Physica A 419 (2015) 408–416

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Community detection by fuzzy clustering

Peng Gang Sun *

School of Computer Science and Technology, Xidian University, Xi'an, 710071, China
Institute of Computational Bioinformatics, Xidian University, Xi'an, 710071, China

HIGHLIGHTS

- Rule I fit the similarity of same groups is stronger than that of different groups.
- Rule II fit the similarity of same groups is weaker than that of different groups.
- Fuzzy clustering adapts to vaguer community detection.

ARTICLE INFO

Article history:
Received 29 April 2014
Received in revised form 1 September 2014
Available online 16 October 2014

Keywords:
Community detection
Fuzzy transitive rule
Edge centrality

ABSTRACT

How to measure the similarity between nodes is of great importance for fuzzy clustering when we use the approach to uncover communities in complex networks. In this paper, we first measure the similarity between nodes in a network based on edge centralities and model the network as a fuzzy relation. Then, two fuzzy transitive rules (Rule I and Rule II) are applied on the relation respectively, by which the similarity information can be transferred from one node to another in the network until the relation reaches a stable state. By choosing different thresholds, our method finally can partition the network into several non-overlapping subgroups. We compare our method with some state of the art methods on the LFR benchmark and real-world networks. We find that our method based on Rule I can correctly identify communities when the similarity between nodes of same groups is greater than that of different groups, while it is just opposite to Rule II. Our method achieves better results than the state of the art methods when the pre-planted communities of the random networks are vaguer.

1. Introduction

Many works showed that most complex networks characterize by groups of nodes, called communities, which have more internal links between nodes of same communities and less external links of different communities [1–4]. Community detection is very important for the understanding of complex systems, since it can uncover the organization features of complex systems as well as the hidden correlations among the components [1–4]. Therefore, more and more scholars from different fields pay more their attentions on community detection [1–4].

In the past several years, many approaches have been proposed for community detection [3,5–38]. Fortunato reviewed in detail the community detection methods and classified them into several categories [3] such as optimization methods [2,5–9], and divisive clustering algorithms [10–12,33,34]. Newman and Girvan [2,5,6] considered the problem of community detection as an objective optimization problem [2,5,6] by maximizing *modularity*. However, the method has a resolution limit problem [13]. Li and Zhang [8] considered a quantitative function, *modularity density* and tried to alleviate this problem by maximizing this function. The second important branch of approaches for community detection is divisive algorithms.

* Correspondence to: School of Computer Science and Technology, Xidian University, Xi'an, 710071, China.
E-mail address: psun@mail.xidian.edu.cn.

http://dx.doi.org/10.1016/j.physa.2014.10.009
0378-4371/

Girvan and Newman (GN) [2,10] proposed a divisive algorithm based on edge betweenness. GN algorithm [2,10] identifies communities by removing the edges with the maximum of edge betweenness in a network. By doing this process repeatedly, the network is broken into smaller and smaller subgroups [2,10]. Furthermore, edge information centrality [11] and edge clustering coefficient [12] were proposed to replace the edge betweenness in the divisive algorithm [2,10] respectively for community detection.

Overlapping communities provide a deep understanding of organizational units in complex networks, since one person may belong to multiple groups, in which they play different roles. Palla et al. [16] first proposed a clique percolation method (CPM) to identify overlapping communities in social networks. Furthermore, many methods that can detect both overlapping and non-overlapping communities have been proposed such as the OSLOM method [36], and the Infomap method [37]. The approach proposed by Ahn et al. [38] is different from the former. They reconstructed communities as groups of edges rather than vertices and showed that this approach can identify overlapping communities. In recent years, fuzzy clustering provides a fresh idea for the understanding of community detection [14,15]. For example, Sun et al. [15] used fuzzy clustering to detect overlapping and non-overlapping communities in complex networks. They treated the network as a fuzzy relation. By fuzzy transitive closure, finally each community is mapped as an equivalence class. They also extended their method to identify overlapping communities. Nevertheless, how to measure the similarity to distinguish the nodes of same communities from that of different communities is of great importance to define the fuzzy relation for a network and detect communities for fuzzy clustering as well. Granovetter [30,31] indicated that weak and strong similarities between nodes play different roles in real-world networks in that the weak similarity keeps the networks global integrated while the strong similarity maintains the communities. Sun et al. [34] used the edge centralities to weight the links of networks and showed that edge centralities indeed can distinguish the links of same groups from that of different groups.

In this paper, we try to distinguish the similarity between nodes of same communities from that of different communities based on edge centralities such as edge betweenness (*EBT*), edge information centrality (*EIC*) and edge clustering coefficient (*ECC*) and construct a fuzzy relation for a network. Then, two fuzzy transitive rules (Rule I and Rule II) are applied on the relation respectively, by which the similarity information can be transferred from one node to another in the network until the relation reaches a stable state. By choosing different thresholds, our method finally can partition the network into several non-overlapping communities with multi-resolutions. Of course, our method based on fuzzy transitive rules for community detection can be easily extended to identify overlapping communities [15]. This paper mainly focuses on the similarity between nodes and two fuzzy transitive rules and tries to elucidate how to assign the similarity between nodes for each of two rules so that they can achieve better clustering results. We compare our method with some state of the art methods of community detection on the Lancichinetti, Fortunato and Radicchi (LFR) benchmark [23] and real-world networks and also discuss the choice of thresholds for community detection.

The rest of the paper is organized as follows. In Section 2, we give an introduction on edge centralities and use them to measure the similarity between nodes. In Section 3, we discuss our method for community detection. Section 4 presents the results on both random networks and real-world networks. The conclusion is provided in Section 5.

2. Measuring similarity between nodes based on edge centrality

Here, we discuss the edge centralities such as edge betweenness (*EBT*) [2,10], edge information centrality (*EIC*) [11] and edge clustering coefficient (*ECC*) [12] and try to use them to distinguish the similarity between nodes of same communities from that of different communities so that we can model a network as a fuzzy similar relation.

2.1. Edge betweenness

Girvan and Newman [2,10] defined edge betweenness centrality based on the work of Freeman [24,25] as the number of shortest paths between vertices that contain the edge, since the shortest paths should go along one of the edges between different communities. Girvan and Newman [2,10] indicated that the edges between communities are always with higher edge betweenness values.

2.2. Edge information centrality

Fortunato et al. [11] defined edge information centrality based on the work of Latora and Marchiori [26,27]. The information centrality, EIC_e for an edge e, is defined as the drop of network efficiency caused by removing the edge in a network.

$$EIC_e = \frac{\Delta NE}{NE} = \frac{NE(G) - NE(G_{\bar{e}})}{NE(G)} \tag{1}$$

$$NE(G) = \frac{1}{n(n-1)} \sum_{v_i \neq v_j \in G} \frac{1}{d_{v_i v_j}} \tag{2}$$

where $G_{\bar{e}}$, a graph is obtained by removing e from G. $NE(G)$, the network efficiency of G. $d_{v_i v_j}$, the length of the shortest path between v_i and v_j. Fortunato et al. [11] showed that the edges connecting different communities are those with higher edge information centrality values.

2.3. Edge clustering coefficient

Radicchi et al. [12] defined edge clustering coefficient based on the work of Watts and Strogatz [28], since the edges within same communities will have a higher likelihood to form cycles, while the edges between different communities will hardly to form cycles. Radicchi et al. [12] showed that the edges between different communities have the greater values of edge clustering coefficient. This measure is defined as follows [12].

$$ECC^{(g)}_{v_i v_j} = (z^{(g)}_{v_i v_j} + 1)/s^{(g)}_{v_i v_j} \tag{3}$$

where $z^{(g)}_{v_i v_j}$, the number of cyclic structures of g on the edge (v_i, v_j), $s^{(g)}_{v_i v_j} = \min(d_{v_i} - 1, d_{v_j} - 1)$. d_{v_i} and d_{v_j}, the degree of v_i and $v_j$respectively. The edge clustering coefficient is defined based on triangles when $g = 3$.

Here, we use $x_{v_i v_j}$ to denote the similarity between node v_i and node v_j, and $x_{v_i v_j}$ equals the edge centrality value, if $(v_i, v_j) \in E$, otherwise $x_{v_i v_j} = 0$. In this paper, $x_{v_i v_j}$ is normalized between 0 and 1 by $x'_{v_i v_j} = x_{v_i v_j}/Max\{x_{v_i v_j}\}$. Here, if $x'_{v_i v_j}$ denotes the similarity between node v_i and node v_j based on *EBT* (edge betweenness), then $\overline{EBT}$ can be denoted by $1 - x'_{v_i v_j}$. Of course, the above is also applicable to *EIC* (edge information centrality) and $\overline{EIC}$ as well as *ECC* (edge clustering coefficient) and $\overline{ECC}$.

We know that nodes connecting different communities are endowed with the greater values by both edge betweenness and edge information centrality, while it is just opposite to the edge clustering coefficient. Therefore, $\overline{EBT}$, $\overline{EIC}$ and *ECC* maintain the nodes within same groups with greater similarity values, while it is just opposite to *EBT*, *EIC* and $\overline{ECC}$.

3. Fuzzy clustering

In this section, we first describe some concepts of fuzzy relation [9,15,29], and then two fuzzy transitive rules. Finally, we discuss our method based on the two rules for community detection.

3.1. Fuzzy relation

Given an unweighted and undirected graph, $G(V, E)$, where V is the set of nodes, and E is the set of links. The graph, G can be denoted by a binary fuzzy relation, R in V, where $R \in \mathcal{F}(V \times V)$, $\mathcal{F}(V \times V)$ is the set of all the fuzzy relations of $V \times V$. $\forall(v_i, v_j) \in V \times V$, $R(v_i, v_j)$ is seen as the membership grade of the pair (v_i, v_j) in R [29].

Definition 1 (*Reflexive Relation*). R is reflexive, if $\forall v_i \in V$, $R(v_i, v_i) = 1$, where $R \in F(V \times V)$. Otherwise, R is irreflexive, if $\forall v_i \in V$, $R(v_i, v_i) = 0$ [29].

Definition 2 (*Symmetric Relation*). R is symmetric, if $\forall(v_i, v_j) \in V \times V$, $R(v_i, v_j) = R(v_j, v_i)$, where $R \in \mathcal{F}(V \times V)$ [29].

Definition 3 (*Inclusion Relation*). $R \subseteq R'$, if $\forall(v_i, v_j) \in V \times V$, $R(v_i, v_j) \le R'(v_i, v_j)$, where $R, R' \in \mathcal{F}(V \times V)$ [29].

3.2. Fuzzy transitive rules

Here, we discuss the two fuzzy transitive rules to fulfill the similarity information transfer from one node to another in a network [9,15,29].

Rule I: $R^t = R^{t-1} \circ R$

$(R^{t-1} \circ R)(v_i, v_j) = \vee_{v_k \in V}(R^{t-1}(v_i, v_k) \wedge R(v_k, v_j))$, $(v_i, v_j) \in V \times V$. If R^{t-1} is reflexive and symmetric, then $R^{k-1} \subseteq R^t$, and $R \subseteq R^2 \subseteq \cdots \subseteq R^t \subseteq \cdots$. If $R^t \supseteq R^{t+1}$, then the information transfer is accomplished in the network by the rule, and $R^*_{max} = R^t$ is the maximal stable state of R based on the rule. "$\circ$" $= (\vee, \wedge)$ is called (*Max*, *Min*) rule. $R^{t-1}(v_i, v_k) \wedge R(v_k, v_j) = Min\left\{R^{t-1}(v_i, v_k), R(v_k, v_j)\right\}$, $R^{t-1}(v_i, v_k) \vee R(v_k, v_j) = Max\left\{R^{t-1}(v_i, v_k), R(v_k, v_j)\right\}$ [9,15,29].

Rule II : $R^t = R^{t-1} \tilde{\circ} R$

$(R^{t-1} \tilde{\circ} R)(v_i, v_j) = \wedge_{v_k \in V}(R^{t-1}(v_i, v_k) \vee R(v_k, v_j))$, $(v_i, v_j) \in V \times V$. If R^{t-1} is irreflexive and symmetric, then $R^t \subseteq R^{t-1}$, and $R \supseteq R^2 \supseteq \cdots \supseteq R^t \supseteq \cdots$. If $R^t \subseteq R^{t+1}$, then the information transfer is accomplished in the network by the rule, and $R^*_{min} = R^t$ is the minimal stable state of R based on the rule. "$\tilde{\circ}$" $= (\wedge, \vee)$ is called (*Min*, *Max*) rule [9,15,29].

Based on the two fuzzy transitive rules, this process, $R \Rightarrow R^2 \Rightarrow \cdots \Rightarrow R^t \Rightarrow \cdots$ can be used to describe the process of community emergence in a network.

3.3. Fuzzy transitive method

Just as we have discussed above, R^*_{max} and R^*_{min} indicate that the information transfer is accomplished. Therefore, our fuzzy transitive method can be described as follows:

(1) $A \Rightarrow R$, transform the adjacency matrix of a graph into a fuzzy relation. Here, R is reflexive and symmetric based on Rule I, or irreflexive and symmetric based on Rule II respectively. Let $A = (a_{ij})_{n\times n}$, $R = (r_{ij})_{n\times n}$, if $a_{ij} = 0$, then $r_{ij} = 0$ based on Rule I, while if $a_{ij} = 0$, then $r_{ij} = 1$ based on Rule II. r_{ij} indicates the similarity between node v_i and node v_j ($R(v_i, v_j) = r_{ij}$), $i, j = 1, 2, \ldots, n$, which can be determined by the edge centralities that we have discussed in Section 2.

$$A = \begin{pmatrix} 0 & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & 0 \end{pmatrix} \Rightarrow R = \begin{cases} \begin{pmatrix} 1 & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & 1 \end{pmatrix}, & \text{Rule I} \\ \begin{pmatrix} 0 & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & 0 \end{pmatrix}, & \text{Rule II} \end{cases} \tag{4}$$

(2) $R \Rightarrow R^*_{max}$ and $R \Rightarrow R^*_{min}$ based on the two transitive rules respectively.

$$R = \begin{cases} \begin{pmatrix} 1 & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & 1 \end{pmatrix} \\ \begin{pmatrix} 0 & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & 0 \end{pmatrix} \end{cases} \Rightarrow \begin{matrix} R^*_{max} = \begin{pmatrix} 1 & \cdots & r'_{1n} \\ \vdots & \ddots & \vdots \\ r'_{n1} & \cdots & 1 \end{pmatrix}, & \text{Rule I} \\ R^*_{min} = \begin{pmatrix} 0 & \cdots & r''_{1n} \\ \vdots & \ddots & \vdots \\ r''_{n1} & \cdots & 0 \end{pmatrix}, & \text{Rule II} \end{matrix} \tag{5}$$

(3) $R^*_{max} \Rightarrow B(R^*_{max})$ and $R^*_{min} \Rightarrow B(R^*_{min})$, transform R^*_{max} and R^*_{min} into *Boolean relations*, $B(R^*_{max})$ and $B(R^*_{min})$ based on the following clustering regulations respectively. Let $R^*_{max} = (r'_{ij})_{n\times n}$, $B(R^*_{max}) = (R^*_{max})_\lambda$, $r'_{ij}(\lambda) = \begin{cases} 1, & r'_{ij} \ge \lambda \\ 0, & r'_{ij} < \lambda \end{cases}$. Let $R^*_{min} = (r''_{ij})_{n\times n}$, $B(R^*_{min}) = (R^*_{min})_\lambda$, then $r''_{ij}(\lambda) = \begin{cases} 1, & r''_{ij} \le \lambda \\ 0, & r''_{ij} > \lambda \end{cases}$, where $\lambda \in [0, 1]$.

(4) $B(R^*_{max}) \to R$, $B(R^*_{min}) \to R$, map the *Boolean relations*, $B(R^*_{max})$ and $B(R^*_{min})$ into R by $R(v_i, v_j) = r_{ij} = r'_{ij}(\lambda)$, or $R(v_i, v_j) = r_{ij} = r''_{ij}(\lambda)$ respectively and induce equivalence classes. Each corresponds to a community in the network. The equivalence class of v_i is the subset of elements in V that are equivalent to v_i : $[v_i]_R = \{v_j \in V | R(v_i, v_j)\}$ [29].

4. Results and discussion

In this section, we first compare our method with some state of the art methods of community detection such as the Louvain method [35], the OSLOM method [36] and the Infomap method [37] on the LFR benchmark [23], and then on real-world networks such as the Zachary club network [32], the college football network [2,16], the Dolphins network [39] and the co-appearance network of characters in the novel Les Miserables [40]. In addition, we give a detailed discussion on the choice of the parameter λ for community detection.

4.1. Testing on the LFR benchmark

Here, we test our method on the LFR benchmark [23] and produce random networks with $N = 200$, $k = 15$, $\max k = 50$, $\min c = 20$, $\max c = 50$. N denotes number of nodes in the networks, k indicates the average node degree, and $\max k$ is the maximum of node degree. mu is the mixing parameter that each vertex shares $1 - mu$ links with the other vertices of its community and mu with the rest nodes in the networks. $\min c$ is the minimum of community sizes, and $\max c$ is the maximum of community sizes [23].

We employ the normalized mutual information (NMI) [22] which can be described by the following equation to evaluate difference between the communities detected by our approach and the real communities in the networks.

$$NMI(A, B) = \frac{2 \sum_{i=1}^{c_A} \sum_{j=1}^{c_B} N_{ij} \log(\frac{N_{i.} N_{.j}}{N_{ij} N})}{\sum_{i=1}^{c_A} N_{i.} \log(\frac{N_{i.}}{N}) + \sum_{j=1}^{c_B} N_{.j} \log(\frac{N_{.j}}{N})} \tag{6}$$

where the real communities are in the rows, and the detected communities are in the columns. The number of vertices shared by the real community i and the detected community j is N_{ij}. c_A and c_B are the numbers of real communities and detected communities respectively, $N_{i.}$ and $N_{.j}$ are the sums over row i and column j of matrix N_{ij} respectively [22].

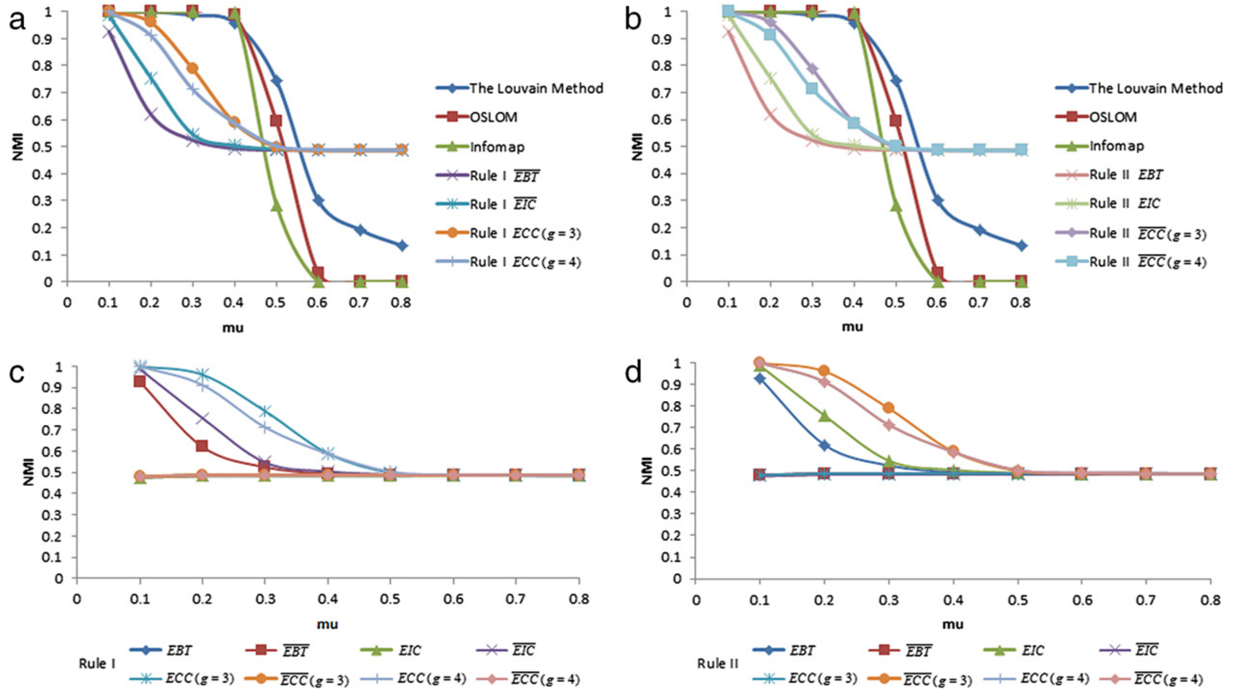

Fig. 1. Testing on the LFR benchmark for community detection. Each point is on average as a function of *mu*. (a) and (b) the results of our method and some state of the art methods for community detection. (c) and (d) show the results of our method based on Rule I and Rule II respectively.

Fig. 1 shows the results on the LFR benchmark for community detection. We find that our method based on Rule I gains better performance when the similarity between nodes of same groups is greater than that of different groups, while it is just opposite to Rule II. The results also show that our method outperforms the state of the art methods such as the Louvain method [35], the OSLOM method [36] and the Infomap method [37] when the mixing parameter of the LFR benchmark, *mu* is close to 0.8, and achieves bad performance when $mu \leq 0.5$. In addition, *ECC* outperforms other edge centralities such as *EBT* and *EIC* on measuring the similarity between nodes for community detection based on fuzzy transitive methods.

Figs. 2 and 3 show the results for the choice of the parameter λ for community detection based on Rule I and Rule II respectively. In Fig. 2, for $\overline{EBT}$ and $\overline{EIC}$, if $mu \to 0$, we obtain better results when $\lambda \to 1$, while for *ECC*, if $mu \to 0$, we achieve better performance when $\lambda \to 0$. In Fig. 3, for *EBT* and *EIC*, if $mu \to 0$, better results are obtained when $\lambda \to 0$, while for $\overline{ECC}$, if $mu \to 0$, better performance is achieved when $\lambda \to 1$.

For the choice of the parameter, λ, we take Rule I as an example, when $\lambda \to 0$, the whole network is a community, and when $\lambda \to 1$, each node is a community, while it is just opposite to the Rule II. Just as we have discussed in the section above, Rule I can correctly identify communities when the similarity between nodes of same communities is endowed with greater values than different communities, while it is just opposite to the Rule II. Granovetter [30,31] indicated that weak and strong similarities between nodes play different roles in real-world networks in that the weak similarity keeps the networks global integrated while the strong similarity maintains the communities. We can see that Rule I is consistent with the results of Granovetter [30,31]. In addition, no criterion can be given to estimate its value for the optimal partition of a network in general, since the choice of the parameter depends on the structure of networks strongly.

4.2. *The Zachary club network*

Zachary [32] considered 34 members of a karate club over two years. In his experiment, a split is observed between *the administrator of the club* (vertex 1) and *the club's instructor* (vertex 33), since the instructor started a new club and took a half of the members of the club with him. Zachary built a network between members for the club.

Here, we want to uncover the factions in the club by our method. The results show that the two well-known communities in Fig. 4 are detected and divided by dashed black lines, which are centered with *the administrator of the club* (vertex 1), and *the club's instructor* (vertex 33). Furthermore, the two primary groups can be subdivided into smaller subgroups, which are separated by dashed red lines. Fig. 4(a)–(c) shows the communities detected by our method with *ECC* $(g = 3)$ based on Rule I on the Karate club network by choosing different λ. Just as we have discussed in the section above, for Rule I, when $\lambda \to 0$, the whole network is a community, and when $\lambda \to 1$, each node is a community, i.e. we can extract more detailed subdivisions for a network when $\lambda \to 1$ based on Rule I, while it is just opposite to the *Rule II*. Fig. 5(a) and (b) shows the results of the choice of the parameter λ on the Karate club network for community detection.

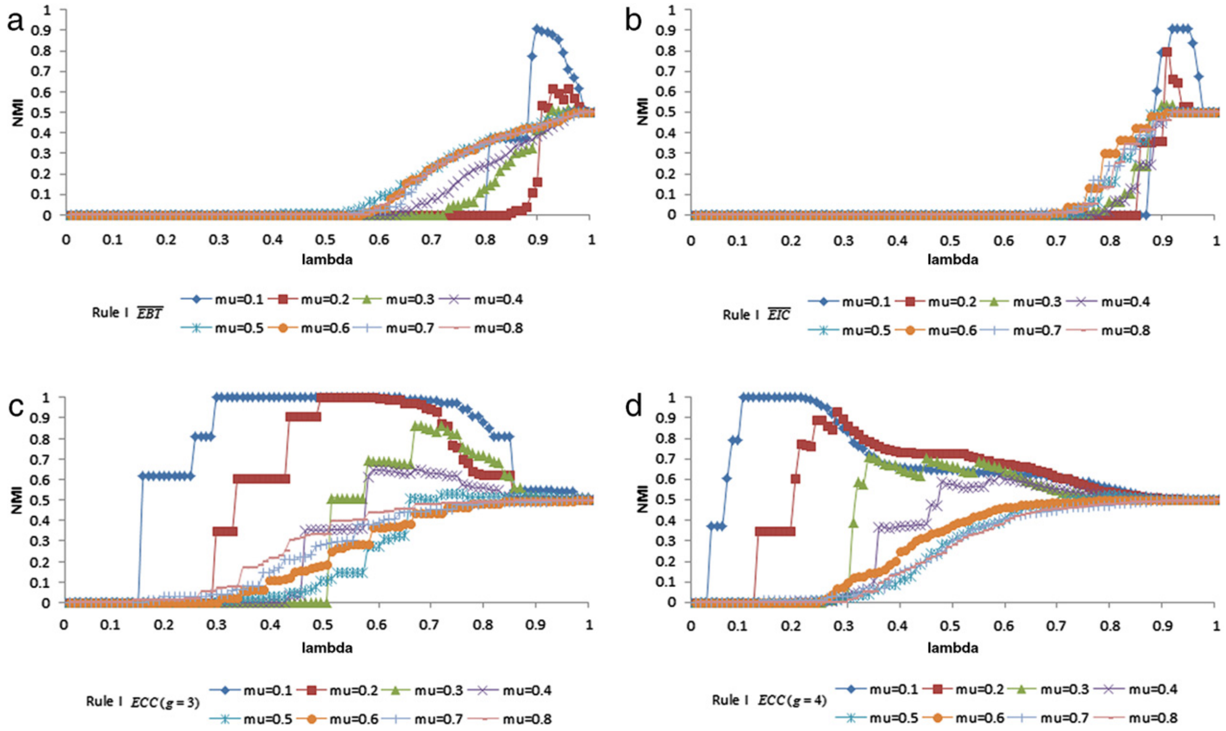

Fig. 2. Results of the choice of the parameter λ on the LFR benchmark for community detection based on Rule I.

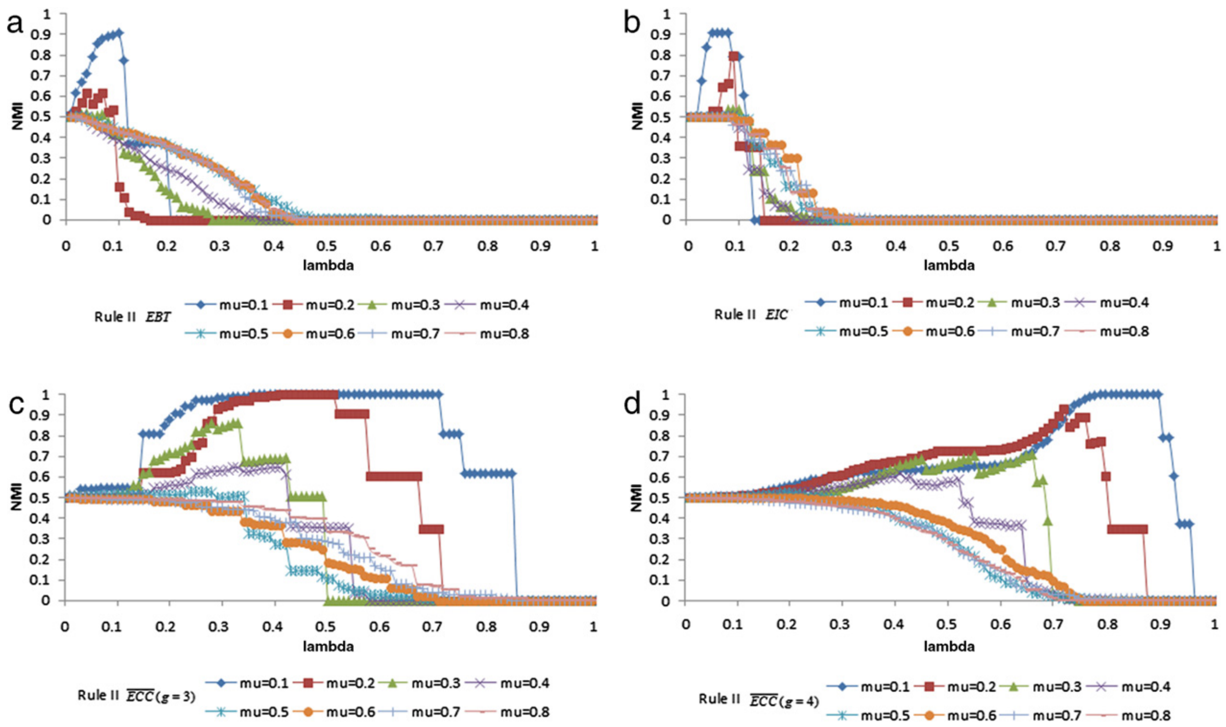

Fig. 3. Results of the choice of the parameter λ on the LFR benchmark for community detection based on Rule II.

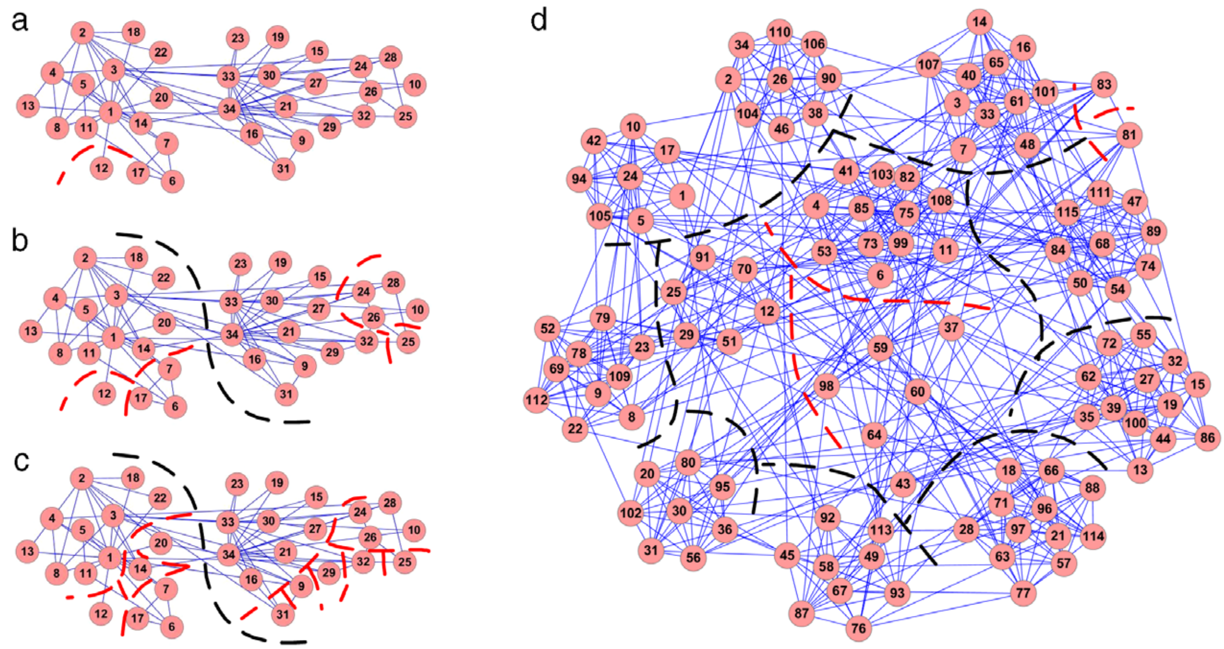

Fig. 4. Testing on the Karate club network and the college football network. (a)–(c) The communities detected by our method on the Karate club network based on Rule I and *ECC* ($g = 3$) with the different choice of λ. (d) The communities detected by our method on the college football network with ECC ($g = 4$) based on Rule I. The primary groups can be subdivided into smaller subgroups, which are separated by dashed red lines.

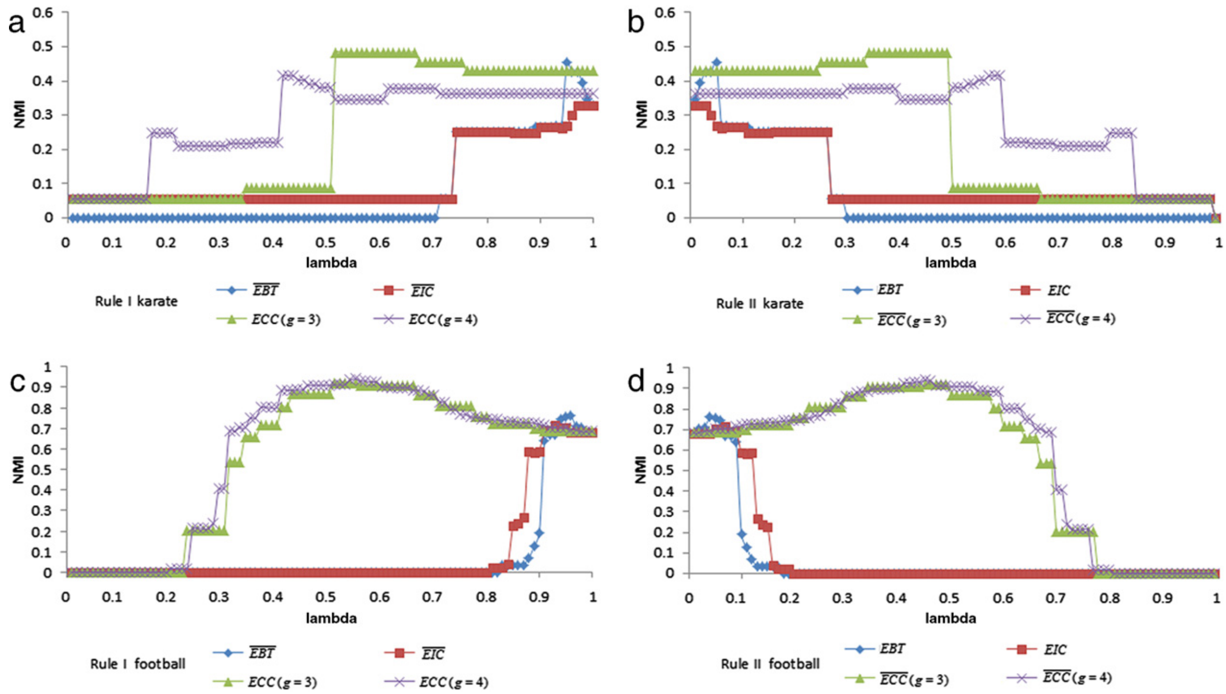

Fig. 5. Results of the choice of the parameter λ on the Karate club network and the college football network for community detection. (a) and (b) show the results on the Karate club network based on Rule I and Rule II respectively. (c) and (d) show the results on the college football network based on Rule I and Rule II respectively.

4.3. The college football network

Here, our approach tries to divide the college football network that represents the game of US college football league [2,16]. The nodes in the network are the 115 teams, and the links denote 616 games in the year. The teams are divided into

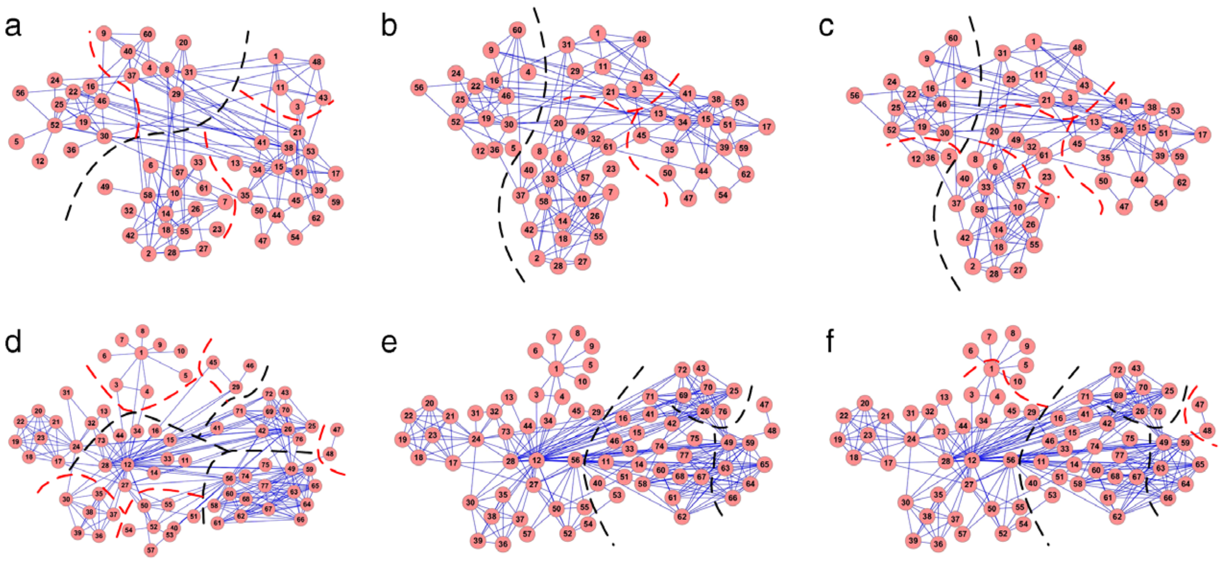

Fig. 6. Results on the Dolphins network and the co-appearance network of characters in the novel Les Miserables. (a) The communities detected by the Infomap method on the Dolphins network. (b) and (c) The communities detected by our method on the Dolphins network with the different choice of λ. (d) The communities detected by the Infomap method on the novel network. (e) and (f) The communities detected by our method on the novel network with the different choice of λ. The primary groups can be subdivided into smaller subgroups, which are separated by dashed red lines.

8–12 conferences and the games are generally more frequent between members of the same conference than different conferences. Fig. 4(d) shows the primary communities detected by our method with *ECC* ($g = 4$) based on *Rule I* on the college football network. Fig. 5(c) and (d) shows the results of the choice of the parameter λ on the college football network for community detection. The primary communities are separated by dashed black lines, which can be subdivided and denoted by dashed red lines.

4.4. The Dolphins network

In this subsection, we discuss the Dolphins network that contains 62 nodes with 159 links, which was studied by the biologist, David Lusseau [39]. The Dolphins form two subgroups, and David Lusseau found that the Dolphins in each subgroup share a similar age. Here, we use our approach to recover the subdivision of the Dolphins. Fig. 6(a) shows the communities identified by the Infomap method, and Fig. 6(b) and (c) shows the results detected by our approach with the different choice of λ. The two subgroups uncovered by both methods approximately agree with the findings observed by David Lusseau. Just as we have discussed on the Karate club network, our method obtains multi-resolutions for a network by choosing different λ, therefore, we can reveal more detailed subdivisions for the Dolphins network when $\lambda \to 1$ based on Rule I.

4.5. The novel network

Here, we also test our approach on the co-appearance network of characters in the novel Les Miserables [40]. In the network, a vertex corresponds to a character, and a link connects two characters if they co-appear in the novel. This network includes 77 nodes and 254 links. Fig. 6 displays the communities detected in the network. The identified subgroups reveal the underlying relationships of characters in the novel. Fig. 6(d) shows the results identified by the Infomap method, and Fig. 6(e) and (f) shows the results detected by our approach with the different choice of λ. The main subgroups revealed by both methods approximately agree with that of in the novel. Just as we have discussed on the Karate club network and Dolphins network, we also can extract different partitions for the novel network by choosing different λ.

5. Conclusions

In this paper, we use fuzzy transitive rules to reveal community structure in complex networks. By choosing different thresholds, our method finally can partition the network into several communities with multi-resolutions. The results show that our method based on Rule I gains better performance when the similarity between nodes of same groups is greater than that of different groups, while it is just opposite to Rule II. Our method achieves better performance than some state of the art methods when *mu* is close to 0.8. We think our results will provide a new insight for the understanding of network partition and community detection. In the future work, we will focus on the distortion problem of fuzzy clustering for community detection.

Acknowledgments

This work is supported by the National Foundation for Studying Abroad, the National Natural Science Foundation of China (Grant Nos. 61202175, 61202174), the Fundamental Research Funds for the Central Universities (Grant Nos. K5051303010, BDY181417, BDY021404), and the Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20120203120015).

References

[1] S.H. Strogatz, Exploring complex networks, Nature 410 (2001) 268–276.
[2] M. Girvan, M.E.J. Newman, Community structure in social and biological networks, Proc. Natl. Acad. Sci. 99 (2002) 7821–7826.
[3] S. Fortunato, Community detection in graphs, Phys. Rep. 486 (2010) 75–174.
[4] A. Lancichinetti, S. Fortunato, Consensus clustering in complex networks, Sci. Rep. 2 (2012) 1–7.
[5] M.E.J. Newman, Detecting community structure in networks, Eur. Phys. J. B 3 (2004) 8321–8330.
[6] M.E.J. Newman, Modularity and community structure in networks, Proc. Natl. Acad. Sci. 103 (2006) 8577–8582.
[7] M. Rosvall, C.T. Bergstrom, An information-theoretic framework for resolving community structure in complex networks, Proc. Natl. Acad. Sci. 104 (2007) 7327–7331.
[8] Z. Li, S. Zhang, R.S. Wang, X.S. Zhang, L. Chen, Quantitative function for community detection, Phys. Rev. E 77 (2008) 036109.
[9] P.G. Sun, L. Gao, Y. Yang, Maximizing modularity intensity for community partition and evolution, Inform. Sci. 236 (2013) 83–92.
[10] M.E.J. Newman, M. Girvan, Finding and evaluating community structure in networks, Phys. Rev. E 69 (2003) 026113.
[11] S. Fortunato, V. Latora, M. Marchiori, Method to find community structures based on information centrality, Phys. Rev. E 70 (2004) 056104.
[12] F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, D. Parisi, Defining and identifying communities in networks, Proc. Natl. Acad. Sci. 101 (2004) 2658–2663.
[13] S. Fortunato, M. Barthélemy, Resolution limit in community detection, Proc. Natl. Acad. Sci. 104 (2007) 36–41.
[14] S.H. Zhang, R.S. Wang, X.S. Zhang, Identification of overlapping community structure in complex networks using fuzzy c-means clustering, Physica A 374 (2007) 483–490.
[15] P.G. Sun, L. Gao, S. Han, Identification of overlapping and non-overlapping community structure by fuzzy clustering, Inform. Sci. 181 (2011) 1060–1071.
[16] G. Palla, I. Derenyi, I. Farkas, T. Vicsek, Uncovering the overlapping community structure of complex networks in nature and society, Nature 435 (2005) 814–818.
[17] J.P. Bagrow, Communities and bottlenecks: Trees and treelike networks have high modularity, Phys. Rev. E 85 (2012) 066118.
[18] G.K. Orman, V. Labatut, H. Cherifi, Comparative evaluation of community detection algorithms: a topological approach, J. Stat. Mech. Theory Exp. 09 (2012) P08001.
[19] J. Xie, S. Kelley, B.K. Szymanski, Overlapping Community Detection in Networks: the State of the Art and Comparative Study, ACM Comput. Surv. 45 (2013) 1–37.
[20] B. Mirkin, S. Nascimento, Additive spectral method for fuzzy cluster analysis of similarity data including community structure and affinity matrices, Inform. Sci. 183 (2012) 16–34.
[21] P. De Meo, E. Ferrara, G. Fiumara, A. Provetti, Enhancing community detection using a network weighting strategy, Inform. Sci. 222 (2013) 648–668.
[22] L. Danon, Jordi Duch, Albert Diaz-Guilera, Alex Arenas, Comparing community structure identification, J. Stat. Mech. Theory Exp. 09 (2005) P09008.
[23] A. Lancichinetti, S. Fortunato, F. Radicchi, Benchmark graphs for testing community detection algorithms, Phys. Rev. E 78 (2008) 046110.
[24] S. Wasserman, K. Faust, Social Network Analysis, Cambridge University Press, Cambridge, 1994.
[25] L. Freeman, A set of measures of centrality based upon betweenness, Sociometry 40 (1977) 35–41.
[26] V. Latora, M. Marchiori, A measure of centrality based on the network efficiency, New J. Phys. 9 (2007) 1–11.
[27] V. Latora, M. Marchiori, Vulnerability and protection of critical infrastructures, Phys. Rev. E 71 (2005) 015103R.
[28] D. Watts, S. Strogatz, Collective dynamics of small-world networks, Nature 393 (1998) 440–442.
[29] M.S. Stachowicz, M.E. Kochanska, Graphic interpretation of fuzzy sets and fuzzy relations, in: A. Ballester, D. Cardus, E. Trillas (Eds.), Mathematics at the Service of Man. Based on materials of Second World Conf., Universidad Politecnica Las Palmas, Spain, 1982.
[30] M.S. Granovetter, The strength of weak ties, Am. J. Sociol. 78 (1973) 1360–1380.
[31] M.S. Granovetter, Economic action and social structure: The problem of embeddedness, The Sociology of Economic Life 91 (1985) 481–510.
[32] W.W. Zachary, An information flow model for conflict and fission in small groups, J. Anthropol. Res. 33 (1977) 452–473.
[33] P.G. Sun, Methods to find community based on edge centrality, Physica A 9 (2013) 1977–1988.
[34] P.G. Sun, Weighting links based on edge centrality for community detection, Physica A 394 (2014) 346–357.
[35] Vincent D Blondel, Jean-Loup Guillaume, Renaud Lambiotte, Etienne Lefebvre, Fast unfolding of communities in large networks, J. Stat. Mech. Theory Exp. 10 (2008) P10008.
[36] A. Lancichinetti, F. Radicchi, J.J. Ramasco, S. Fortunato, Finding statistically significant communities in networks, PLoS ONE 6 (2011) e18961.
[37] M. Rosvall, C.T. Bergstrom, Multilevel compression of random walks on networks reveals hierarchical organization in large integrated systems, PLoS ONE 6 (4) (2011) e18209.
[38] Y.Y. Ahn, J. Bagrow, S. Lehmann, Link communities reveal multiscale complexity in networks, Nature 466 (2010) 761–764.
[39] D. Lusseau, K. Schneider, O.J. Boisseau, P. Haase, E. Slooten, S.M. Dawson, The bottlenose dolphin community of Doubtful Sound features a large proportion of long-lasting associations, Behav. Ecol. Sociobiol. 54 (2003) 396–405.
[40] D.E. Knuth, The Stanford GraphBase: A Platform for Combinatorial Computing, Addison-Wesley, Reading, MA, 1993.