub $k$ with level $s$. |
| $P^s_{n_k^s,k}$ | Probability of $n$ flow units in the queue at hub $k$ with level $s$. |
| $Lq_k^s$ | Length of queue at hub $k$ with level $s$. |
| $W_{ij}$ | Flow unit between node $i$ and $j$. |
| $O_i = \sum_j W_{ij}$ | Total amount of flow unit originating at node $i$. |
| $D_i = \sum_j W_{ji}$ | Total amount of flow unit delivered at node $i$. |
| $K_k^s$ | Finite capacity of queue at hub $k$ with level $s$. |

***Decision variables***

| | |
|---|---|
| $X_{ik}$ | 1 if node $i$ is allocated to node $k$; 0 otherwise. |

Based on the above notations, the arrival rate of the flow unit to hub $k$ is calculated as Eq. (1).

$$\lambda_k = \sum_i (O_i + D_i) X_{ik} \tag{1}$$

Alternatively, according to Gross and Harris [46], the total service time ($W_k^s$), which is the sum of waiting time in the queue ($Wq_k^s$) and processing time ($\frac{1}{\mu_k^s}$) is calculated by:

$$W_k^s = Wq_k^s + \frac{1}{\mu_k^s} \tag{2}$$

In order to calculate the waiting time of the arrival flow unit into the hub $k$ with capacity level $s$ ($Wq_k^s$), Eq. (3) is presented.

$$Wq_k^s = \frac{Lq_k^s}{\lambda_k \left(1 - P^s_{n_k^s,k}\right)} \tag{3}$$

where $P^s_{n_k^s,k}$, $Lq_k^s$, $P^s_{0k}$, $a_k^s$ and $\rho_k^s$ are calculated as Eqs. (4)–(8), respectively.

$$P^s_{n_k^s,k} = \frac{(\lambda_k)^{K_k^s}}{K_k^s!\left(\mu_k^s\right)^{K_k^s}} P^s_{0k} \tag{4}$$

$$Lq_k^s = \frac{\left(a_k^s\right)^{c_k^s}\left(\rho_k^s\right)}{c_k^s!\left(1-\rho_k^s\right)^2} P^s_{0k}\left[1 - \left(\rho_k^s\right)^{K_k^s - c_k^s + 1} - \left(1-\rho_k^s\right)\left(K_k^s - c_k^s + 1\right)\left(\rho_k^s\right)^{K_k^s - c_k^s}\right] \tag{5}$$

$$P^s_{0k} = \left[\frac{\left(a_k^s\right)^{c_k^s}\left(1 - \left(a_k^s\right)^{K_k^s - c_k^s + 1}\right)}{c_k^s!\left(1 - a_k^s\right)} + \sum_{v=0}^{c_k^s - 1} \frac{\left(a_k^s\right)^v}{v!}\right]^{-1} \tag{6}$$

$$a_k^s = \frac{\lambda_k}{\mu_k^s} \tag{7}$$

$$\rho_k^s = \frac{\lambda_k}{c_k^s \mu_k^s} \tag{8}$$

### 3.3. *Model formulation*

This section proposes a new mathematical model for the HLP. The model is developed utilizing the modeling framework of Section 3.1 and the queuing system representation in Section 3.2. Hereafter, the main assumptions and some extra notations are explained to propose the mathematical model.

#### 3.3.1. *Assumptions*

- The number of hubs to be located in the network is given.
- Each non-hub node is allocated to exactly one hub (i.e., single-allocation).
- Hubs are fully interconnected.
- Different transportation modes can be utilized at each *hub-hub* link.
- Transportation mode $t = 1$ is considered for *node-hub* and *hub-node* links.
- Establishing hubs and links in the network imposes a fixed cost to the system.
- Transportation cost and time at the links are uncertain parameters.

*3.3.2. Notations*

Parameters:

*Fuzzy parameters*

$\tilde{C}_{ij}^{t}$ Unit transportation cost between node i and j by transportation mode t.

$\widetilde{CS}_{k}^{t}$ The cost of utilizing transportation mode $t$ at hub $k$.

$\tilde{F}_{k}^{s}$ Fixed cost of establishing hub $k$ with capacity level $s$.

$\tilde{T}_{ij}^{t}$ Travel time between nodes $i$ and $j$ with transportation mode $t$.

*Deterministic parameters*

$P$ Number of hubs that must be located in the network.

$\sigma$ *Node-hub* transportation cost discount factor.

$\beta$ *Hub-hub* transportation cost discount factor.

$\delta$ *Hub-node* transportation cost discount factor (i.e., $0 < \alpha, \beta, \delta < 1$; $\beta < \alpha, \delta$).

***Variables:***

$X_{ijkl}^{t}$ 1 if the flow unit from node $i$ to node $j$ is sequentially transferred through hub $k$ to hub $l$ using transportation mode $t$; 0 otherwise.

$Z_{k}^{s}$ 1 if a hub is established at node $k$ with capacity level $s$; 0 otherwise.

$SL_{k}^{t}$ 1 if hub $k$ utilizes transportation mode $t$.

$\varphi$ Maximum travelling time between each pair of O-D nodes.

***Mathematical Formulation***:

$$\min \sum_{i}\sum_{j}\sum_{k}\sum_{l}\sum_{t} W_{ij}\left(\sigma\tilde{C}_{ik}^{1} + \beta\tilde{C}_{kl}^{t} + \delta\tilde{C}_{lj}^{1}\right)X_{ijkl}^{t} + \sum_{k}\sum_{s}\tilde{F}_{k}^{s}Z_{k}^{s} + \sum_{k}\sum_{t}\widetilde{CS}_{k}^{t}SL_{k}^{t} \tag{9}$$

$$\min \varphi \tag{10}$$

s.t.

$$\sum_{\substack{k \\ i \neq k}} X_{ik} + \sum_{s} Z_{i}^{s} = 1 \qquad \forall i \tag{11}$$

$$\sum_{k}\sum_{s} Z_{k}^{s} = P \tag{12}$$

$$\sum_{k}\sum_{l}\sum_{t} X_{ijkl}^{t} = 1 \quad \forall i, j \tag{13}$$

$$\left(\tilde{T}_{ik}^{1} + W_{k}^{s} + \tilde{T}_{kl}^{t} + W_{l}^{s} + \tilde{T}_{lj}^{1}\right)X_{ijkl}^{t} \leq \rho \quad \forall i, j, k, l, t, s \tag{14}$$

$$\sum_{t} X_{ijkl}^{t} \geq X_{ik} + X_{jl} - 1 \qquad \forall i, j, k, l; i \neq k, j \neq l \tag{15}$$

$$\sum_{l}\sum_{t} X_{\ ijkl}^{\ t} \leq \sum_{s} Z_{k}^{s} \qquad \forall i, j, k \tag{16}$$

$$\sum_{k}\sum_{t} X_{\ ijkl}^{\ t} \leq \sum_{s} Z_{l}^{s} \qquad \forall i, j, l \tag{17}$$

$$X_{ik} \leq \sum_{s} Z_{k}^{s} \qquad \forall i, k \tag{18}$$

$$\sum_{s} Z_{k}^{s} \leq 1 \qquad \forall k \tag{19}$$

$$X_{ijkl}^{t} \leq SL_{k}^{t} \qquad \forall i, j, k, l, t \tag{20}$$

$$X_{ijkl}^{t} \leq SL_{l}^{t} \qquad \forall i, j, k, l, t \tag{21}$$

$$SL_{k}^{t} \leq \sum_{s} Z_{k}^{s} \qquad \forall k, t \tag{22}$$

$$X^t_{ijkl},\ X_{ik},\ Z^s_k,\ SL^t_k \in \{0,1\} \qquad \forall i,j,k,l,t,s \tag{23}$$

$$\varphi \geq 0 \tag{24}$$

Objective function (9) minimizes the total transportation cost of the hub network. Objective function (10) with constraints (14) and (24) the maximum transportation time between each pair of O-D nodes. Constraint (11) indicates the single-allocation assumption. Constraint (12) requires that a total number of $P$ hubs are located in the network. Constraint (13) selects one hub pair $(k, l)$ for each O-D pair $(i, j)$ and a specific mode $t$. Constraint (15) ensures that there is a path going through the hubs $k$ and $l$ with a specific mode $t$ if there exist links between $i$ to $k$ and $l$ to $j$. Constraints (16) and (17) ensure the correct paths going through the located hubs. Constraint (18) guarantees that a *node-hub* link exists if and only if the hub has been located with a specific capacity level. Constraint (19) ensures at most one capacity level for the hubs. If any of the $X^t_{ijkl}$ variables in Constraints (20) and (21) is equal to 1, the corresponding $SL^t_k$ and $SL^t_l$ variables should be set equal to 1. Constraint (22) ensures that any assignment of a *hub-hub* link with a specific mode $t$ for an O-D pair is limited to located hubs. Constraint (24) is a domain constraint.

## 4. Proposed solution approach

The proposed model for HLP is a fuzzy multi-objective non-linear programming (FMONLP) one. To convert this model into its equivalent crisp counterpart, there a numbers of methods that can be adopted [47,48], from which a two-phase approach is proposed. In the first phase, the original model is converted into an equivalent auxiliary crisp multi-objective model by applying an efficient method introduced by Jiménez et al. [49]. Then, in the second phase, the fuzzy aggregation function developed by Torabi and Hassini [50] is utilized to solve the resulting crisp multi-objective model by replacing it with a single-objective parametric model to find the final preferred compromise solution.

### *4.1. Equivalent auxiliary crisp model*

This section presents a well-known approach to transform the proposed FMONLP model into an equivalent auxiliary crisp model for HLP under demand and supply uncertainties. In this paper, we adopt an approach introduced by Jiménez et al. [49] that can be used for solving mathematical programming problems, in which, all the coefficients are fuzzy numbers in general. Jiménez et al. [49] introduced a resolution method for this type of problems that permits the interactive participation of the Decision Maker (DM) in all the steps of the decision making process. Assume the following mathematical programming problem (25) with fuzzy parameters:

$$\begin{aligned}&\text{Min } Z = \tilde{C}^t x\\ &\text{s.t.}\\ &X \in N\left(\tilde{A},\tilde{b}\right) = \left\{x \in R^n \,|\, \tilde{a}_i x \geq \tilde{b}_i, i = 1,\ldots,m, x \geq 0\right\}\end{aligned} \tag{25}$$

where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n)$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$ and $\tilde{b} = (\tilde{b}_1, \tilde{b}_1, \ldots, \tilde{b}_n)^t$ represent fuzzy parameters involved in the objective function and constraints, respectively. In addition, the fuzzy number $x = (x_1, x_1, \ldots, x_n)$ is the crisp decision vector which represents the possibility distribution for fuzzy parameters. Similar to Jiménez [51], we use a fuzzy relationship to compare fuzzy numbers. Accordingly, the model (25) is transformed into the crisp equivalent parametric linear programming problem defined as (26) in which $\alpha$ denotes the degree that at least all the constraints are fulfilled (i.e., $\alpha$ is the feasibility degree of decision $x$).

$$\begin{aligned}&\min\ EV(\tilde{c})x\\ &\text{s.t.}\\ &\left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \qquad \forall i;\ x \geq 0;\ \alpha \in [0,1]\end{aligned} \tag{26}$$

where the expected value of a triangle fuzzy number ($EV(\tilde{c})$) is the half point of its expected interval as Eq. (27) [52]. Moreover, $[E_1^a, E_2^a]$ and $[E_1^b, E_2^b]$ are the expected intervals of $\tilde{a}$ and $\tilde{b}$ [49]. Besides, in case there is trapezoidal fuzzy number ($\tilde{c}$) in the model, the expected interval is easily calculated as Eq. (28).

$$\mathrm{EI}(\tilde{c}) = [\mathrm{E}_1^c, \mathrm{E}_2^c] = \left[\int_0^1 f_A^{-1}(\alpha)d\alpha - \int_0^1 g_A^{-1}(\alpha)d\alpha\right] = \left[\frac{1}{2}(c_1+c_2), \frac{1}{2}(c_3+c_4)\right] \tag{27}$$

$$\mathrm{EV}(\tilde{c}) = \frac{\mathrm{E}_1^c + \mathrm{E}_1^c}{2} = \frac{c_1+c_2+c_3+c_4}{4} \tag{28}$$

where $EI(\tilde{c})$ is defined as the expected interval of the trapezoidal fuzzy number $\tilde{c}$ (see Fig. 2).

In order to cope with fuzzy parameters in the constraints, the following constraints are presented for less than or equal (i.e., Constraint (29)) and equality (i.e., Constraints (30) and (31)) type of constraints, respectively.

$$\left[(1-\alpha)E_1^{a_i} + \alpha E_2^{a_i}\right]x \leq \alpha E_1^{b_i} + (1-\alpha)E_2^{b_i} \qquad \forall i = 1,\ldots,m;\ x \geq 0;\ \alpha \in [0,1] \tag{29}$$

$$\left[\left(1-\frac{\alpha}{2}\right)E_1^{a_i} + \frac{\alpha}{2}E_2^{a_i}\right]x \leq \frac{\alpha}{2}E_1^{b_i} + \left(1-\frac{\alpha}{2}\right)E_2^{b_i} \qquad \forall i = 1,\ldots,m;\ x \geq 0;\ \alpha \in [0,1] \tag{30}$$

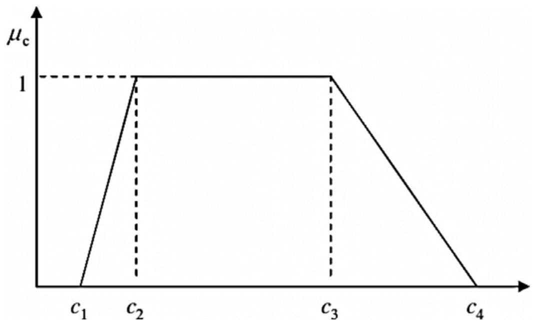


**Fig. 2.** Trapezoidal fuzzy number $\tilde{c}$.

$$\left[\left(1-\frac{\alpha}{2}\right)E_2^{a_i}+\frac{\alpha}{2}E_1^{a_i}\right]x \le \frac{\alpha}{2}E_2^{b_i}+\left(1-\frac{\alpha}{2}\right)E_1^{b_i} \qquad \forall i=1,\dots,m;\ x\ge 0;\ \alpha\in[0,1] \tag{31}$$

Regarding to the explained method, the proposed auxiliary crisp FMONLP model with trapezoidal fuzzy parameters can be presented as follows:

$$\begin{aligned}
&\mathrm{Min}\sum\nolimits_i\sum\nolimits_j\sum\nolimits_k\sum\nolimits_l\sum\nolimits_t W_{ij}\left(\sigma\frac{C_{ik1}^1+C_{ik2}^1+C_{ik3}^1+C_{ik4}^1}{4}+\beta\frac{C_{kl1}^t+C_{kl2}^t+C_{kl3}^t+C_{kl4}^t}{4}+\delta\frac{C_{lj1}^1+C_{lj2}^1+C_{lj3}^1+C_{lj4}^1}{4}\right)X_{ijkl}^t\\
&\quad+\sum_k\sum_s\frac{F_{k1}^s+F_{k2}^s+F_{k3}^s+F_{k4}^s}{4}\times Z_k^s+\sum_k\sum_t\frac{CS_{k1}^t+CS_{k2}^t+CS_{k3}^t+CS_{k4}^t}{4}SL_k^t\\
&\min\varphi
\end{aligned}\tag{32}$$

s.t.

$$\begin{aligned}
&\Bigg((1-\alpha)\frac{T_{ik1}^1+T_{ik2}^1}{2}+(\alpha)\frac{T_{ik3}^1+T_{ik4}^1}{2}+W_k^s+(1-\alpha)\frac{T_{kl1}^t+T_{kl2}^t}{2}+(\alpha)\frac{T_{kl3}^t+T_{kl4}^t}{2}+W_l^s\\
&\quad+(1-\alpha)\frac{T_{lj1}^1+T_{lj2}^1}{2}+(\alpha)\frac{T_{lj3}^1+T_{lj4}^1}{2}\Bigg)X_{ijkl}^t\le\varphi \qquad \forall i,j,k,l,t,s
\end{aligned}\tag{33}$$

s.t.: (11)–(13), (15)–(24)

### 4.2. Proposed solution approach

Various approaches have been developed to solve the crisp multi-objective programming models in the literature including priori, interactive and posteriori methods [53]. Among these methods, fuzzy interactive methods are one of the most attractive approaches in this area, mainly for their ability in measuring and adjusting the satisfaction levels of each objective function based on the DM preferences in an interactive and progressive way [50]. In this paper, to solve the developed model, a hybrid solution approach is utilized by combining the presented method in the previous section with the TH method [50]. Contrary to the classical multi-objective programming methods such as the weighted sum method [54–56] that may result in weakly efficient solutions; the TH method guarantees to find merely efficient solutions. In this way, the steps of the proposed hybrid method can be summarized as follows:

**Step 1:** Identify all uncertain variables and acquire the related probability values.
**Step 2:** Formulate a specific multi-objective HLP mathematical model.
**Step 3:** Convert the imprecise objective functions into their crisp ones by using the expected value of corresponding imprecise parameters.
**Step 4:** Convert the fuzzy constraints into their crisp ones, and formulate the equivalent auxiliary crisp multi-objective model.
**Step 5:** Determine the Positive Ideal Solution (*PIS*) and the Negative Ideal Solution (*NIS*) for each objective function. To reach the *PIS*, i.e., $(\mathcal{Z}_1^{PIS}, x_1^{PIS})$ and $(\mathcal{Z}_2^{PIS}, x_2^{PIS})$, the equivalent crisp multi-objective model should be solved for each objective function separately, and then the *NIS* for each objective function can be estimated as follows:

$$\mathcal{Z}_1^{NIS}=\mathcal{Z}_1\left(x_2^{PIS}\right),\ \mathcal{Z}_2^{NIS}=\mathcal{Z}_2\left(x_1^{PIS}\right),$$

**Step 6:** Determine a linear membership function for each objective function as Eqs. (34) and (35):

$$\mu_1(\mathcal{Z})=\begin{cases}1 & \text{if } \mathcal{Z}_1<\mathcal{Z}_1^{PIS}\\ \frac{\mathcal{Z}_1^{NIS}-\mathcal{Z}_1}{\mathcal{Z}_1^{NIS}-\mathcal{Z}_1^{PIS}} & \text{if } \mathcal{Z}_1^{PIS}\le\mathcal{Z}_1\le\mathcal{Z}_1^{NIS}\\ 0 & \text{if } \mathcal{Z}_1>\mathcal{Z}_1^{NIS}\end{cases}\tag{34}$$

$$\mu_2(\mathcal{Z}) = \begin{cases} 1 & \text{if } \mathcal{Z}_2 < \mathcal{Z}_2^{PIS} \\ \frac{\mathcal{Z}_2^{NIS} - \mathcal{Z}_2}{\mathcal{Z}_2^{NIS} - \mathcal{Z}_2^{PIS}} & \text{if } \mathcal{Z}_2^{PIS} \le \mathcal{Z}_2 \le \mathcal{Z}_2^{NIS} \\ 0 & \text{if } \mathcal{Z}_2 > \mathcal{Z}_2^{NIS} \end{cases} \tag{35}$$

In fact, $\mu_k(x)$ denotes the satisfaction degree of the $k$th objective function.

**Step 7:** Transform the equivalent crisp multi-objective model into a single-objective model using the aggregation function [50]. It is worth to mention that the TH method ensures to obtain only efficient (i.e., Pareto-optimal) solutions. The modified TH aggregation function is computed by:

$$\max \mathbb{Q}(W) = \vartheta \lambda_0 + (1 - \vartheta) \sum_{\tau} \phi_\tau \mu_\tau(\mathcal{Z}) \tag{36}$$

s.t.

$$\lambda_0 \le \mu_\tau(\mathcal{Z}) \qquad \forall \tau \in \{1, 2\} \tag{37}$$

$$x \in \mathcal{Z}(x), \qquad \lambda_0, \phi_\tau, \vartheta \in [0, 1] \tag{38}$$

Where $\mathcal{Z}(x)$ represents the feasible region involving some constraints of the equivalent crisp model. In addition, $\phi_\tau$ and $\vartheta$ symbolize the importance of the $k$th objective function and the coefficient of compensation, respectively ($\sum_{\tau} \phi_\tau = 1;\ \phi_\tau > 0$). The optimal value of variable $\lambda_0 = \min_{\tau}\{\mu_\tau(x)\}$ indicates the minimum satisfaction degree of the objective functions and the TH aggregation function actually looks for a compromise value between the min operator and the weighted sum operator based on the value of $\vartheta$. In other words, the DM can reach balanced and unbalanced compromised solution via manipulating the value of parameters $\phi_\tau$ and $\vartheta$ on the basis of their preferences (see [57] for details).

**Step 8:** Specify the values of $\phi_\tau$ and $\vartheta$, then solve the respective single-objective model. If the DM is satisfied with the current solution, stop; otherwise, provide another compromise solution by changing the values of $\vartheta$ (and if necessary, the value of $\phi_\tau$) [56-58].

## 5. Solution methods

The model presented in Section 4 is a mixed-integer nonlinear programming (MINLP) optimization problem due to the nonlinear form of Constraint (33). Solving such an MINLP problem is computationally challenging, especially for large-sized instances.

The proposed model is coded in GAMS 22.9 software. The reliability and validity of the proposed model are tested on some randomly generated data sets of the problem sizes up to ten nodes and five hubs on a Pentium 4 computer with 2.3 GHz CPU and 4 GB RAM. The optimal solutions related to these problems are successfully achieved through using the non-linear branch-and-reduce approach in BARON solver. However, the required computational time to solve the larger instances (up to 15 nodes) significantly increases even up to several days. Further attempts to solve the larger instances (more than 15 nodes) are unsuccessful due to huge required computational time. To overcome this limitation, the authors developed an efficient meta-heuristic algorithm, namely differential evolution (DE), to find efficient enough (i.e., near-optimal) solutions in a reasonable computational time especially for real-sized instances. Furthermore, in order to evaluate the performance of the proposed DE algorithm, a comparison between the obtained solutions of the proposed DE algorithm with those of exact solutions of GAMS software in some small-sized test problems is conducted. For large-sized instance, the superiority of the proposed DE algorithm is shown in comparison to a well-known meta-heuristic algorithm, namely simulated annealing (SA) [59,60]. In the following sub-sections, we first introduce a solution representation and short descriptions of the proposed DE algorithm.

### 5.1. Solution representation

The solution representation of the proposed model contains three different steps including *Location-Allocation* (LA), *Transportation Mode Selection* (TMS), and *Hub Capacity Level Selection* (HCLS). The LA and TMS parts are considered based on what proposed by Mohammadi et al. [16] and the HCLS part has been only explained in this paper.

This section develops a continuous solution representation for the HCLS part of the proposed model. The HCLS part contains a $(1 \times I)$ matrix, in which the $I$ is the number of nodes in the hub network. This matrix is filled with random numbers belonging to [0, 1]. All bits of the matrix are multiplied by the number of capacity levels ($S$) and are rounded up. The value of each bit shows the selected capacity level for the corresponding node if that node is a hub. For example, consider a hub network with six nodes and three capacity levels, in which, nodes 2 and 5 are considered as hubs. Fig. 3 illustrates that the second and third capacity levels are considered for hubs 2 and 5, respectively.

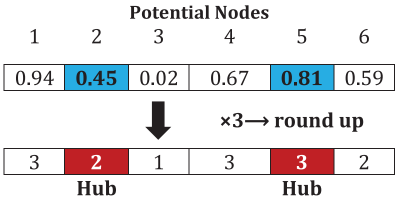


**Fig. 3.** HCLS representation.

### 5.2. DE algorithm

In order to find global optima for numerous unconstrained problems, Storn and Price [61] developed a simple and efficient heuristic algorithm named Differential Evolution (DE). Similar to other evolutionary computational algorithms, DE involves the evolution of a population of solutions with the *PS* size using some operators (e.g., mutation, crossover, and selection) [62]. The initial population is often randomly generated over the variables' domain. Each solution vector in the population has to be selected once as the target vector so that totally *Pop* competitions take place in one generation. A new solution vector is generated by the DE's mutation operator, in which the weighted difference between two population vectors is added to the third vector. Hence, this algorithm is named differential evolution. Note that these three vectors are randomly selected and must be different from the target vector; therefore, *Pop* must be at least 4. Let $\omega_i$, $i = 1, \ldots, Pop$, be the target vector. Then, a mutated vector is generated according to the following equation.

$$\mu_i = \omega_{v1} + F(\omega_{v2} - \omega_{v3}) \tag{39}$$

where $v1$, $v2$, and $v3$ are mutually different random indices in the interval $\{1, 2, \ldots, Pop\}$, and are not equal to $i$. $F$ in the Eq. (39) is a real-valued constant $\in [0, 2]$, which controls the amplification of the differential variation between the second and the third randomly chosen population vectors (i.e., $\omega_{v2} - \omega_{v3}$). Each mutated vector shares its information with a target vector using the crossover operation in order to create a new solution $\tau_i = \{\tau_{i1}, \ldots, \tau_{ij}, \ldots, \tau_{iD}\}$ as follows:

$$\tau_{ij} = \begin{cases} \mu_{ij} if\ rand(j) \le CR\ and\ j = rnbr(i) \\ \omega_{ij} if\ rand(j) > CR\ and\ j \ne rnbr(i) \end{cases} \tag{40}$$

where rand($j$) is the $j$th component of a $D$-dimensional uniform random number $\in [0, 1]$ and rnbr($i$) is a randomly chosen index $\in \{1,\ldots,D\}$ to ensure that at least one mutated dimensional value is used in the new created solution.

If the new created solution yields a lower cost function value than the target vector for a minimization problem, then the new solution is replaced by the target vector in the following generation. This constitutes the selection operation. The algorithm can be terminated with a predetermined maximum number of generations and/or a predetermined maximum number of function evaluations.

## 6. Computational experiments

In this section, the validity of the proposed model is first evaluated by solving several test problems. Next, the conflict between two objective functions is reported based upon the result of a sample test problem. Afterward, a performance comparison is done between the proposed DE algorithm and GAMS software results for some small-sized test problems utilizing the TH aggregation function (Eq. (36)). Afterwards, several sensitivity analyses are done to investigate the sensitivity of the objective functions to the input parameters. Finally, in order to better validate the proposed model and the solution approaches, a real transportation case is studied.

### 6.1. Validating the correctness of the proposed model

To validate the correctness of the proposed mathematical model and the efficiency of the proposed solution approach, the model is implemented on several numerical experiments and the related results are reported in this section. The details of the parameters' distribution functions and the size of test problems are listed in Table 1. The summary of the results on test problems for different values of $\vartheta$ and $\varphi$, are listed in Table 2. It should be noted that the DM provides the relative importance of objective functions directly. However, multiple attribute decision-making (MADM) methods (e.g., Analytic Hierarchy Process (AHP)) can be used for setting the objective weights in a more precise manner.

According to Jimenez et al. [49], to generate the trapezoidal fuzzy parameters, first the four prominent points are determined. For this purpose, consider a trapezoidal parameter like $\tilde{A} = [a^1, a^2, a^3, a^4]$. Two values of $a^2$ and $a^3$ are first provided randomly by utilizing the uniform distributions specified in Table 1 in a form that $a^2 < a^3$. Then, without loss of generality, two random numbers $(r_1, r_2)$ are generated between 0.2 and 0.8 using a uniform distribution by which the $a^1$ and $a^4$ are then calculated as follows:

$$a^1 = (1 - r_1)a^2;\ a^4 = (1 + r_2)a^3$$

**Table 1**
Sources of random generation of the parameters.

| Parameters | Values Problem 1 | Problem 2 | Problem 3 |
|---|---|---|---|
| $I$ | 5 | 7 | 10 |
| $M$ | 2 | 2 | 3 |
| $S$ | 2 | 2 | 2 |
| $P$ | 2 | 2 | 3 |
| $\tilde{C}_{ij}^{t}$ | (20,80) | (50,120) | (200,300) |
| $\widetilde{CS}_{k}^{t}$ | $(10^2,2\times10^2)$ | $(10^3,2\times10^3)$ | $(10^4,2\times10^4)$ |
| $\tilde{F}_{k}^{s}$ | $(10^3,2\times10^3)$ | $(10^4,2\times10^4)$ | $(10^5,2\times10^5)$ |
| $\tilde{T}_{ij}^{t}$ | (5,15) | (10,30) | (20,50) |
| $K_{k}^{s}$ | (500,700) | (800,1000) | (3000,4000) |
| $\mu_{k}^{s}$ | Poison(1000) | Poison (1500) | Poison (2500) |
| $c_{k}^{s}$ | 2 | 2 | 3 |
| $W_{ij}$ | Poison(100) | Poison (200) | Poison (500) |
| $\sigma$ | 0.95 | 0.95 | 0.95 |
| $\beta$ | 0.75 | 0.75 | 0.75 |
| $\delta$ | 0.95 | 0.95 | 0.95 |

**Table 2**
Summary of the results for the test problem 1 ($\alpha = 0.5$).

| Problem No. | $\vartheta$ | $\varphi$ | $\mathcal{Z}_1$ | $\mu_1(\mathcal{Z})$ | $\mathcal{Z}_2$ | $\mu_2(\mathcal{Z})$ |
|---|---|---|---|---|---|---|
| 1 | 0.6 | 0.3,0.7 | 81166.36 | 0.912 | 140.51 | 0.971 |
| | 0.6 | 0.5,0.5 | 78078.61 | 0.953 | 148.11 | 0.931 |
| | 0.6 | 0.7,0.3 | 76045.22 | 0.980 | 150.39 | 0.919 |
| | 0.4 | 0.3,0.7 | 82597.27 | 0.893 | 138.42 | 0.982 |
| | 0.4 | 0.5,0.5 | 79735.45 | 0.931 | 146.21 | 0.941 |
| | 0.4 | 0.7,0.3 | 75066.17 | 0.993 | 159.89 | 0.869 |
| 2 | 0.6 | 0.3,0.7 | 323489.80 | 0.888 | 386.45 | 0.913 |
| | 0.6 | 0.5,0.5 | 303532.10 | 0.916 | 394.37 | 0.893 |
| | 0.6 | 0.7,0.3 | 270744.45 | 0.962 | 412.58 | 0.847 |
| | 0.4 | 0.3,0.7 | 359841.32 | 0.837 | 369.42 | 0.956 |
| | 0.4 | 0.5,0.5 | 337745.30 | 0.868 | 382.88 | 0.922 |
| | 0.4 | 0.7,0.3 | 281436.07 | 0.947 | 408.23 | 0.858 |
| 3 | 0.6 | 0.3,0.7 | 752740.58 | 0.914 | 481.52 | 0.952 |
| | 0.6 | 0.5,0.5 | 728990.01 | 0.933 | 491.56 | 0.926 |
| | 0.6 | 0.7,0.3 | 696489.23 | 0.959 | 492.72 | 0.923 |
| | 0.4 | 0.3,0.7 | 773991.09 | 0.897 | 471.49 | 0.978 |
| | 0.4 | 0.5,0.5 | 738990.25 | 0.925 | 487.70 | 0.936 |
| | 0.4 | 0.7,0.3 | 678988.81 | 0.973 | 503.91 | 0.894 |

As can be seen in Table 2, changing the values of $\vartheta$ changes the value of both objective functions. As a result, satisfaction degrees representing each objective function are altered correspondingly. For example, it can be seen that by altering the value of $\vartheta$ for the test problem 1, the satisfaction degree of the first objective function increases from 0.893 to 0.980. Similarly, by manipulating the value of $\vartheta$ for the test problem 1, the satisfaction degree of the second objective function increases from 0.869 to 0.982. This event similarly happens to the other two test problems. This fact demonstrates that the DM can easily adjust the value of $\vartheta$ in order to maximize the satisfaction degree of each objective function. Moreover, a higher value for $\vartheta$ means that higher weights are assigned to acquire a higher lower bound for the satisfaction degree of objectives ($\lambda_0$) and correspondingly more balanced compromise solutions. In contrast, a lower value for $\vartheta$ means that more attention is paid to obtain a solution with a high satisfaction degree for the fuzzy goals with higher relative importance, without paying attention to the satisfaction degree of other fuzzy goals (i.e., yielding unbalanced compromise solutions).

According to Table 2, it also can be realized that by increasing the value of $\varphi$ for each objective function, the satisfaction degrees of all objective functions are also increased. This issue explains that the DM can set the value of $\varphi$ based on the objective functions preferences in order to obtain the best fitted solution under current decision making situation. Furthermore, it means that by increasing the value of $\varphi$ corresponding to each objective function, the DM gains better value for that objective function. In addition, the Pareto frontier of problems 1–3 are depicted in Figs. 4–6, in which white and black marks corresponds to $\vartheta = 0.4$ and $\vartheta = 0.6$, respectively. It can be conceived from Figs. 4–6 that since both objective functions need to be minimized, Pareto frontiers for problems 1 to 3 are concave.

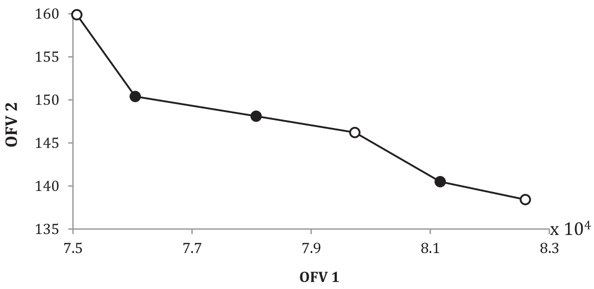


**Fig. 4.** Pareto frontier of problem number 1.

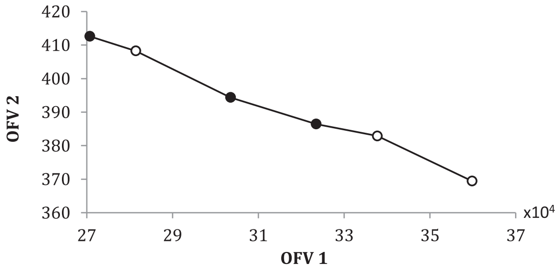


**Fig. 5.** Pareto frontier of problem number 2.

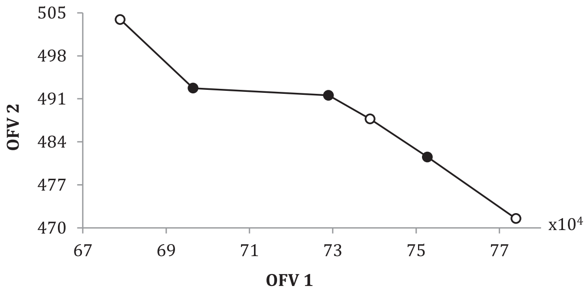


**Fig. 6.** Pareto frontier of problem number 3.

**Table 3**
DE parameter settings.

| Parameter | Population size | Crossover rate | F | NFC |
|---|---|---|---|---|
| Settings | 300 | 0.3 | 0.8 | 40000 |

### 6.2. *Comparative study*

In this section, the performance of the proposed DE algorithm in terms of both solution quality and computational time is evaluated in a comparative study using a number of randomly generated data sets. These data sets contain 30 test problems ranging from small- to large-sized instances (up to 200 nodes). Besides, for each test problem, five different intervals of the parameters were created for better comparison. Therefore, 150 problem instances are tested in total. The required parameters in the proposed DE algorithm are tuned using the response surface methodology (RSM) [16,63] which is a well-known tool to do so. The results of the RSM are tabulated in Table 3.

The SA and DE solutions are compared with the optimal solutions obtained from the GAMS software for small size test problems up to 10 nodes. Hence, for bigger test problems, just the comparison between SA and DE is reported as in Table 4. Consequently, for small size problems, a gap between SA and DE algorithms with GAMS is presented through the *percentage of relative gap* measure which is calculated based on $[100 \times (G_{GAMS} - G_{Meta})/G_{Meta}]$, where $G_{GAMS}$ and $G_{Meta}$ are the TH objective function values of the obtained solution over the GAMS software and meta-heuristic algorithm, respectively. Similarly, for problems of larger sizes, a gap between SA and DE algorithms is presented based on $[100 \times (\mathbb{Q}_{DE} - \mathbb{Q}_{SA})/\mathbb{Q}_{SA}]$ in which, $G_{DE}$ and $G_{SA}$ are the TH objective function value of DE and SA algorithms, respectively. Since the proposed DE algorithm dominates the SA algorithm in terms of solution quality (i.e., optimality gap), the mean gap of five different replications is reported for the SA algorithm in

**Table 4**
Average % relative gaps and CPU Time for small-sized problems ($\alpha = 0.5$).

| Data set # | $I$ | $P$ | DE Replications 1 | 2 | 3 | 4 | 5 | DE Gap (%) | DE Time (s) | SA Gap (%) | SA Time (s) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 2 | .00 | .00 | .00 | .00 | .00 | .000 | 33 | .000 | 26 |
| 2 | | 3 | .00 | .00 | .00 | .00 | .00 | .000 | 32 | .010 | 26 |
| 3 | 7 | 2 | .00 | .00 | .00 | .00 | .00 | .000 | 48 | .092 | 32 |
| 4 | | 3 | .00 | .00 | .01 | .00 | .00 | .002 | 49 | .130 | 33 |
| 5 | | 4 | .01 | .00 | .02 | .01 | .02 | .010 | 49 | .270 | 34 |
| 6 | 10 | 2 | .05 | .06 | .00 | .03 | .05 | .040 | 68 | 1.22 | 45 |
| 7 | | 3 | .05 | .05 | .04 | .06 | .04 | .050 | 66 | 1.40 | 45 |
| 8 | | 4 | .06 | .07 | .07 | .06 | .06 | .065 | 68 | 1.88 | 44 |
| 9 | | 5 | .07 | .08 | .08 | .08 | .08 | .080 | 68 | 2.16 | 46 |

**Table 5**
Average % relative gaps and CPU Time for large-sized problems ($\alpha = 0.5$).

| Data set # | $I$ | $P$ | DE Time (s) | SA Gap (%) | SA Time (s) |
|---|---|---|---|---|---|
| 10 | 15 | 2 | 120 | 3.14 | 98 |
| 11 | | 3 | 121 | 3.22 | 97 |
| 12 | | 4 | 122 | 3.44 | 103 |
| 13 | | 5 | 120 | 3.52 | 103 |
| 14 | | 6 | 121 | 3.64 | 107 |
| 15 | 50 | 8 | 721 | 6.64 | 602 |
| 16 | | 12 | 722 | 6.74 | 595 |
| 17 | | 16 | 723 | 6.92 | 606 |
| 18 | | 20 | 722 | 7.08 | 603 |
| 19 | 100 | 10 | 854 | 9.16 | 673 |
| 20 | | 15 | 856 | 9.10 | 674 |
| 21 | | 20 | 855 | 9.30 | 671 |
| 22 | | 25 | 854 | 9.66 | 674 |
| 23 | 150 | 15 | 1034 | 13.4 | 831 |
| 24 | | 20 | 1034 | 13.6 | 832 |
| 25 | | 25 | 1036 | 14.4 | 836 |
| 26 | | 30 | 1033 | 15.0 | 832 |
| 27 | 200 | 20 | 1247 | 19.3 | 971 |
| 28 | | 30 | 1248 | 19.3 | 972 |
| 29 | | 35 | 1251 | 19.8 | 973 |
| 30 | | 40 | 1250 | 21.3 | 977 |

Tables 4 and 5. The results reveal that DE outperforms SA in all test problems in terms of solution quality as shown in Tables 4 and 5.

The percentage of relative gaps for the five replications of each test problem were averaged and recorded as the "Gap (%)" column. The values in the "Gap (%)" column obtained by solving the problem instances using SA and DE and the corresponding average computational times are listed in Tables 4 and 5, respectively, for small- and large-sized problems. The required computational times for the SA and DE are also given in the "Time (s)" column in Tables 4 and 5. Despite being relatively small, the computation times of the proposed meta-heuristic algorithms increase as the problem size becomes larger. These results demonstrate that both SA and DE approaches can find near optimal solutions in a reasonable time while DE outperforms the SA algorithm in terms of solution quality.

In order to see whether a significant difference exists between the CPU times related to the DE and SA algorithms, a paired $t$ test was conducted. The test showed that there is no significant statistical difference between the CPU times of the DE and SA algorithms. Therefore, this similarity in the CPU time and the outperformance of DE compared to the SA algorithm in terms of objective function values reveal that DE outperforms the SA algorithm.

### *6.3. Sensitivity analysis*

In order to recognize the most significant parameters of the proposed model, several sensitivity analyses are carried out and the impact of parameters alteration on objective functions is investigated as Figs. 7–10. It should be noted that all experiments are done for problem number 3 and the all trends can be generalized for other sizes of the problem. Figs. 7 and 8 depict the sensitivity of two objective functions upon demand increase, in which both OFs are getting worse regarding to increased demand. The second objective function is more sensitive to demand alteration, where a little increase in demand leads to higher value of

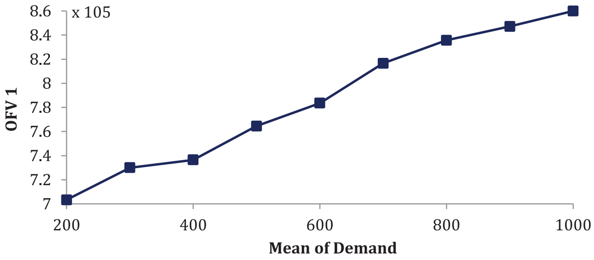


**Fig. 7.** First objective function value vs. mean of flow unit.

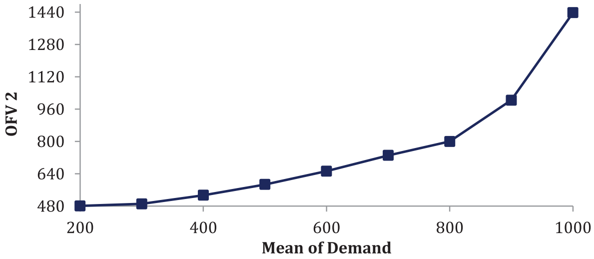


**Fig. 8.** Second objective function value vs. mean of flow unit.

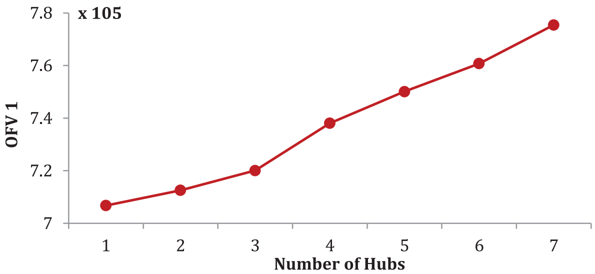


**Fig. 9.** First objective function value vs. number of hubs ($P$).

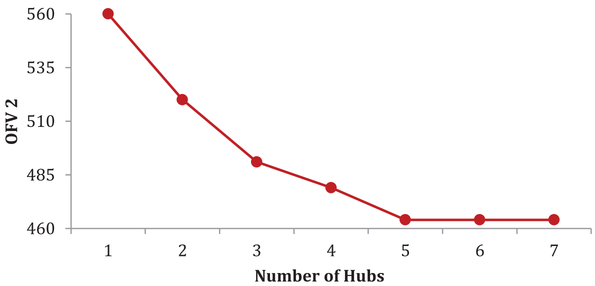


**Fig. 10.** Second objective function value vs. number of hubs ($P$).

waiting time in the hubs. These results are more useful for cargo transport organizations that use current infrastructure of the country and try to maximize responsiveness of their services as well as minimum transportation cost.

Figs. 9 and 10 illustrate the alteration of first and second objective functions with respect to increase in the number of hubs ($P$), respectively. Although the first objective function is increased where the higher number of hubs needs higher fixed cost, but second objective function is decreased where higher number of hubs leads to lower congestion at the hubs, and consequently lower waiting time of the flow units. Besides, by using a higher number of hubs, more flow units benefit from economies of scales of hub-to-hub links to be transferred to their destination with lower transportation cost. It can be also seen that the second objective function is not changed when the number of hubs exceeds 5, where no better solution is found. Therefore, a decision maker is expected to make a trade-off between the first and second objectives, by adjusting the number of hubs.

Finally, Figs. 11 and 12 illustrate the sensitivity of second objective function to the alteration of mean service rate ($\mu$) of the hubs and finite capacity of the queue ($K$) formed by flow units. In Fig. 11, it can be shown that by increasing the mean value of service rate, maximum transportation time between each pair of O-D nodes is decreased where higher value of service rate leads

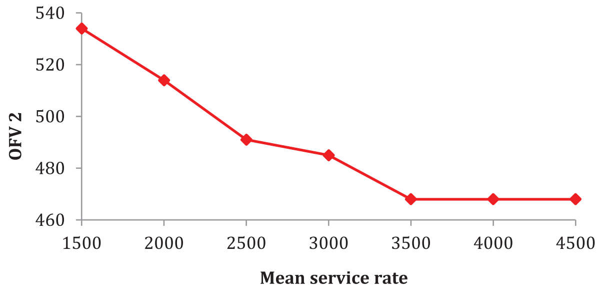


**Fig. 11.** Second objective function value vs. mean service rate ($\mu$).

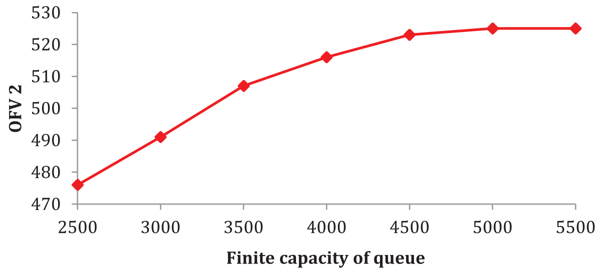


**Fig. 12.** Second objective function value vs. finite capacity of the queue ($K$).

to lower value of waiting time. Fig. 12 shows that by increasing the finite capacity of the queue, more flow unit is allowed to enter the hubs and consequently, the waiting time at hubs is increased that leads to higher value of second objective function.

### 6.4. Case study

In this section, a real case of passenger transportation in Iran is studied to validate the performance of the proposed model and the solution approaches. We use a special instance on hub airport location of Iranian aviation between 37 cities, which are the active airports (see Fig. 13). This instance is called Iranian Aviation Dataset (IAD) and has been prepared by Karimi and Bashiri [64]. Since the flow unit data originates from two criteria, including tourism and industrial, they calculated the importance of cities by TOPSIS (Technique for Order Preference by Similarity to Ideal Solution). Then, considering the importance of the population of each city, data flow units between cities were computed. The distance and fixed cost data have been taken from the Iranian airport corporation office.

In this case, two roads and air transportation modes have been considered. The unit transportation cost ($\tilde{C}_{ij}^t$) and Travel time ($\tilde{T}_{ij}^t$) for both road and air modes can be calculated based on the distance between cities. We also considered $\sigma$, $\beta$, and $\delta$ equal to 0.9, 0.75 and 0.9, respectively. We extended the data set by defining three capacity levels: small ($n_i^1$), medium ($n_i^2$), and large ($n_i^3$). Two cases have been also considered for the finite capacity of the queue at the nodes, namely tight capacity and excess capacity. The hub capacities ($n_i^s$) and fixed costs ($F_i^s$) are: $n_i^1 = \frac{1}{P}\bar{W}$, $n_i^2 = \frac{P-1}{P}\bar{W}$, $n_i^3 = \frac{P+1}{P}\bar{W}$, $F_i^1 = \bar{F}_i$, $F_i^2 = \sqrt{P-2}\bar{F}_i$, $F_i^3 = \sqrt{P}\bar{F}_i$, $P = 5, 6$ for the tight capacity case and $n_i^1 = \bar{W}$, $n_i^2 = 2\bar{W}$, $n_i^3 = 3\bar{W}$, $F_i^1 = \sqrt{P}\bar{F}_i$, $F_i^2 = \sqrt{2P}\bar{F}_i$, $F_i^3 = \sqrt{3P}\bar{F}_i$, $P = 2, 3$ for the excess capacity case, where $\bar{W} = \sum_{i,j>i} W_{ij}$ and $\bar{F}_i$ is the data obtained from IAD.

In order to generate trapezoidal fuzzy parameters according to Section 6.1, the four prominent points for each parameter at each level are determined based on the data obtained by IAD. For instance, consider the fixed cost of opening a hub at node $i$ with level $s$, $F_i^s$. The trapezoidal parameter $\tilde{F}_i^s$ is generated as Eq. (41).

$$\tilde{F}_i^s = \left[(1-r_1)\left(1-\frac{r_1+r_2}{4}\right)F_i^s,\ \left(1-\frac{r_1+r_2}{4}\right)F_i^s,\ \left(1+\frac{r_1+r_2}{4}\right)F_i^s,\ (1+r_2)\left(1+\frac{r_1+r_2}{4}\right)F_i^s\right] \tag{41}$$

where $r_1$ and $r_2$ are randomly generated from the interval [0.2,0.8] using a uniform distribution.

The parameters are set in a way that lead to different system characteristics with two capacity levels (tight and excess) and two values of $P$. Consequently, 4 real scenarios are investigated. After implementing the proposed model and solution approaches on the IAD data set using $\vartheta = \phi_\tau = 0.5$ and $\alpha = 0.5$, the results obtained by the DE algorithm are illustrated in Figs. 14–17.

Fig. 14 demonstrates the transportation network with 4 hubs and tight capacity. It can be seen that hubs number 10, 12, 15, 19 and 31 have been located with capacity levels 3, 2, 3, 1 and 3, respectively. Fig. 15 depicts the hub network with 5 hubs and tight capacity strategy. It can be discerned that hubs number 10, 12, 15, 19, 30 and 31 have been located with capacity levels 3, 2, 3, 1, 2 and 3, respectively. In the case of excess capacity, Fig. 16 illustrates the hub network with 2 hubs at nodes 15 and 31 with


| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Abadan | 2 | Ahvaz | 3 | Arak | 4 | Ardabil | 5 | Bandar Abbas |
| 6 | Birjand | 7 | Bojnurd | 8 | Bushehr | 9 | Chabahar | 10 | Esfahan |
| 11 | Gorgan | 12 | Hamedan | 13 | Ilam | 14 | Iranshahr | 15 | Kerman |
| 16 | Kermanshah | 17 | Khark | 18 | Khoramabad | 16 | Mashhad | 20 | Nowshahr |
| 21 | Ramsar | 22 | Rasht | 23 | Sabzevar | 24 | Sanandaj | 25 | Sari |
| 26 | Shahrekord | 27 | Shahrud | 28 | Shiraz | 29 | Sirjan | 30 | Tabriz |
| 31 | Tehran | 32 | Urmia | 33 | Yasooj | 34 | Yazd | 35 | Zabol |
| 36 | Zahedan | 37 | Zanjan | | | | | | |

**Fig. 13.** Iran cities in the IAD data set.

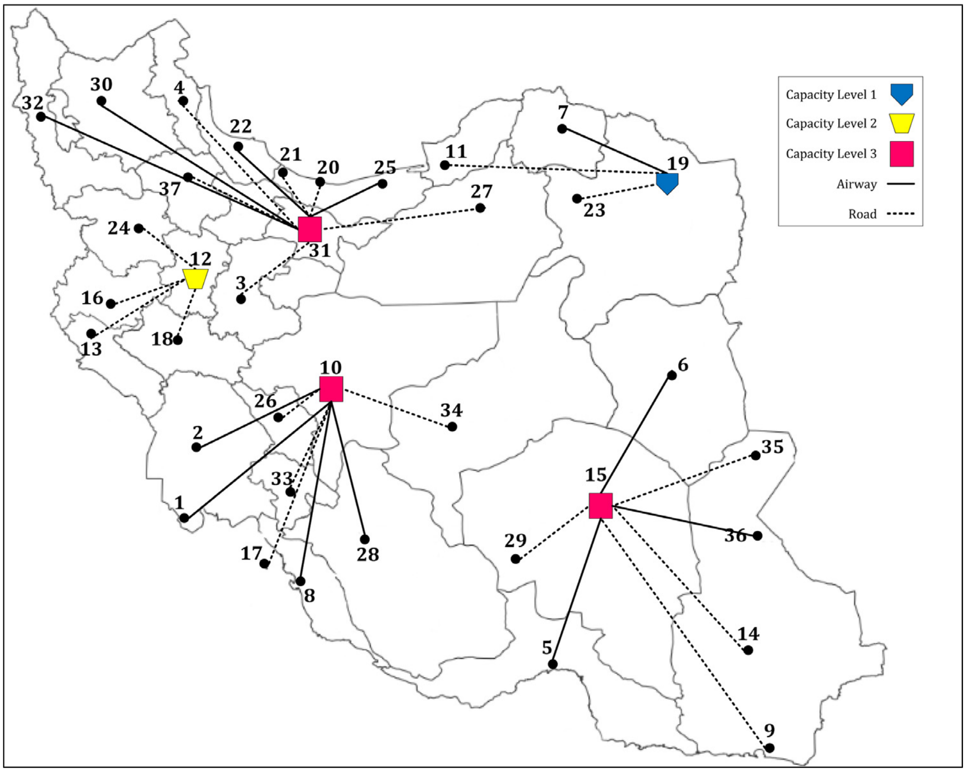


**Fig. 14.** Hub topography for tight capacity and $P$=5.

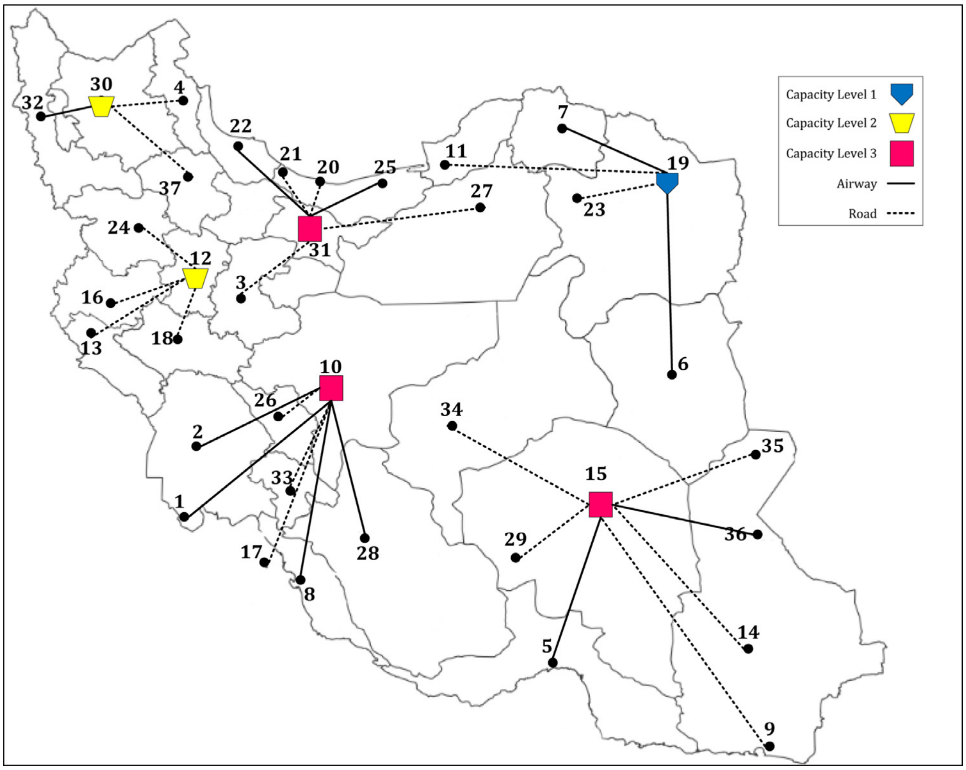


**Fig. 15.** Hub topography for tight capacity and $P$=6.

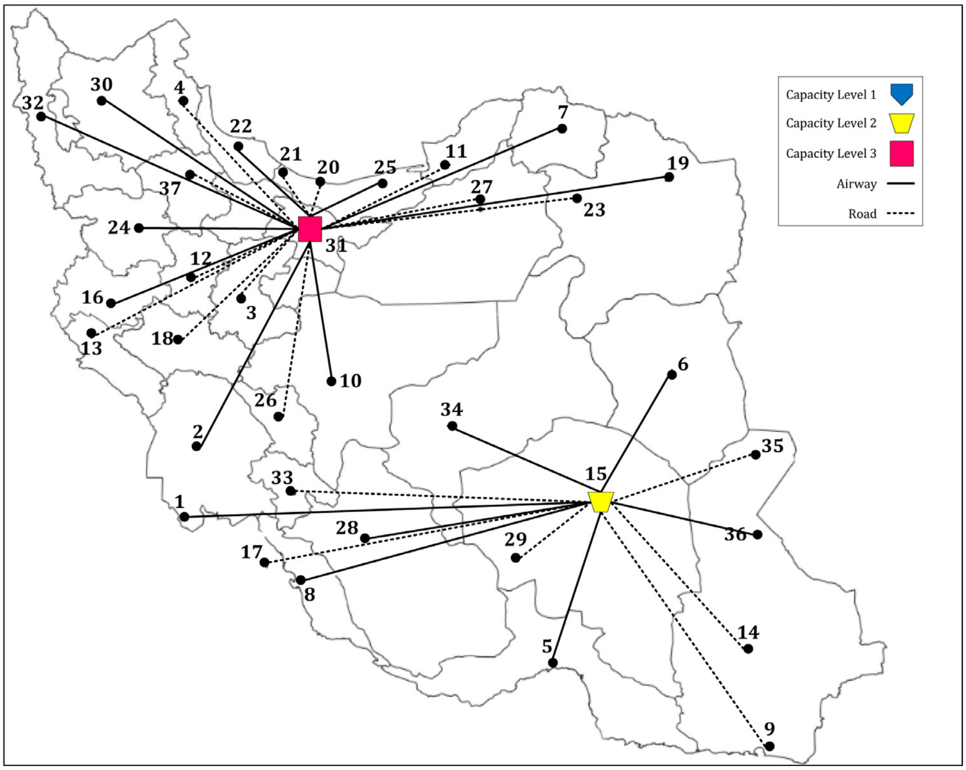


**Fig. 16.** Hub topography for excess capacity and $P$=2.

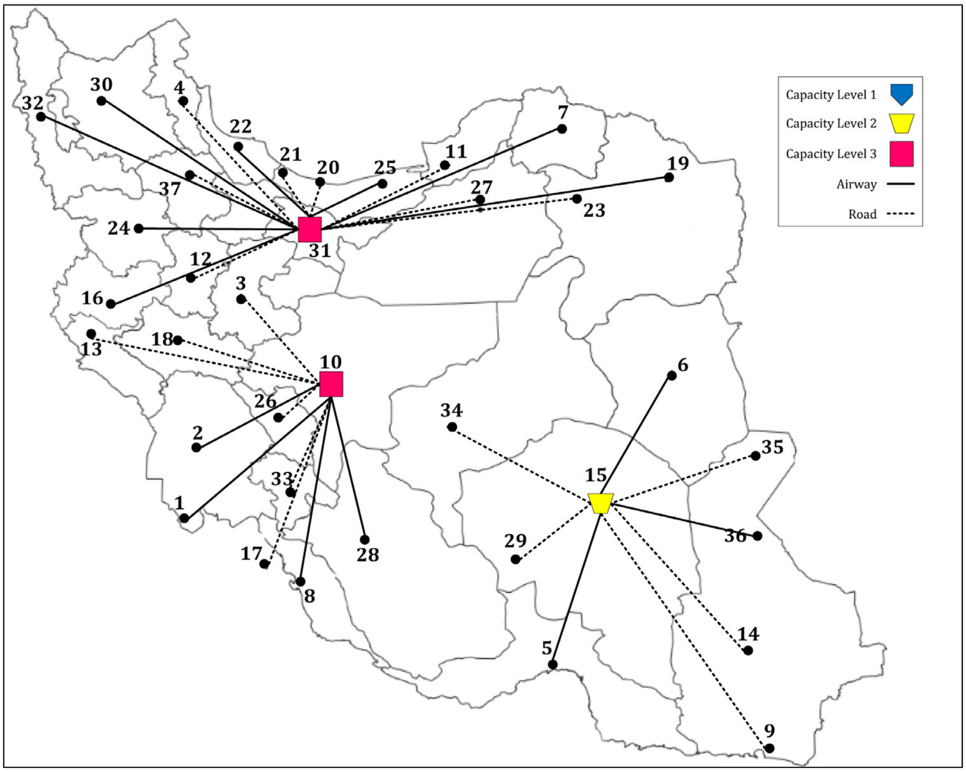


**Fig. 17.** Hub topography for excess capacity and $P$=3.

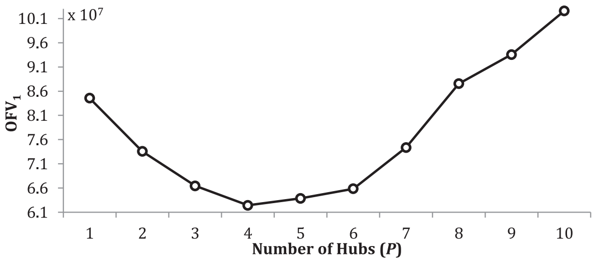


**Fig. 18.** $OFV_1$ vs. Number of hubs ($P$).

**Table 6**
Details of real HLP case.

| Input | | Output | | |
|---|---|---|---|---|
| Capacity | $P$ | Hub locations | OFV $_1$ | OFV $_2$ |
| Tight | 5 | 10, 12, 15, 19, 31 | 56845320 | 834 |
| | 6 | 10, 12, 15, 19, 30, 31 | 59728425 | 702 |
| Excess | 2 | 15, 31 | 73568351 | 623 |
| | 3 | 10, 15, 31 | 64254880 | 623 |

capacity levels 2 and 3, respectively. Similarly, Fig. 17 shows the results of excess capacity strategy with 3 hubs at nodes 10, 15 and 31 with capacity levels 3, 2 and 3, respectively.

The detailed results of four real scenarios are listed in Table 6, in which the first and the second columns are capacity strategy and number of hubs that should be located ($P$), and the third to fifth columns are the location of hubs and values of the first and the second objective functions, respectively.

What appears later is the effect of number of hubs on the objective function values that is investigated in the real case with excess capacity as Fig. 18. It can be explained that by increasing the number of hubs, the value of the first objective function ($OFV_1$) is decreased for $P = 4$ and is increased for $P > 4$. It can be concluded that having four hubs leads us to the minimum cost

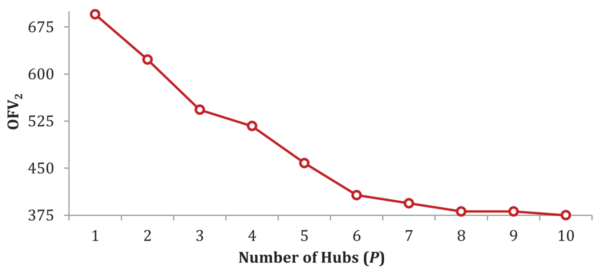


**Fig. 19.** $OFV_2$ vs. Number of hubs ($P$).

in the hub network. In spite of the first objective, the value of the second objective function ($OFV_2$) is decreased by increasing the number of hubs (see Fig. 19). It can be proved that $OFV_2$ is decreased with higher rate for all $P \leq 6$; however, the number of hubs becomes more than six, $OFV_2$ is slightly decreased.

## 7. Conclusion

This paper proposed a multi-objective, multi-modal, hub location problem (HLP) considering congestion in the hubs. A modeling framework for the operations of a multi-modal HLP was developed using a queuing system approach in order to minimize the total transportation cost and the maximum transportation time between each pair of O-D nodes in the network. Since the capacity of hub facilities is limited, the entering flow unit to a hub should wait to receive services, in which a queue is made at each hub. In this paper, a maximum allowable capacity is considered for the arrival flow to the hubs. Therefore, an M/M/c/K queuing system was developed to analyze the waiting time of flow units at each hub. On the other hand, for each hub, several capacity levels were considered while just one of them can be selected. In addition, due to environmental impacts, significant parameters were considered as fuzzy numbers. In order to cope with this uncertainty, an efficient hybrid approach was developed. Furthermore, in order to solve large-sized instances of the presented mathematical model, a well-known meta-heuristic, namely differential evolution (DE), was applied on the model. The performance of the proposed DE algorithm for solving small-sized instances was evaluated by comparing the obtained optimal solutions with those of GAMS commercial software. For large-sized instances, the superiority of the proposed DE algorithm was shown in comparison to simulated annealing (SA) algorithm. Finally, a real case of passenger transportation in Iran was studied to better validate the performance of the proposed model and solution approach.

## Reference

[1] I. Contreras, E. Fernandez, A. Marin, Tight bounds from a path based formulation for the tree of hub location problem, Comput. Oper. Res. 36 (2009) 3117–3127.
[2] H.A. Eiselt, V. Marianov, A conditional p-hub location problem with attraction functions, Comput. Oper. Res. 36 (2009) 3128–3135.
[3] M.R. Silva, C. Cunha, New simple and efficient heuristics for the uncapacitated single allocation hub location problem, Comput. Oper. Res. 36 (2009) 3152–3165.
[4] M. Mohammadi, R. Tavakkoli-Moghaddam, H. Tolouei, M. Yousefi, Solving a hub covering location problem under capacity constraints by a hybrid algorithm, J. Appl. Oper. Res. 2 (2) (2010) 109–116.
[5] M. Mohammadi, R. Tavakkoli-Moghaddam, H. Rostami, A multi-objective imperialist competitive algorithm for a capacitated hub covering location problem, Int. J. Indus. Eng. Comput. 2 (3) (2011) 671–688.
[6] M. Mohammadi, R. Tavakkoli-Moghaddam, A. Ghodratnama, H. Rostami, Genetic and improved shuffled frog leaping algorithms for a 2-stage model of a hub covering location network, Int. J. Indus. Eng. Prod. Res. 22 (3) (2011) 171–179.
[7] R. Ghodsi, M. Mohammadi, H. Rostami, Hub covering location problem under capacity constraints In Proceedings of the Fourth Asia International Conference on Mathematical/Analytical Modelling and Computer Simulation (IEEE). (2010) 204-208.
[8] S.H. Owen, M.S. Daskin, Strategic facility location: a review, Eur. J. Oper. Res. 111 (1998) 423–447.
[9] S. Alumur, B.Y. Kara, Network hub location problems: the state of the art, Eur. J. Oper. Res. 190 (1) (2008) 1–21.
[10] J.F. Campbell, AT. Ernst, M. Krishnamoorthy, Hub location problems, in: Z. Drezner, H. Hamacher (Eds.), Facility Location: Applications And Theory, Springer, Berlin, 2002.
[11] R. Zanjirani Farahani, M. Hekmatfar, A. Boloori Arabani, E. Nikbakhsh, Hub location problems: a review of models, classification, techniques and application, Comput. Ind. Eng. 64 (2013) 1096–1109.
[12] N. Vidyarthi, S. Jayaswal, R. Tyagi, Hub and spoke network system design for freight transportation with priority consignment classes, CIRRELT, Nov. 2013.
[13] P.Z. Tan, B.Y. Kara, A hub covering model for cargo delivery systems, Networks 49 (1) (2007) 28–39.
[14] C.C. Lin, J.Y. Lin, Y.C. Chen, The capacitated p-hub median problem with integral constraints: an application to a Chinese air cargo network, Appl. Math. Model. 36 (6) (2012) 2777–2787.
[15] R. Ishfaq, C.R. Sox, Design of intermodal logistics networks with hub delays, Eur. J. Oper. Res. 220 (2012) 629–641.
[16] M. Mohammadi, F. Jolai, R. Tavakkoli-Moghaddam, Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm, Appl. Math. Model. 37 (24) (2013) 10053–10073.
[17] M. Mohammadi, S.A. Torabi, R. Tavakkoli-Moghaddam, Sustainable hub location under mixed uncertainty, Trans. Res. Part E 62 (2014) 89–115.
[18] M. Mohammadi, S. Dehbari, B. Vahdani, Design of a bi-objective reliable healthcare network with finite capacity queue under service covering uncertainty, Trans. Res. Part E 72 (2014) 15–41.
[19] B. Zahiri, R. Tavakkoli-Moghaddam, M. Mohammadi, P. Jula, Multi-objective design of an organ transplant network under uncertainty, Transp. Res. Part E 72 (2014) 101–124.
[20] F. Niakan, B. Vahdani, M. Mohammadi, A multi-objective optimization model for hub network design under uncertainty: an inexact rough-interval fuzzy approach, Eng. Opt. 47 (2015) 1670–1688.

Please cite this article as: Y. Rahimi et al., Multi-objective hub network design under uncertainty considering congestion: An M/M/c/K queue system, Applied Mathematical Modelling (2015), http://dx.doi.org/10.1016/j.apm.2015.11.019

[21] V. Rodriguez, M.J. Alvarez, L. Barcos, Hub location under capacity constraints, Trans. Res. Part E: Logis. Trans. Rev. 43 (2007) 495–505.
[22] M. Costa, M. Captivo, J. Climaco, Capacitated single allocation hub location problem – A bi-criteria approach, Comput. Oper. Res. 35 (11) (2008) 3671–3695.
[23] L.V. Snyder, Facility location under uncertainty: a review, IIE Trans. 38 (2006) 537–554.
[24] C. Negoita, L. Zadeh, H. Zimmermann, Fuzzy sets as a basis for a theory of possibility, Fuzz. Set. Syst. 1 (1978) 3–28.
[25] M. Bashiri, M. Mirzaei, M. Randall, Modeling fuzzy capacitated p-hub center problem and a genetic algorithm solution, Appl. Mat. Model. 37 (5) (2013) 3513–3525.
[26] k. Yang, Y. Liu, G. Yang, Optimizing fuzzy p-hub center problem with generalized value-at-risk criterion, Appl. Math. Model. 38 (15-16) (2014) 3987–4005.
[27] M.E. O'Kelly, The location of interacting hub facilities, Trans. Sci. 20 (2) (1986) 92–106.
[28] J.F. Campbell, Integer programming formulations of discrete hub location problems, Eur. J. Oper. Res. 72 (2) (1994) 387–405.
[29] A.T. Ernst, M. Krishnamoorthy, Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem, Eur. J. Oper. Res. 104 (1) (1998) 100–112.
[30] D. Skorin-Kapov, J. Skorin-Kapov, M.E. O'Kelly, Tight linear programming relaxations of uncapacitated p-hub median problems, Eur. J. Oper. Res. 94 (3) (1996) 582–593.
[31] V. Marianov, D. Serra, Probabilistic, maximal covering location-allocation models for congested systems, J. Regi. Sci. 38 (3) (1998) 401–424.
[32] V. Marianov, D. Serra, Location–allocation of multiple-server service centers with constrained queues or waiting times, Ann. Oper. Res. 111 (2002) 35–50.
[33] V. Marianov, D. Serra, Location models for airline hubs behaving as M/D/c queues, Comput. Oper. Res. 30 (7) (2003) 983–1003.
[34] S. Elhedhli, F.X. Hu, Hub-and-spoke network design with congestion, Comput. Oper. Res. 32 (6) (2005) 1615–1632.
[35] O. Berman, M.J. Hodgson, D. Krass, Flow-interception problems, in: Z. Drezner (Ed.), Facility Location: A Survey Of Applications And Methods, Springer, New York, 1995.
[36] S.H.A. Rahmati, A. Ahmadi, M. Sharifi, A. Chambari, A multi-objective model for facility location–allocation problem with immobile servers within queuing framework, Comput. Ind. Eng. 74 (2014) 1–10.
[37] M. Mohammadi, F. Jolai, H. Rostami, An M/M/c queue model for hub covering location problem, Math. Comput. Model. 54 (2011) 2623–2638.
[38] T. Van Woensel, F.R.B. Cruz, A stochastic approach to traffic congestion costs, Comput. Oper. Res. 36 (2009) 1731–1739.
[39] T. Van Woensel, N. Vandaele, Empirical validation of a queueing approach to uninterrupted traffic flows, 4OR 4 (1) (2006) 59–72.
[40] T. Van Woensel, B. Wuyts, N. Vandaele, Validating state-dependent queueing models for uninterrupted traffic flows using simulation, 4OR 4 (2) (2006) 159–174.
[41] M. Peterson, D. Bertsimas, A. Odoni, Models and algorithms for transient queuing congestion at airports, Manag. Sci. 41 (8) (1995) 1279–1295.
[42] T. Aykin, Lagrangian-relaxation based approaches to capacitated hub-and-spoke network design problem, Eur. J. Oper. Res. 79 (3) (1994) 501–523.
[43] J. Ebery, M. Krishnamoorthy, A. Ernst, N. Bolanf, The capacitated multiple allocation hub location problem: Formulations and algorithm, Eur. J. Oper. Res. 120 (3) (2000) 614–631.
[44] M. Sasaki, M. Fukushima, On the hub-and-spoke model with arc capacity constraints, J. Oper. Res. Soc. Jpn. 46 (4) (2003) 409–428.
[45] S. Elhedhli, H. Wu, A lagrangian heuristic for hub-and-spoke system design with capacity selection and congestion, INFORMS J. Comput. 22 (2) (2010) 282–296.
[46] D. Gross, C.M. Harris, Fundamentals of Queuing Theory, Wiley, New York, 1974.
[47] B. Zahiri, R. Tavakkoli-Moghaddam, M.S. Pishvaee, A robust possibilistic programming approach to multi-period location–allocation of organ transplant centers under uncertainty, Comput. Ind. Eng. 74 (2014) 139–148.
[48] B. Zahiri, S.A. Torabi, M. Mousazadeh, S.A. Mansouri, Blood collection management: methodology and application, Appl. Math. Model. in Press, doi:10.1016/j.apm.2015.04.028.
[49] M. Jimenez, M. Arenas, A. Bilbao, M.V. Rodriguez, Linear programming with fuzzy parameters: an interactive method resolution, Eur. J. Oper. Res. 177 (2007) 1599–1609.
[50] S.A. Torabi, E. Hassini, An interactive possibilistic programming approach for multiple objective supply chain master planning, Fuzz. Set Syst. 159 (2008) 193–214.
[51] M. Jimenez, Ranking fuzzy numbers through the comparison of its expected intervals, Int. J. Uncertain. Fuzzy Knowl. Base Syst. 4 (4) (1996) 379–388.
[52] S. Heilpern, The expected valued of a fuzzy number, Fuzz. Set. Sys. 47 (1992) 81–86.
[53] C.L. Hwang, A. Masud, Multiple objective decision making, Methods and applications: a state of the art survey, Lecture Notes In Economics And Mathematical Systems, 164, Springer-Verlag, Berlin, 1979.
[54] S.M. Guu, Y.K. Wu, Two phase approach for solving the fuzzy linear programming problems, Fuzz. Set. Sys. 107 (1999) 191–195.
[55] Y.J. Lai, C.L. Hwang, A new approach to some possibilistic linear programming problems, Fuzz. Set. Sys. 49 (1993) 121–133.
[56] M. Sakawa, H. Yano, T. Yumine, An interactive fuzzy satisfying method for multi objective linear-programming problems and its application, IEEE Trans. Syst. M. Cyber. SMC-17 (1987) 654–661.
[57] J. Wang, Y.F. Shu, A possibilistic decision model for new product supply chain design, Eur. J. Oper. Res. 177 (2007) 1044–1061.
[58] M. Mousazadeh, S.A. Torabi, B. Zahiri, A robust possibilistic programming approach for pharmaceutical supply chain network design, Comput. Chem. Eng. 82 (2015) 115–128.
[59] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, E. Teller, Equations of state calculations by fast computing machines, J. Chem. Phys. 21 (1953) 1087–1092.
[60] S. Kirkpatrick, C.D. Gelatt, M.P. Vecchi, Optimization by simulated annealing, Science 220 (1983) 671–680.
[61] R. Storn, K. Price, Differential evolution – A simple and efficient heuristic for global optimization over continuous spaces, J. Glob. Optim. 11 (1997) 341–359.
[62] P. Calegari, G. Coray, A. Hertz, D. Kobler, P. Kuonen, A taxonomy of evolutionary algorithms in combinatorial optimization, J. Heur. 5 (1999) 145–158.
[63] A. Hassanzadeh, A. Jafarian, M. Amiri, Modeling and analysis of the causes of bullwhip effect in centralized and decentralized supply chain using response surface method, Appl. Math. Model. 38 (9) (2014) 2353–2365.
[64] H. Karimi, M. Bashiri, Hub covering location problems with different coverage types, Sci. Iran. 18 (6) (2011) 1571–1578.