Spectral Efficiency of OFDM Systems With Random Residual CFO

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) over wireless channels is sensitive to carrier frequency offset (CFO), which destroys orthogonality amongst sub-carriers, giving rise to inter-carrier interference (ICI). Different techniques are available for estimating and compensating for the CFO at the receiver. However, in practice, a residual CFO remains at the receiver after CFO estimation, where the estimation accuracy depends primarily on the fractions of time and power used by the estimator. In this paper, we propose to measure the efficiency of OFDM systems with CFO estimation errors in terms of the spectral efficiency, which accounts for both, the degradation in signal-to-interference plus noise ratio (SINR) due to the residual CFO, and the penalty of the extra power and spectral resources allocated to achieve the desired CFO estimation accuracy. New accurate expressions are derived for the spectral efficiency of wireless OFDM systems in the presence of residual CFO and frequency-selective multipath fading channel. These are used to compare between two common CFO estimation methods in wireless OFDM systems, namely, the cyclic prefix based and the training symbols based CFO estimation techniques for fixed and variable pilot power. These results are further extended to include OFDM systems with transmit diversity techniques. In addition, the impact of imperfect channel estimation on the overall spectral efficiency is also included. Numerical results reveal that the cyclic prefix based CFO technique is more efficient than the training symbols based CFO technique when perfect channel state information (CSI) is known blindly at the receiver. Furthermore, fixed pilot power results in a spectral efficiency ceiling as SNR increases, whereas spectral efficiency increases with SNR without bound in the equal pilot and signal powers case.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), signal to interference plus noise ratio (SINR), inter-carrier interference (ICI), carrier frequency offset (CFO), multi-path fading channels, spectral efficiency.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is the method of choice for combating frequency-selective multipath fading channels, and achieving high spectral efficiency in wireless communication systems. With the aid of sufficient cyclic prefix, inter-symbol interference (ISI) due to multipath fading is completely avoided without the need for complicated equalizers.

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Since OFDM systems divide the available wide-band amongst a set of orthogonal overlapping sub-carriers, it is supremely susceptible to time and frequency synchronization errors which results in destroying the orthogonality amongst sub-carriers, bringing about inter-carrier interference (ICI) which can deteriorate the performance of OFDM systems.

The estimation and removal of carrier frequency offset (CFO) from the received symbols is critical in OFDM systems. Several data aided (DA) and non data aided (NDA) CFO estimation techniques have been proposed in the literature (e.g., [1]–[8]). DA CFO estimation base the estimate on a training set of data. Therefore, this kind of estimators lead to higher estimation accuracy and less receiver complexity, at the expense of reduced spectral efficiency because of the periodic transmission of known pilot sequences. On the other hand, NDA CFO estimation blindly estimates the CFO based on the statistics of the received data without the aid of training sequence. Therefore, NDA estimators lead to lower estimation accuracy and higher receiver complexity at the merit of higher system throughput. However, despite the fact that CFO estimation and compensation is performed at the receiver, a random residual CFO still exists which increases at low number of training symbols and/or low to medium signal to noise ratio (SNR) region. Further, this residual CFO might degrade the OFDM system performance.

It is common in most previous research to measure the performance degradation due to non-random CFO in terms of the average signal-to-interference plus noise ratio (SINR) (e.g., [2], [9], [10]). A lower and upper bound on the average SINR analysis in the presence of non-random CFO for multipath fading channels is presented in [2], [10], respectively. Besides, [9] derived an exact expression for evaluating the average SINR in the presence of non-random CFO over multipath fading channels.

Recent analysis on bit error rate (BER) deterioration due to random and non-random CFO is considered in [11]–[18]. In [16], accurate ICI model in additive white Gaussian noise (AWGN) channels based on infinite series approximations and two dimensional characteristic functions for BER of OFDM systems in the presence of non-random CFO is addressed. In [15], BER of OFDM systems in the presence of non-random CFO for multipath fading channels is presented, the ICI model is approximated by a summation of two independent complex Gaussian random variables (RVs). The authors in [17] presented approximate ICI models which consider only the correlation from the two adjacent sub-carriers, while modeling the ICI from other sub-carriers as a pure Gaussian RVs ignoring their correlation with the useful signal component. In [19], an accurate characterization of the effects of non linear devices in OFDM signals is addressed, which then used to study the accuracy of using the Gaussian approximation of OFDM signals with low and high number of sub-carriers. BER analysis of binary phase shift keying (BPSK) OFDM with random residual frequency offset has been addressed in [18] based on the assumption that the residual CFO being very small.

This paper proposes to quantify the performance of wireless OFDM systems with different practical CFO estimation methods in terms of the spectral efficiency, which is a measure of the average number of information bits that can be transmitted over the channel per unit time per unit bandwidth (with negligible error probability). We present accurate expressions for the spectral efficiency of OFDM wireless systems which take into account both, the degradation in SINR due to the residual CFO estimation errors, and the penalty due to the addition of cyclic prefix as well as the periodic transmission of training symbols used by the CFO estimators (in the DA case).

More relevant to our work are [20]–[25], where capacity and spectral efficiency of OFDM systems were presented. Athaudage *et al.* [25] addressed the capacity of OFDM systems in the presence of non-random CFO, whereas [20] considered the OFDM system capacity in slow fading channels with infinite number of sub-carriers. In addition, [22] focused on the effect of non-random CFO on the spectral efficiency of adaptive M-ary quadrature amplitude modulation (M-QAM) in OFDM systems with imperfect channel state information. Further, in [24], spectral efficiency of multi-carrier code division multiple access (MC-CDMA) with non-random CFO over multipath fading is presented. However, all existing research tackles spectral efficiency when influenced by a given non-random CFO. In practice, the CFO affecting the performance of OFDM systems is random. For instance, when transceivers use stable crystal oscillator and omit CFO estimation. Different oscillators (for different users) could have slightly different frequencies which can also depend on their temperatures [18]. Our goal is to find the ensemble average spectral efficiency of a user (not specific user with constant CFO).

The main contributions of this paper are summarized as follows:

- Motivated by [26], a new modified formula for evaluating the spectral efficiency of OFDM systems is presented. This formula accounts for the extended system resources of power and bandwidth wasted while substituting not only for the cyclic prefix inserted to counteract delay spread, but also for the training sequence transmitted to estimate and compensate for the CFO. It is to be emphasized that increasing the cyclic prefix and pilot sequence size assures avoiding ISI and decreases estimation error, respectively. However, these are considered as a loss in the system resources as they do not constitute useful information. Therefore, too little training symbols and the CFO is improperly learned, too much training is a wast of resources. Hence, the trade-off between these two conflicting factors is explicit in the modified formula for the spectral efficiency of OFDM systems.
- Taking into account accurate statistical models for the ICI, new exact expressions for the spectral efficiency of OFDM systems in the presence of residual CFO and multipath fading channels are derived, namely, we address three cases for the frequency offset; the classical non-random CFO, random CFO, random residual CFO when applying a CFO estimator. In addition, the impact of imperfect channel estimation on the spectral efficiency of OFDM systems is considered. Also, OFDM systems with transmit diversity is addressed. As a result, the new expression for the OFDM system spectral efficiency requires only a single numerical integration in the case of non-random CFO, which can be accurately simplified to a finite summation using the well known Gauss-Laguerre quadrature method. This is a huge reduction in the required computational complexity when compared with the direct method, which requires as much as the number of sub-carriers numerical integrations.
- Two well known CFO estimators are chosen from the literature and used to estimate their spectral efficiency. Here, each estimator variance depends on the training sequence size and the pilot power. In this paper, variable and fixed pilot power are considered. When the pilot power equals the signal power, the estimator accuracy improves as SNR increases. This is contrary to most previously published work where they measure the performance analysis of a system with fixed estimation error variance assuming that the pilot power is fixed regardless of the signal power. This assumption is commonly used when no explicit expression is available for the estimator variance.

The structure of the rest of the paper is as follows. In Section II, we introduce the system model. In Section III, the instantaneous SINR is addressed. In Section IV, spectral efficiency analysis is considered. In Section V we include imperfect channel estimation. OFDM systems with transmit diversity is introduced in Section VI. numerical results are provided in Section VII. Finally, Section VIII concludes the paper.

II. System Model

The transmitted baseband OFDM signal can be written as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j2\pi \frac{n}{T} t}, \qquad t \in [-T_G,\, T] \tag{1}$$

where N is the useful number of sub-carriers, T_G is the length of the cyclic prefix (CP), and T is the length of the OFDM symbol duration. Here $T = N/R$ and R is the symbol rate of the input data to be transmitted. Without loss of generality, we normalize the time such that $R = 1$, and therefore, $T = N$.

In (1), a_n is the M-ary complex information bearing symbol on the n^{th} sub-carrier. It is assumed that a_n, $n \in [0, N-1]$ are zero mean complex RVs with $\mathbb{E}[a_n a_m^*] = \delta(n-m)\mathcal{E}_n \,\forall\, n$, where $\delta(.)$ denotes the Dirac's delta function, $\mathbb{E}(.)$ is the expectation operator, and $\mathcal{E}_n$ is the energy of the transmitted symbol on the n^{th} sub-carrier.

The channel impulse response of the time-invariant frequency-selective multipath Rayleigh fading channel is

$$h(\tau) = \sum_{l=0}^{L-1} g_l \delta(\tau - \tau_l) \tag{2}$$

where g_l and τ_l are the complex amplitude and propagation delay of the l^{th} path, respectively, $\tau_0 < \tau_1 < \ldots < \tau_{L-1}$. It is assumed that g_l, $l \in [0, L-1]$ are zero mean complex Gaussian random variables (RVs) with $\mathbb{E}[g_l g_k^*] = \delta(l-k)\gamma_l \, \forall \, l$, where $\gamma_l = \mathbb{E}[|g_l|^2]$ assuming that the power is normalized such that $\sum_{l=0}^{L-1} \gamma_l = 1$.

Inter-symbol interference is avoided by choosing the cyclic prefix duration $T_G > \tau_{L-1}$. Therefore, assuming perfect time synchronization and a normalized residual CFO of Δ, the received signal after removing the cyclic prefix is

$$r(t) = \frac{1}{\sqrt{N}} e^{j2\pi \frac{\Delta}{T} t} \sum_{l=0}^{L-1} g_l \sum_{n=0}^{N-1} a_n e^{j2\pi \frac{n}{T}(t-\tau_l)} + w(t) \tag{3}$$

where $w(t)$, $t \in [0, T]$ is the complex additive white Gaussian noise (AWGN) with zero mean and two sided power spectral density $\mathcal{N}_0/2$ per dimension.

The N-point FFT samples at the receiver are

$$\begin{aligned} y_p &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r\left(k\frac{T}{N}\right) e^{-j2\pi \frac{k}{N} p}, \quad p = 0, 1, \ldots, N-1 \\ &= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} g_l a_n e^{j\frac{2\pi}{N}(n-p+\Delta)k} e^{-j2\pi n \frac{\tau_l}{T}} + w_p \end{aligned} \tag{4}$$

where w_p is a zero mean complex Gaussian RV with variance $\mathcal{N}_0$.

A. CFO Estimators

Different DA and NDA algorithms for CFO estimation have been proposed in the literature (e.g., [1]–[8]). Schmidl *et al.* [8] divided the training symbols into two identical sets and used them to estimate the CFO. Authors in [3], [5] based the CFO estimation on dividing the pilot sequence into $K \geq 2$ identical parts which lead to a better estimation accuracy at the cost of higher complexity. On the other hand, [1] used the cyclic prefix to estimate CFO without the aid of training symbols. Further, [4] and [6] blindly estimate the CFO without the aid of training data based on the statistics of the received symbols. Therefore, these CFO estimators trade-off between throughput and estimation accuracy.

The asymptotic property of the maximum likelihood estimation (MLE) reveals that if the regularity conditions are satisfied [27], then the MLE of an unknown parameter Θ is asymptotically Gaussian distributed as $\widehat{\Theta} \sim \mathcal{N}\left(\Theta, \mathcal{F}^{-1}(\Theta)\right)$, where $\mathcal{F}(\Theta)$ is the Fisher information. Note that this distribution holds only when the training sequence size approaches infinity. However, this is not the case in most practical estimation problems and CFO estimation is of no exception. Hence, in most practical scenarios, the estimator variance is higher than the Cramer-Rao bound (CRB), and asymptotically attains the bound as the training symbols and/or pilot power approaches infinity [27, eqs. (3.19-3.20)]. Therefore, assuming that the CFO estimator is unbiased, the distribution of the normalized residual CFO (Δ) estimator adopted in this paper is $\Delta \sim \mathcal{N}\left(0, \sigma_\Delta^2\right)$, which came from the assumption that $f_c = \widehat{f_c} + \Delta$, where f_c is the true normalized carrier frequency offset, $\widehat{f_c}$ is the estimated normalized carrier frequency offset. The estimator variance, σ_Δ^2 depends on the method of CFO estimation used. For instance, when the cyclic prefix is used for CFO estimation, the CFO estimator variance introduced by [1], [2] is given as

$$\sigma_\Delta^2 = \frac{1}{\pi^2 T_G \, \mathrm{SNR}_E} \tag{5}$$

where SNR_E is the signal-to-noise ratio during the estimation phase. On the other hand, when a training sequence is used for CFO estimation, the CFO estimator variance given by [3] is given as

$$\sigma_\Delta^2 = \frac{3}{2\pi^2 \nu N \left(1 - \frac{1}{K^2}\right) \mathrm{SNR}_E} \tag{6}$$

where ν is the number of OFDM blocks used for CFO estimation,[1] and K is the number of identical parts in the training sequence. Observe that when $K = 2$ in equation (6), it reduces to the same estimator introduced by [8].

It is to be emphasized that these are just two examples considered from the literature to introduce the trade-off between the effect of the loss (in terms of spectral efficiency) in the cyclic prefix and the periodic transmission of training symbols used for CFO estimation, which depend on the chosen method of CFO estimation. It is evident from (5) and (6) that increasing the training symbols length and/or SNR_E improves the estimation accuracy. However, periodic transmission of long cyclic prefixes as well as training symbols does not constitute information. Hence, resulting in spectral efficiency loss. Therefore, a new expression for the evaluation of spectral efficiency is proposed, which takes into account the trade-off between these conflicting factors.

Note that both CFO and channel are jointly estimated in practice. Therefore, channel estimation errors and CFO estimation errors should be jointly addressed in the performance analysis of OFDM systems. However, for simplicity, many CFO and channel estimators published in the literature treat the two problems separately. For instance, CFO is estimated assuming perfect channel estimation, and channel is estimated assuming perfect CFO estimation. In addition, it is seen that CFO and channel estimators can be decoupled and treated separately when addressed together. Therefore, frequency offset must be estimated and compensated for first, then performing channel estimation. Recently, joint channel and CFO estimation have been addressed. However, no explicit estimator variances have been found due to the complexity of the system as an iterative

[1] In general, ν OFDM training blocks is sent every V OFDM blocks to estimate the CFO first, then estimating the channel after compensating for the CFO.

estimation techniques were proposed. Therefore, since the two chosen CFO estimators assume perfect channel estimation, we address the effect of frequency offset estimation errors on the performance of OFDM systems. Moreover, the effect of joint estimation errors on the performance of OFDM systems is discussed in Section V, where the joint estimation error variance for the CFO and channel are approximated by the CRB.

III. The Instantaneous SINR

Equation (4) can be rewritten in the following compact form

$$y_p = a_p c\,(p,\,p)\,H_p + \sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n,\,p) H_n + w_p \tag{7}$$

where $\{H_0,\,H_1,\,\ldots,\,H_{N-1}\}$ are jointly complex Gaussian random variables with zero means and correlations $\mathbb{E}[H_n H_m^*] = \sum_{l=0}^{L-1} \gamma_l e^{-j2\pi(n-m)\frac{\tau_l}{T}}$. The frequency correlation function $\mathbb{E}[H_n H_m^*]$ in the case of exponential power delay profile is given by [28]

$$\mathbb{E}\left[H_n H_m^*\right] = \frac{1}{1 - j2\pi\sigma\,(n-m)} \tag{8}$$

where σ is the mean delay spread.

The function $c(n,\,p)$ in (7) is given by

$$\begin{aligned} c(n,\,p) &= \frac{1}{N}\sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(n-p+\Delta)k} \\ &= \frac{\operatorname{sinc}\,(n-p+\Delta)}{\operatorname{sinc}\left(\frac{1}{N}\,(n-p+\Delta)\right)} e^{j\pi\left(1-\frac{1}{N}\right)(n-p+\Delta)}. \end{aligned} \tag{9}$$

It is worth noting that in the ICI free case $\Delta = 0$, and therefore $c(n,\,p) = \delta(n-p)$.

The first term in (7) includes the desired signal, whereas the second term is the ICI due to CFO. Therefore, assuming one tap equalizer and perfect channel state information (CSI) at the receiver (the effect of imperfect CSI is considered in Section V), the p^{th} decision variable may be written as

$$\widehat{a_p} = a_p + \frac{\sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n,\,p) H_n + w_p}{c(p,\,p) H_p}. \tag{10}$$

It can be verified that the N-dimensional random vector $\boldsymbol{H} = [H_0,\,H_1,\,\ldots,\,H_{N-1}]^T$ is a proper complex Gaussian random vector having a probability density function (PDF) given as

$$f_{\boldsymbol{H}}(\boldsymbol{x}) = \frac{1}{\pi^N\,|\Lambda|} \exp\left(-\boldsymbol{x}^\dagger \Lambda^{-1} \boldsymbol{x}\right) \tag{11}$$

where $\dagger$ denotes the complex conjugate transposition (Hermitian), and $\Lambda = \mathbb{E}[\boldsymbol{H}\boldsymbol{H}^\dagger]$ is the complex covariance matrix with entries given in (8).

Most previously published work on the performance analysis of OFDM systems approximate the ICI component $\sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n,\,p) H_n$ by pure complex Gaussian RVs and ignore their cross-correlation with the useful signal component. In this paper, the ICI term $\sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n,\,p) H_n$ is modeled as conditionally complex Gaussian RVs (with random variance) when conditioned on $\boldsymbol{H}$ and Δ. This approximation is justified by the central limit theorem which is valid here since the ICI term $\sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n,\,p) H_n$ becomes a sum of independent RVs when $\boldsymbol{H}$ and Δ are given. However, it is to be emphasized at this point that this is completely distinct from the classical Gaussian assumption approximation methods where this sum is usually approximated by a pure Gaussian RVs with non-random variance.

Therefore, when the receiver treats the ICI as an additional AWGN when conditioned on $\boldsymbol{H}$ and Δ, the decision variables (7) become conditionally complex Gaussian RVs. The conditional variances is

$$\operatorname{Var}\left(\widehat{a_p} \mid \boldsymbol{H},\,\Delta\right) = \frac{\sum_{n\neq p} \mathcal{E}_n\,|c(n,\,p)|^2\,|H_n|^2 + \mathcal{N}_0}{|c\,(p,p)|^2\,\left|H_p\right|^2} \tag{12}$$

where $\sum_{n\neq p} = \sum_{\substack{n=0\\n\neq p}}^{N-1}, p = 0,\,1,\,\ldots,\,N-1.$

The instantaneous SINR over the p^{th} sub-carrier for a given realization of $\boldsymbol{H}$ and Δ becomes a RV given as

$$\text{SINR}_p = \frac{|c\,(p,\,p)|^2\,\left|H_p\right|^2}{\sum_{n\neq p} |c(n,\,p)|^2\,|H_n|^2 + \frac{1}{\text{SNR}_D}} \tag{13}$$

where $\text{SNR}_D = \frac{N}{N+T_G+T_E}\text{SNR}$ is the effective normalized useful symbol signal-to-noise ratio, T_E is the effective per OFDM block training symbols length used to estimate the CFO,[2] and SNR is the signal-to-noise ratio in AWGN channel in the absence of ICI.

Note that for conservation of time and energy, $(N + T_G + T_E)$ SNR $= T_G\text{SNR}_G + T_E\text{SNR}_E + N\,\text{SNR}_D$, where SNR_G is the signal-to-noise ratio wasted while substituting for the cyclic prefix. Note also that $\text{SNR}_D = \frac{\mathcal{E}}{\mathcal{N}_0}$ assuming that[3] $\mathcal{E}_1 = \mathcal{E}_2 = \ldots = \mathcal{E}_{N-1} = \mathcal{E}$. [Equation (14), shown at the bottom of the page].

[2]To introduce the loss in the extended time due to the addtion of cyclic prefix and training symbols per OFDM block, we equally divide the OFDM training sequence length among the $(V-\nu)$ OFDM data blocks and consider that to be the extended time, $T_E = \frac{\nu N}{V-\nu}$.

[3]Note that without channel state information at the transmitter (CSIT), the transmitted power should be spread uniformly amongst all sub-carriers.

$$\mathbb{E}\left[\log_2\left(1 + \frac{|c\,(p,\,p)|^2\,\left|H_p\right|^2}{\sum_{n\neq p} |c(n,\,p)|^2\,|H_n|^2 + \frac{1}{\text{SNR}_D}}\right)\right] = \log_2 e \int_0^\infty \frac{1}{z}\mathbb{E}\left(e^{-z\sum_{n\neq p}|c(n,p)|^2|H_n|^2} - e^{-z\sum_{n}|c(n,p)|^2|H_n|^2}\right) e^{-z/\text{SNR}_D} dz \tag{14}$$

IV. Spectral Efficiency Analysis

Spectral efficiency is the average number of information bits which can be transmitted with an arbitrary small error probability per unit time per unit Hertz. Therefore, inspired by [26], the new modified spectral efficiency formula accounts for the extended system resources of power and bandwidth wasted while substituting for both, the cyclic prefix inserted to mitigate ISI, and the training sequence transmitted to estimate and compensate for the CFO. Therefore, assuming perfect CSI is available only at the receiver, and that each sub-carrier is decoded independently, the spectral efficiency of OFDM systems for a given CFO $\triangle$ is

$$\mathcal{G}(\triangle) = \frac{1-\alpha}{N+T_G}\sum_{p=0}^{N-1}\mathcal{R}_p(\triangle) \tag{15}$$

where α is the fraction of time resources allocated to CFO and/or channel estimation,[4] $\mathcal{R}_p(\triangle)$ is the conditional average (ergodic) mutual information over the p^{th} sub-carrier for a given $\triangle$

$$\begin{aligned}\mathcal{R}_p(\triangle) &= \mathbb{E}_{\boldsymbol{H}|\triangle}\left\{\log_2\left(1+\mathrm{SINR}_p\right)\right\}\\ &= \mathbb{E}\left[\log_2\left(1+\frac{|c(p,\,p)|^2\left|H_p\right|^2}{\sum\limits_{n\neq p}|c(n,\,p)|^2\,|H_n|^2+\frac{1}{\mathrm{SNR}_D}}\right)\right].\end{aligned} \tag{16}$$

Note that Shannon's Gaussian channel capacity formula is used in (16). In reality where practical codes and modulation schemes are used, it is common to use the formula $\log_2\left(1+\frac{\mathrm{SINR}_p}{\Gamma}\right)$ instead, where Γ is the SNR gap which denotes the amount of extra coding gain needed to achieve Shannon capacity.

As far as the evaluation of the average in (16) is concerned, direct methods to calculate such averages which involve N RVs require at least N-fold integrals over $\boldsymbol{H}$. In order to reduce the required computational complexity, we rely on the following lemma to transform (16) into a more convenient form which facilitates the calculation of the required average by using known results from the theory of Gaussian quadratic forms.

Lemma 1 [29, eq. (6)]: For any $x \geq 0$ then

$$\log(1+x) = \int_0^\infty \frac{1}{z}\left(1-e^{-zx}\right)e^{-z}dz. \tag{17}$$

[4]The loss induced by the addition of training symbols may be seen as $\frac{V-\nu}{V}$. Therefore, by letting $\alpha=\frac{\nu}{V}$, the loss reduces to $(1-\alpha)$. Note that $\alpha=0$ if no training sequence was transmitted for CFO and channel estimation.

Corollary 2: A useful corollary to Lemma 1, for any x, $y \geq 0$ then

$$\log\left(1+\frac{x}{y}\right) = \int_0^\infty \frac{1}{z}\left(e^{-zy}-e^{-z(x+y)}\right)dz. \tag{18}$$

Now, using (18), we can express (16) to get (14).

In (14), the interchange of the integration and expectation operations is justified by the Fubini's theorem [30, pp. (200)], which is applicable because the integrand is non negative.

When comparing equations (16) and (14), it is seen that the proposed formula has the desirable property that the RVs $\left\{|c(n,\,p)|^2|H_n|^2,\ n=0,\,1,\,\ldots,\,N-1\right\}$ appear only as a linear combinations at the exponent. Let us define the joint moment generating function (MGF) of these RVs as [9]

$$\begin{aligned}\mathcal{M}(z_0,\,z_1,\,\ldots,\,z_{N-1}) &= \mathbb{E}\left[e^{-\sum\limits_n z_n|c(n,\,p)|^2|H_n|^2}\right]\\ &= \frac{1}{\left|I+\mathrm{diag}\left(z_0,\,z_1,\,\ldots,\,z_{N-1}\right)\Lambda_p\right|}\end{aligned} \tag{19}$$

where $\sum\limits_n = \sum_{n=0}^{N-1}$, I is the $N\times N$ identity matrix, diag(.) is the $N\times N$ diagonal matrix, $\Lambda_p = C_p\Lambda$ where $C_p = \mathrm{diag}\left(|c(0,\,p)|^2,\,|c(1,\,p)|^2,\,\ldots,\,|c(N-1,\,p)|^2\right)$, and Λ is the complex covariance matrix of the channel vector with entries given in (8).

Plugging (19) into (14) yields the new explicit expression for the conditional mutual information (20), shown at the bottom of the page.

To summarize, (15) and (20) give the exact conditional spectral efficiency of OFDM system for a given CFO, which involves only one single integration.

Note that to gain valuable insights into the impact of model parameters on the spectral efficiency of OFDM system, one can apply Jensen's inequality to (16) for a closed-form upper bound. Therefore, for the sake of a closed-form expression of the average $\mathbb{E}\{\mathrm{SINR}_p\}$, Jensen's inequality can be applied to get an upper bound to this average as given in [10, eq. (33)]. Therefore, the conditional average mutual information over the p^{th} sub-carrier for a given $\triangle$ can be upper bounded as

$$\begin{aligned}\mathcal{R}_p(\triangle) &\leq \log_2\left(1+\mathbb{E}\left\{\mathrm{SINR}_p\right\}\right)\\ &\leq \log_2\left(\frac{\mathrm{SNR}_D+1}{\mathrm{SNR}_D\left(1-\left|\frac{\sin(\pi\triangle)}{N\sin\left(\frac{\pi}{N}\triangle\right)}\right|^2\right)+1}\right).\end{aligned} \tag{21}$$

In practice, the CFO deteriorating the system spectral efficiency is a random variable. Our aim is to find the ensemble average spectral efficiency of a user. Therefore, in the absence

$$\mathcal{R}_p(\triangle) = \log_2 e\int_0^\infty \frac{1}{z}\left(\mathcal{M}(z,\,\ldots,\,\underset{p}{0},\,\ldots,\,z) - \mathcal{M}(z,\,z,\,\ldots,\,z)\right)e^{-z/\mathrm{SNR}_D}dz \tag{20}$$

of CFO estimation, CFO could be modeled as a Gaussian RV. However, as a worst case performance, CFO is considered as uniformly distributed RV according to $\Delta \sim \mathcal{U}(-a, a)$. Further, in the presence of CFO estimation, based on MLE asymptotic property, the residual CFO is modeled as a Gaussian random variable according to $\Delta \sim \mathcal{N}\left(0, \sigma_\Delta^2\right)$.

Let us first define the spectral efficiency in general over any probability density function (PDF) for the CFO, Δ. The overall average mutual information over the p^{th} sub-carrier is the conditional average mutual information over the p^{th} sub-carrier for a given CFO Δ which is given in (16) averaged over the defined CFO PDF, given as

$$\mathcal{R}_p = \mathbb{E}_\Delta\left[\mathbb{E}_{\boldsymbol{H}|\Delta}\left\{\log_2\left(1+\text{SINR}_p\right)\right\}\right]. \tag{22}$$

The simplified overall average mutual information over the p^{th} sub-carrier is written as (23), shown at the bottom of the page.

Therefore, when perfect CSI is available only at the receiver, and that each sub-carrier is decoded independently, the overall spectral efficiency of the OFDM system may be estimated by

$$\mathcal{G} = \frac{1-\alpha}{N+T_G}\sum_{p=0}^{N-1}\mathcal{R}_p. \tag{24}$$

Equations (23) and (24) give the exact overall spectral efficiency of OFDM systems for any given probability density function for the CFO, Δ.

As far as the computation of the spectral efficiency in the presence of ICI due to CFO over correlated fading channels is concerned, we have been able to derive new exact expressions for the spectral efficiency which requires only between one (non-random CFO) to two (random CFO) fold integrations to evaluate (20) and (23), respectively. Note that direct methods to calculate such averages (16) and (22) would require at least N (non-random CFO) to $N+1$ (random CFO) fold integrations, respectively. Hence, there is a huge reduction in computational complexity for the evaluation of spectral efficiency of OFDM systems.

Note that the conditional mutual information in (20) can be evaluated using the Gauss-Laguerre quadrature rules (25), shown at the bottom of the page, where $\lambda_i = \text{SNR}_D\beta_i$, α_i and β_i are the i^{th} abscissa and weight, respectively, of the I^{th} order Lagurre polynomial, which tabulated in [31, eq. (25.4.45)]. The remainder $\mathcal{R}_\mathcal{N}$ is sufficiently small for $I \geq 15$.

V. Imperfect Channel Estimation

In practice, a training sequence is used to estimate the OFDM channel after compensating for the frequency offset. Let H_p and $\widehat{H}_p$ denote the actual channel and the estimated channel over the p^{th} sub-carrier, respectively. It is assumed that H_p and $\widehat{H}_p$ are jointly ergodic and stationary Gaussian processes. In addition, it is also assumed that the channel estimate $\widehat{H}_p$ and estimation error ε are orthogonal. Furthermore, the accuracy of the estimator depends on the training sequence length and pilot power. Therefore, in the case of imperfect channel estimation, the p^{th} sub-carrier channel can be decomposed as

$$H_p = \widehat{H}_p + \varepsilon \tag{26}$$

where ε is the channel estimation error, distributed according to $\varepsilon \sim \mathcal{CN}(0, \sigma_\varepsilon^2)$. It is to be emphasized that the parameter σ_ε^2 captures the quality of the channel estimation, which depends on the method of estimation, the training sequence length, and SNR_E.

From (7), assuming one tap equalizer and imperfect CSI at the receiver, the received symbol over the p^{th} sub-carrier may be rewritten as

$$y_p = a_p c(p, p)\widehat{H}_p + a_p c(p, p)\varepsilon + \sum_{\substack{n=0\\n\neq p}}^{N-1} a_n c(n, p) H_n + w_p. \tag{27}$$

Therefore, from the independence between $\widehat{H}_p$ and ε, the instantaneous SINR over the p^{th} sub-carrier for a given realization of $\boldsymbol{H}$ and Δ becomes a RV given as

$$\text{SINR}_p = \frac{|c\,(p, p)|^2\left|\widehat{H}_p\right|^2}{|c\,(p, p)|^2\sigma_\varepsilon^2 + \sum\limits_{n\neq p}|c(n, p)|^2|H_n|^2 + \frac{1}{\text{SNR}_D}}. \tag{28}$$

As far as the variance of the estimation errors are concern, a joint frequency offset and channel estimation for OFDM systems has been studied in [32]. The joint Cramer-Rao lower bound gives a lower bound for the variance of an unbiased estimator, the joint modified Cramer-Rao bound of the CFO and the channel are (respectively) given as [32]

$$\sigma_\Delta^2 \geq \frac{3N}{\pi^2\text{SNR}_E(N-1)(N+1)} \tag{29}$$

$$\mathcal{R}_p = \log_2 e\int_0^\infty \frac{1}{z}\,\mathbb{E}_\Delta\left(\mathcal{M}(z, \ldots, \underset{p}{0}, \ldots, z) - \mathcal{M}(z, z, \ldots, z)\right)e^{-z/\text{SNR}_D}dz \tag{23}$$

$$\mathcal{R}_p\,(\Delta) = \log_2 e\sum_{i=1}^{I}\frac{\alpha_i}{\beta_i}\left(\mathcal{M}(\lambda_i, \ldots, \underset{p}{0}, \ldots, \lambda_i) - \mathcal{M}(\lambda_i, \lambda_i, \ldots, \lambda_i)\right) + \mathcal{R}_\mathcal{N} \tag{25}$$

and

$$\sigma_\varepsilon^2 \geq \frac{1}{N\mathrm{SNR}_E}\left(2T_G + \frac{3(N-1)^2}{\left(N^2-1\right)}\right). \tag{30}$$

In a similar manner to the derivations of (20), we arrive at the following new expression for the conditional mutual information over the p^{th} sub-carrier for a given $\triangle$, shown in (31), shown at the bottom of the page, where here the covariance matrix $\Lambda_p = C_p\Lambda$ is similar to that of (19) except for the element $\Lambda(p, p)$ which is replaced by $\Lambda(p, p) - \sigma_\varepsilon^2$.

VI. Transmit Diversity

In this section, the analysis of Section IV is extended to the case of OFDM systems with transmit diversity. Let the number of transmit antennas be M, and let the total transmit power be distributed equally among the transmit antennas. The conditional average mutual information over the p^{th} sub-carrier for a given $\triangle$ (16) can be modified to

$$\mathcal{R}_p(\triangle) = \mathbb{E}\left[\log_2\left(1 + \frac{\frac{1}{M}\sum\limits_m |c(p, p)|^2 \left|H_p^m\right|^2}{\sum\limits_{n\neq p}\frac{1}{M}\sum\limits_m |c(n, p)|^2\left|H_n^m\right|^2 + \frac{1}{\mathrm{SNR}_D}}\right)\right] \tag{32}$$

where $\sum\limits_m = \sum_{m=1}^M$, $c(n, p)H_n^m$ is the ICI component due to the n^{th} sub-carrier and the m^{th} antenna element, where $m = 1, \ldots, M$.

It can be shown that in the case of independent diversity channels with perfect CSI at the receiver following a similar manner to the derivations of (20), we get the following new expression for the conditional mutual information, given in (33), shown at the bottom of the page.

VII. Numerical Results

We consider a multipath fading channel with exponential power delay profile. First, with the aid of (8), elements of the complex covariance matrix Λ can be evaluated, where the mean delay spread is chosen as $\sigma = 1$. The simulation results are based on the classical method of generating correlated random numbers. Therefore, since the specified covariance matrix Λ is positive semi-definite and its eigen-values are always non-negative, the Cholesky decomposition method is used to transform a set of uncorrelated Gaussian random numbers to a set of correlated numbers of a predefined correlation matrix Λ with entries given by[5] (8). As a result, the correlation matrix is decomposed to give a lower triangle matrix L. Therefore, to get a correlated random numbers with a correlation matrix Λ, simply apply the matrix L to a vector of normalized independent random variables. Finally, the overall simulated average mutual information over the p^{th} sub-carrier (22) can be evaluated by averaging 10^5 Monte Carlo simulations of the instantaneous mutual information $\log_2(1 + \mathrm{SINR}_p)$, where SINR_p is given by; perfect CSI (13), imperfect CSI (28), and transmit diversity (32). This is then substituted in (24) to find the overall spectral efficiency. It is to be emphasized that at high number of subcarriers, elements of the correlation matrix Λ become insignificant when $|n - m| > 20$ [9]. On the other hand, for the theoretical analysis results, Λ is used to find the MGF by (19), where this is used to evaluate the simplified overall average mutual information over the p^{th} sub-carrier given in (23), which is then substituted in (24) to find the overall spectral efficiency.

In Figs. 1–5, Monte Carlo simulations of the overall average mutual information over the p^{th} sub-carrier which is given by (22), where the instantaneous SINR over the p^{th} sub-carrier is given by (13), with the aid of the overall spectral efficiency formula (24), is used to validate the new analytical expression (24), where the overall average mutual information over the p^{th} sub-carrier is given by (23). It is seen that the simulation and proposed analytical expression provide a perfect match which corroborate the exactness of the proposed analytical expressions. Note that in Figs. 1–7, it is assumed that perfect CSI is available at the receiver, and that the spectral efficiency is evaluated without taking into account the penalty of the extra power and spectral resources allocated to channel estimation assuming that blind channel estimation is performed instead.

In Fig. 1, spectral efficiency (exact and upper bound) against SNR is shown for non-random $\triangle$. In the presence of CFO, the spectral efficiency increases with increasing the SNR (until a ceiling is reached beyond which the spectral efficiency becomes independent of SNR), and decreases with increasing the CFO.

[5]Note that we are alternating between the covariance and correlation matrix since they are equal because the mean of the channel RVs is zero. The Cholesky decomposition of the predefined covariance matrix Λ is given as: $\Lambda = U\Sigma U^\dagger = \left(U\sqrt{\Sigma}\right)\left(U\sqrt{\Sigma}\right)^\dagger$. Therefore, a correlated channel vector of random numbers is given by $\boldsymbol{H} = L\boldsymbol{x}$ where $\boldsymbol{x} \sim \mathcal{CN}(0, I_N)$, then $\boldsymbol{H} \sim \mathcal{CN}(0, LL^\dagger)$, where $L = U\sqrt{\Sigma}$.

$$\mathcal{R}_p(\triangle) = \log_2 e \int_0^\infty \frac{1}{z}\left(\mathcal{M}(z, \ldots, \underset{p}{0}, \ldots, z) - \mathcal{M}(z, z, \ldots, z)\right) e^{-z\{|c(p,p)|^2\sigma_\varepsilon^2 + 1/\mathrm{SNR}_D\}} dz \tag{31}$$

$$\mathcal{R}_p(\triangle) = \log_2 e \int_0^\infty \frac{1}{z}\left(\mathcal{M}^M(z, \ldots, \underset{p}{0}, \ldots, z) - \mathcal{M}^M(z, z, \ldots, z)\right) e^{-zM/\mathrm{SNR}_D} dz. \tag{33}$$

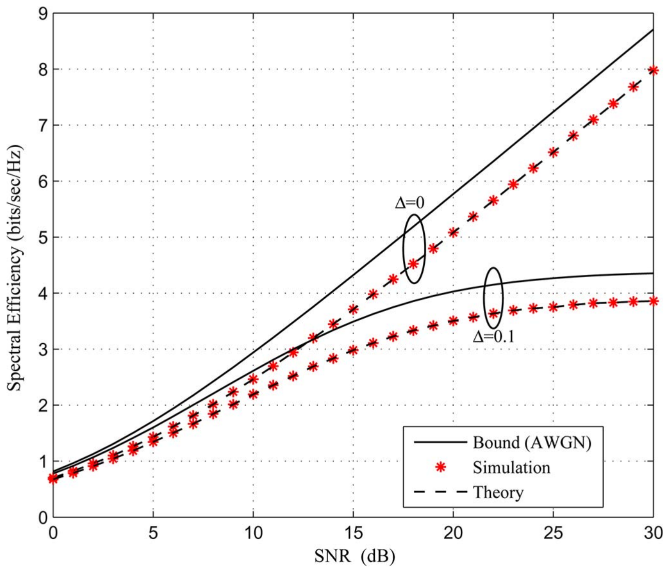

Fig. 1. Spectral efficiency against SNR for a given non-random CFO with $N = 64$, $\alpha = 0$, $T_G = 8$ and $\sigma = 1$.

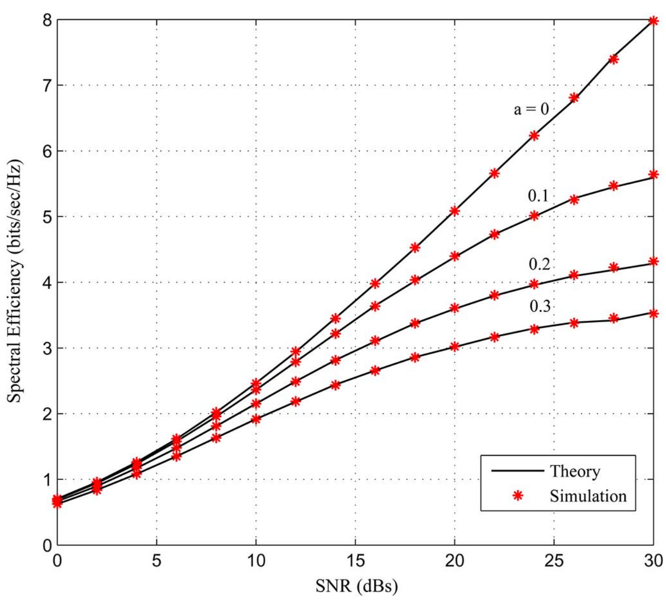

Fig. 2. Spectral efficiency against SNR with uniform random CFO, $\Delta \sim \mathcal{U}(-a,\, a)$, for $N = 64$, $\alpha = 0$, $T_G = 8$ and $\sigma = 1$.

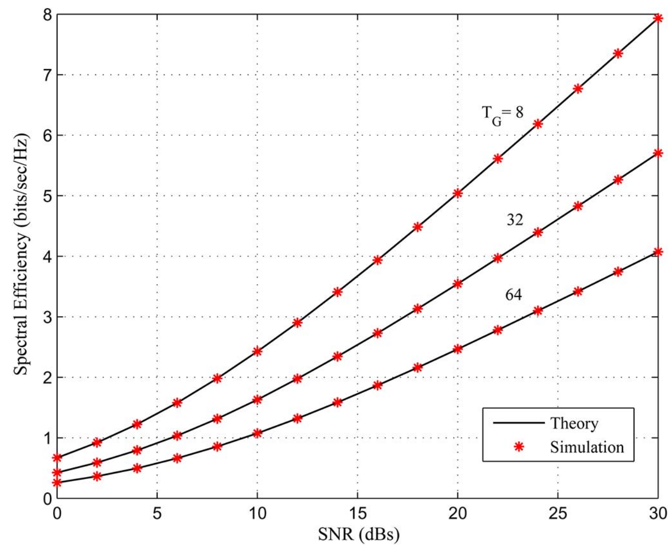

Fig. 3. Spectral efficiency against SNR for the cyclic prefix based (CB) CFO estimation with $\text{SNR}_E = \text{SNR}$, $N = 64$, $\alpha = 0$, and $\sigma = 1$.

In addition, spectral efficiency degradation due to CFO is more severe at high SNR values since the SINR at high SNR is approximated as signal to interference ratio (SIR) from the ICI due to CFO. This can easily be seen since $\text{SINR} = \frac{1}{\frac{1}{\text{SIR}} + \frac{1}{\text{SNR}}}$. Therefore, for high SNR, SNR $\gg$ SIR, SINR $\approx$ SIR, and for low SNR, SNR $\ll$ SIR, SINR $\approx$ SNR.

In Fig. 2, spectral efficiency against SNR is plotted for random Δ. The CFO Δ is a uniform RV distributed according to $\Delta \sim \mathcal{U}(-a,\, a)$, which is chosen as a worst case scenario when no CFO estimation is performed. For instance, in mobile systems CFO might be introduced from mobility due to Doppler shift. It is seen from Fig. 2 that the spectral efficiency increases with increasing the SNR until a ceiling is reached, and decreases with increasing the CFO variance parameter a. For instance, to compensate for the loss in spectral efficiency due to random CFO, SNR should be increased from 20 dB to 24 dB in order to achieve 5 bits/sec/Hz when $a = 0.1$. Further, spectral efficiency degradation becomes more severe when the CFO variance parameter a exceeds 0.02.

In Fig. 3, spectral efficiency against SNR is drawn when (5) is used as the variance of the CP-based CFO estimator Δ. Therefore, without the aid of training sequence in CFO estimation, $\nu = 0$ since the length of the cyclic prefix is used for CFO estimation, hence, $\alpha = 0$ and $T_E = 0$. Fig. 3 reveals that as the cyclic prefix[6] T_G increases, a degradation in spectral efficiency is seen when the number of sub-carriers $N = 64$. Therefore, even though increasing the cyclic prefix length decreases the CFO estimation error variance and improves accuracy, the system resources loss induced during the estimation phase decreases the spectral efficiency. Hence, there is a trade-off between the loss in spectral efficiency due to residual CFO and the loss in system resources while estimating the CFO. In order to gain more insight into the system, we provide an approximate closed form expression to the spectral efficiency with a Gaussian RV Δ. From (21), with the aid of Taylor series expansion, we get $\left|\frac{\sin(\pi\Delta)}{N\sin(\frac{\pi}{N}\Delta)}\right|^2 \approx 1 - \frac{(\pi\Delta)^2}{3}$ [33]. Therefore, when the residual CFO is distributed as $\Delta \sim \mathcal{N}\left(0,\, \sigma_\Delta^2\right)$, the overall average mutual information over the p^{th} sub-carrier can be approximated by

$$\mathcal{R}_p \approx \log_2\left(\frac{\text{SNR}_D + 1}{\text{SNR}_D \frac{\pi^2}{3}\sigma_\Delta^2 + 1}\right). \tag{34}$$

[6]Note that the loss induced by the cyclic prefix comprises two components: 1) the time to transmit the cyclic prefix and 2) the power which is used to transmit the cyclic prefix. It is to be emphasized that both these factors are explicitly included in the spectral efficiency (15) and (16).

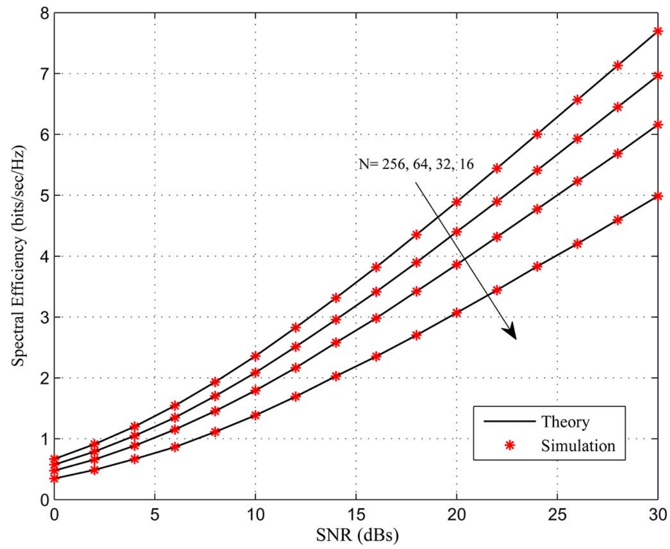

Fig. 4. Spectral efficiency against SNR for the training based (TB) CFO estimation with equal pilot and signal power, $\alpha = \frac{1}{9}$, $T_G = 8$ and $\sigma = 1$.

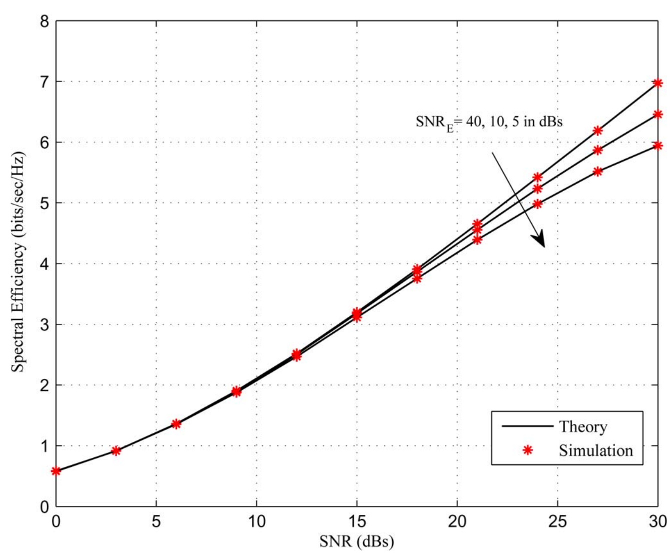

Fig. 5. Spectral efficiency against SNR for the training based (TB) CFO estimation with fixed pilot power, $N = 64$, $\alpha = \frac{1}{9}$, $T_G = 8$ and $\sigma = 1$.

In the CP-based (CB) CFO estimation, we have

$$\mathcal{R}_p \approx \log_2\left(\frac{\frac{N}{N+T_G}\text{SNR}+1}{\frac{N}{3T_G(N+T_G)}+1}\right). \tag{35}$$

In Figs. 4 and 5, spectral efficiency against SNR is shown for the training based (TB) CFO estimation with equal pilot and signal power, and fixed pilot power, respectively. The estimator variance is given by (6), for $T_G = 8$ and $\sigma = 1$. Here, the OFDM training block is divided into two identical parts, $K = 2$ in (6). It is assumed here that one OFDM block is used for CFO estimation $\nu = 1$ from every nine OFDM blocks $V = 9$, assuming that the coherence time is at least higher than nine OFDM blocks so that the channel (which is assumed perfectly known here) is considered constant each nine OFDM symbols. Therefore, the fraction of time resources allocated to estimation is $\alpha = \frac{1}{9}$, and the effective per OFDM block training symbol length is $T_E = \frac{N}{8}$. Hence, it is seen from Fig. 4 that the higher the number of sub-carriers, N and/or SNR, the higher the spectral efficiency. When comparing Figs. 3 and 4 for $N = 64$ and $T_G = 8$, it is seen that spectral efficiency with CP-based CFO estimation (Fig. 3) is higher than that of training based CFO estimation (Fig. 4). In addition, note that in the case of equal pilot and signal power, in contrast to Figs. 1 and 2, spectral efficiency increases with SNR without bound as there is no ceiling in this case. This is justified by the fact that increasing SNR decreases the ICI through CFO estimation as SNR$_E$ increases. Furthermore, as SNR$_E$ approaches infinity, ICI approaches zero. However, it is also seen that in the fixed pilot power case Fig. 5, at low SNR$_E$, there is a spectral efficiency ceiling as SNR increases. To gain more insight into the system settings, substituting this estimator variance in (34), in the training based (TB) CFO estimation with equal pilot and signal power, we have

$$\mathcal{R}_p \approx \log_2\left(\frac{\frac{N}{N+T_G+T_E}\text{SNR}+1}{\frac{2}{3(N+T_G+T_E)}+1}\right). \tag{36}$$

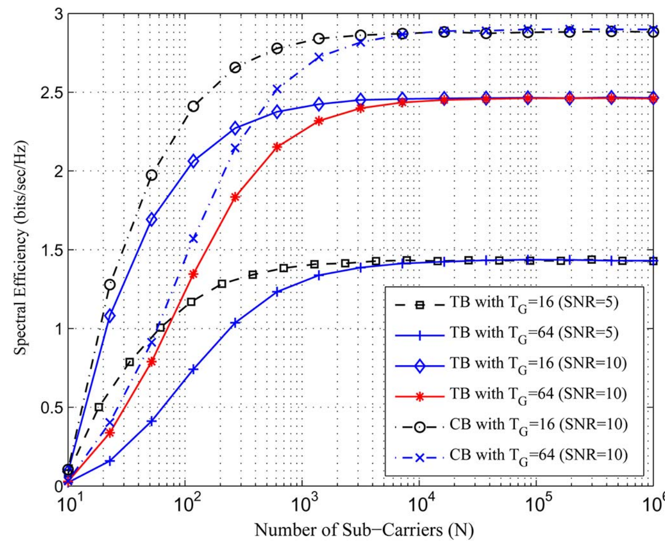

Fig. 6. Spectral efficiency against the number of sub-carriers, N, for the training based (TB) and cyclic prefix based (CB) CFO estimation with equal pilot and signal power, SNR $= 5\,\&\,10$ dB, $\alpha = \frac{1}{9}$ for TB, $\alpha = 0$ for CB and $\sigma = 1$..

In Fig. 6, spectral efficiency against the number of sub-carriers, N is shown for both, the cyclic prefix based (CB) CFO estimation, and the training based (TB) CFO estimation, with equal pilot and signal power, for SNR $= 5\,\&\,10$ dB and $\sigma = 1$. It is seen that the spectral efficiency increases monotonically with increasing the number of sub-carriers N until a ceiling is reached, beyond which the spectral efficiency becomes independent of the number of sub-carriers. Fig. 6 shows that at low number of sub-carriers $N \leq 10^4$, less number of cyclic prefix T_G has higher spectral efficiency. However, they both match at higher number of sub-carriers. As far as the spectral efficiency is concern, it is seen that the cyclic prefix based method outperforms the training based method when perfect CSI is known blindly at the receiver.

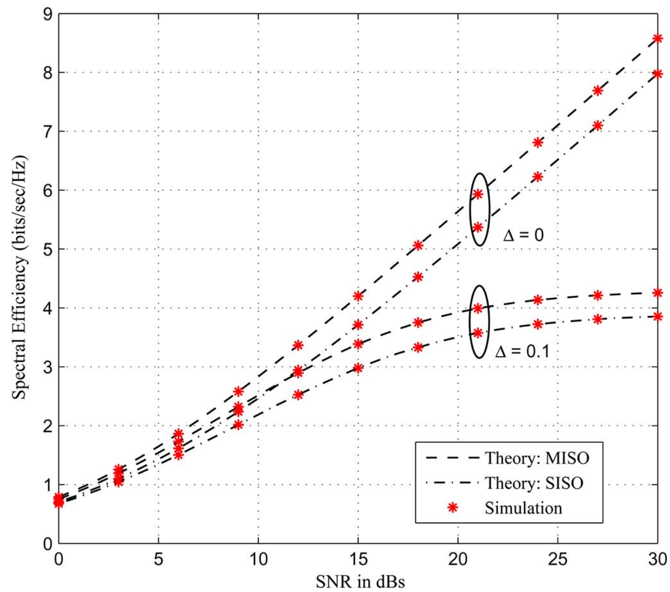

Fig. 7. Transmit diversity spectral efficiency against SNR for a given non-random CFO with $M = 5$, $\alpha = 0$, $N = 64$, $T_G = 8$ and $\sigma = 1$.

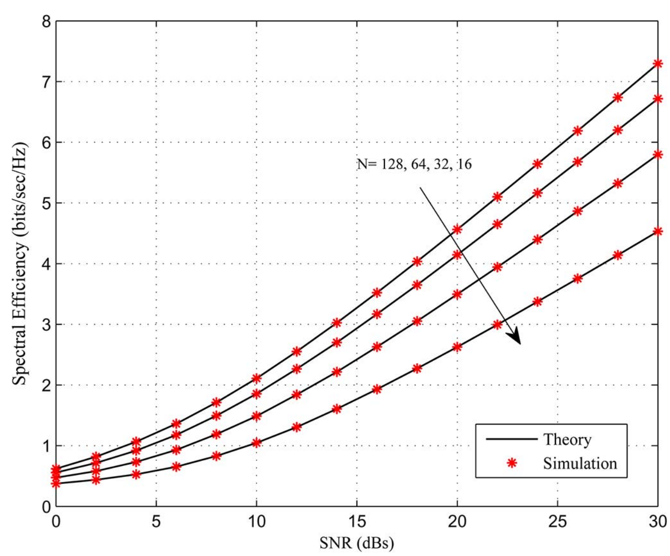

Fig. 8. Spectral efficiency against SNR for the training based CFO estimation with imperfect CSI for equal pilot and signal power, $\alpha = \frac{1}{9}$, $T_G = 8$ and $\sigma = 1$.

In Fig. 7, spectral efficiency of OFDM systems with transmit diversity against SNR for non-random CFO is drawn when $M = 5$, $N = 64$, $T_G = 8$, $\alpha = 0$, and $\sigma = 1$. As expected, it is seen that there is a little improvement in spectral efficiency with transmit diversity. This is due to the fact that spectral efficiency is improved by spatial multiplexing. It is well known that transmit and receive diversity is used to combat the severe channel fading and improve transmission reliability. Therefore, diversity improves error rate and does not provide huge improvement to the spectral efficiency.

In Fig. 8, spectral efficiency against SNR for the training based CFO estimation with imperfect CSI is shown for equal pilot and signal power with $\alpha = \frac{1}{9}$, $T_G = 8$ and $\sigma = 1$. Here, since the impact of imperfect channel estimation is taken into account jointly with the CFO, it is seen clearly that Fig. 8 has a worse spectral efficiency than Fig. 4 where perfect CSI is assumed.

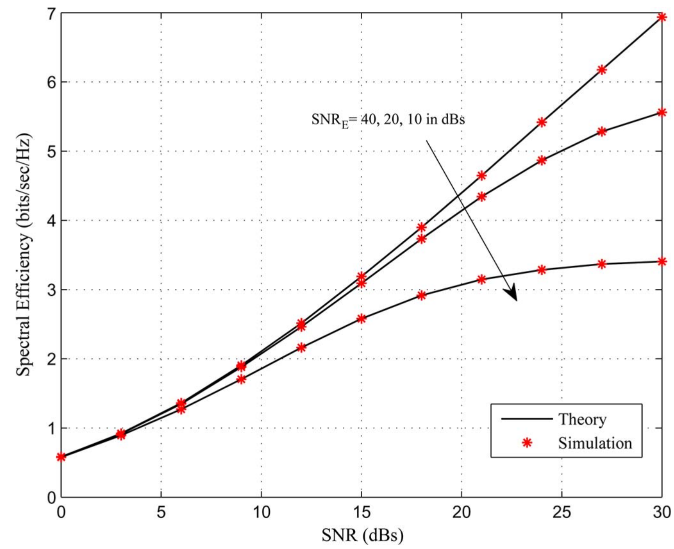

Fig. 9. Spectral efficiency against SNR for the training based CFO estimation with imperfect CSI for fixed pilot power, $N = 64$, $\alpha = \frac{1}{9}$, $T_G = 8$ and $\sigma = 1$.

In Fig. 9, spectral efficiency against SNR for the training based CFO estimation with imperfect CSI is plotted for fixed pilot power with $N = 64$, $\alpha = \frac{1}{9}$, $T_G = 8$ and $\sigma = 1$. Compared with Fig. 5 where perfect CSI is assumed, a worse spectral efficiency is seen as SNR_E decreases. This is justified by the increase in estimation error variance due to the joint CFO and channel estimation.

VIII. Conclusion

The performance of the OFDM systems is susceptible to degradation when failing to attain perfect time and frequency synchronization. Even though CFO estimation is performed at the receiver, a residual frequency offset still exists and introduces ICI which degrades the OFDM system performance over multipath wireless channels. New precise expressions for the spectral efficiency of OFDM systems in the presence of residual CFO and multipath fading channels are presented, which accurately accounts for both the degradation in SINR due to the residual CFO, and the penalty of the extra power and spectral resources required to achieve the desired CFO estimation accuracy, as well as the periodic transmission of the cyclic prefix. Numerical results demonstrated that the cyclic prefix based CFO technique is more efficient than the training symbols based CFO technique when perfect CSI is known blindly at the receiver. However, the training symbols based CFO estimation technique is more efficient than the cyclic prefix based CFO technique if a training sequence is sent for channel estimation. In addition, low fixed pilot power results in a spectral efficiency ceiling as SNR increases, whereas in the equal pilot and signal powers case, spectral efficiency increases with SNR without bound. Further, it is also seen that when joint CFO and channel estimation is performed at the receiver, this results in a worse estimation error variance for the CFO and channel.

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