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Energy demand in production systems: A Queuing Theory perspective

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ABSTRACT

Production systems are usually organised in departments consisting of machines, each one characterised by specific patterns of energy demand over time. This work proposes an analytical approach, based on the application of Queuing Theory, to model the power request and the consequent energy use in a production system. Despite the industrial context addressed, the model may be easily applied to small units (e.g., civil buildings) and other energy sources (e.g., thermal energy), thus giving more relevance to the approach proposed. The model can efficiently support *green-field* cases, particularly avoiding or integrating the traditional assumptions, such as load and coincidence factors (usually employed to determine the contractual electrical power), which provide a static view of the power needs of the system. In fact, the proposed queuing model considers the arrivals as the statistical distribution of the switch-on of machines and service completions as the statistical distribution of the processing times at the machines themselves, thus offering a dynamic view of the power loads. Therefore, the model may be helpful while assessing the contract with the energy supplier or planning the production schedule of plants with significant energy-related constraints, including plant services. A numerical example shows the application of the proposed approach and its results are compared to those determined by the traditional design methodology.

1. Introduction

Production systems are usually modelled in order to enable managers to look at the system in terms of inventory level, WIP, throughput and service level, or any other indicator helpful when evaluating the system performances, as well as when appreciating the potential impacts of the interventions that can be planned and implemented.

When the focus goes to the so-called *logistic performances* (e.g. costs, lead times, inventory level and service level), the system is modelled to capture the behaviour of the main flows (such as products and information) and the model itself is used as a tool to describe the production system, to monitor its current performance and to focus on potential improvements. However, when attention is paid to the *energy requirements* of the production system, energy and power level concepts may replace, in some ways, inventory and service levels. In fact, when monitoring the inventory level, two main purposes are followed and grasped: the first is to assess the maximum inventory level, e.g. to respond to some physical constraint of the storage area, and the second one is to optimise the inventory holding cost of the stored items, according to the planned demand. In the energy-supply view, the first purpose is to dimension the contractual power level (i.e., the maximum value allowed for the supply) keeping into account, at the same time, information on energy usage and related costs. Furthermore, the inventory level is used to evaluate the service level reached by the productive system, thus being a performance indicator of it; similarly, the energy used per unit of output can be considered as an indicator of the efficiency of the production system.

Nowadays, the efficient use of energy cannot be neglected or undervalued by the managers of manufacturing systems: they should include it among the most relevant issues to be addressed by the company, being a strategic key-element to guarantee competitiveness and to improve the overall company results. This issue is relevant not only in terms of economic outcome, but also in terms of environmental and social responsibility.

So as to offer an easy-to-implement tool, enabling managers to make more conscious decisions on energy use and related issues, this work presents a model that may be used to represent different production scenarios under different electricity supply contracts. Thanks to this approach, it is possible to appreciate the power level and the related energy use, as a consequence of manufacturing operations, under different scenarios. The focus of the study will be on production systems, e.g. paying attention to the correct design of the electricity supply, set according to demand power

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levels and foreseeable energy use; however, the approach may be easily extended to different environments and energy sources.

The paper is organised as follows: Section 2 presents the system considered, the problem to be solved and its main assumptions. In Section 3, the methodology is shown and its formulation is detailed in Section 4. Section 5 offers a numerical example and, finally, Section 6 focuses on hints for discussion and conclusions.

2. Problem description

One of the most important managerial actions aiming at reducing energy costs is the proper fitting to the plant requirements of energy, i.e. electricity and the supply contracts related. Manufacturing systems are usually organised in departments, where each resource/machine presents its own demand pattern of energy over time. Energy requirements usually differ from machine to machine, depending on the power of the machines and on the persistence at the different machining states (such as idle, standby, load level, machining parameters, maximum speed) as well as on production volumes, planned mix and batch sizes.

As discussed, the problem examined stems from the fact that access to electricity supply (as well as other energy sources) is limited and costly, being related to contracts with energy suppliers fixing: (1) the maximum power level (*contractual power level*) that, when exceeded, determines substantial penalties to the user, and (2) energy costs, associated with the energy usage and its profile over time (as in the cases of peaks and time windows set during the day).

In order to accomplish a reduction of the energy cost, different tools can be implemented. The cost component may be reduced by a negotiation with the energy supplier, thus considering both the contractual power level and the cost per energy unit, eventually related to the power level itself. On the other hand, the energy usage can be reduced by (1) technological improvements, e.g. limiting both the machining power required and working times, or (2) managerial actions, aimed at increasing the efficiency of the energy usage.

The problem of the energy usage can be tackled by a monitoring tool, which allows managers to appreciate the relationships between power level and energy use. By this approach, it is possible to determine the power level supplied and, at the same time, to evaluate the total energy costs.

Monitoring tools allow the user to rearrange the statistical distributions representing the energy requirements at a manufacturing resource/machine (Fig. 1 shows the data of electricity power levels taken from an industrial case study). The same data may be reorganised as in Fig. 2, which shows the statistical distribution of the electricity use.

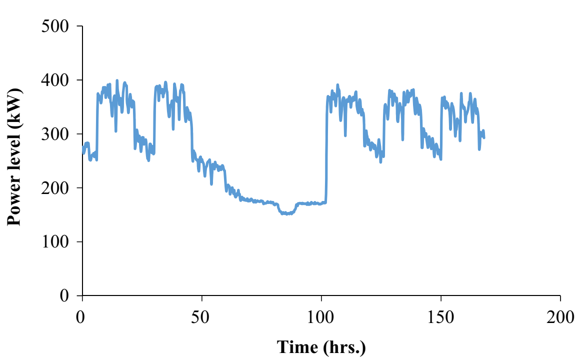

Fig. 1. Electricity power level monitoring tool: output graph.

Even if the monitoring tool approach may be very useful to decision makers, e.g. to set the contractual power of the production system, it is quite unusual to find it in a real production context, regardless of the increasing attention that the managers themselves are paying to the control and use of energy. According to this fact, the present contribution aims at offering an alternative or supplementary decision-making tool, particularly addressed to the determination of the contractual power for a production facility (or a manufacturing department) to be designed (*green-field* case) or when the revamping and/or new production projects are under examination.

When *measured* data (i.e., the monitoring system records) or historical data are not available (*green-field* case), the statistical distribution of the energy requirements may be *modelled* on the basis of estimated production data: the approach may be useful to compare different productive scenarios, too. In particular, production data are necessary as referred to *what* should be produced (i.e., type of items, production volumes, production mix, batch sizes, etc.) and *how* it should be produced (i.e., resources involved in the production process, values set for the processing parameters, etc.).

In this context, Queueing Theory could be used to describe the system behaviour and its performance with reference to the *jobs* that are manufactured by the system, considering their routings, resource information (number, type and capacity) and demand. Based on the results of the *job-queueing* model, a further model can be developed by additional data on the requirements of power at the resources involved in the manufacturing process, i.e. a *power-queueing* model. As *job-queueing* parameters are essential to compute the *power-queueing* related ones, average service times, i.e. production times at the different departments, are considered in the *power-queueing* model, too, and the average energy requirement of the system can be computed (Fig. 3).

Within this framework, the proposed model encompasses both *energy availability* and *energy efficiency* issues:

- Energy availability, as related to power availability. It may be defined as the mean power level required multiplied by the mean time of energy usage, as compared to the theoretical power availability. For example, when the mean power level is close to the contractual one, the residual power (and energy)

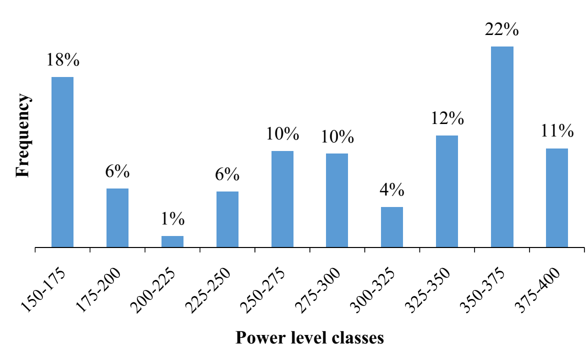

Fig. 2. Statistical distribution of the electricity use.

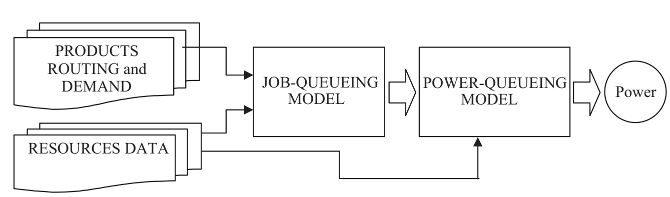

Fig. 3. Job-queueing and power-queueing models relationship.

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availability is limited; therefore, managers can decide to negotiate a new contract with a more suitable power level.

- Energy efficiency, as related to energy availability (computed as the contractual power level multiplied by the reference time interval). This index allows the computation of the energy usage ratio, i.e. the ratio between energy usage and energy availability. In fact, energy usage is a basic and necessary knowledge, required to be aware of energy requirements and to consequently assess the improvements to be implemented to gain energy efficiency. Energy usage can be computed by the mean power level and time, also when only production data are known.

The approach proposed differs from the existing ones, such as event-driven simulation tools, as it does not require decision-makers' expertise in simulation techniques. In addition, specific software or computing applications are not necessary, as the model allows the computation of the main performance indices (mean power level, mean time, etc.) by a simple formulation: the interpretation of results is straightforward. Moreover, the model formulation does not require parameter settings (as necessary for simulation tools), as it explicitly considers the uncertain nature of the modelled quantities. Finally, the proposed model demonstrates how Queuing Theory can be applied to assess both current situations and *green-field* cases: it may be employed as a *monitoring tool* (e.g., when KPI computation is required to evaluate current system performance) as well as a *design tool* (e.g., when specific issues, such as energy availability, usage and efficiency, are addressed).

In conclusion, the present work shows a simple modelling tool, based on elementary production data (such as production times, technological cycles, bills of material and so on), which describes the production system from the viewpoints of the energy power level and the energy use profile. Its basic, though novel, structure may be of interest both to practitioners and academicians.

3. Methodology

3.1. Literature review

Devoldere et al. (2007) presented a methodology for the determination of energy use related to the machining of discrete parts, with a special focus on the importance of energy requirements during non-production times. Based on a simulation tool, Hesselbach et al. (2008) presented an integrated approach to support both the energy efficient design and the management of production facilities, together with technical building services. Dietmair and Verl (2009) introduced a general method to model the energy requirements of machines, based on a statistical discrete-event formulation. The parameter information, required to describe the discrete events, can be obtained by a small number of simple measurements or, with a certain degree of uncertainty, from the machine and component documentations. Avram and Xirouchakis (2011) presented a methodology to evaluate the phased use of energy requirements in a machine tool system; in particular, exact power levels and energy use for a specific part manufacturing are determined, enabling a comparison of different milling part programs with respect to their energy consumption levels. Diaz and Dornfeld (2011) focused their attention on the characterisation of the energy consumption and its consequent reduction for a Mori Seiki milling machine.

Bruzzone et al. (2012) proposed the integration of an EAS module (Energy-Aware Scheduling) within an advanced planning and scheduling (APS) system, incorporating a model to control the shop-floor power peak for a given detailed schedule. The goal of the EAS is to optimise the given schedule from the viewpoint of energy consumption, while keeping the given assignment and sequencing fixed. The problem is discussed by minimising the shop-floor power peak and concurrently limiting the deterioration of two scheduling objectives (tardiness and makespan minimisation) set; the technique adopted is a Mixed Integer Programming (MIP) problem. He et al. (2012) proposed a modelling method for task-oriented energy consumption in a machining manufacturing system. The energy consumption features, driven by the task flows of the manufacturing system itself, are analysed, thus describing how energy requirements dynamically depend on the flexibility and variability of the task flows in the production processes. The results provide a valuable insight of energy consumption in machining systems, supporting robust decisions on energy efficient strategies.

Prabhu et al. (2012) proposed a queuing model to predict energy savings in serial production lines, where idling machines are switched to a low-power state in a scenario consisting of machines with Poisson arrivals and exponential service times. Jeon and Prabhu (2012) extended their previous model and generalised their “energy aware” queuing model to a re-entrant structure, presenting an application to a semiconductor factory. Fernandez et al. (2013) discussed a non-linear programming model, with the goal of minimising both inventory holding and energy costs, considering power and energy-consumption costs and distinguishing between off-peak and on-peak configurations.

Mouzon et al. (2007) proposed a model for the development of operational methods for a machine scheduling oriented to the reduction of energy requirements: they presented a neural network model based on the arrival process of incoming jobs, that replaces statistical methods of arrival process forecasting when probability distribution of job inter-arrival times is not available. A recent model that explicitly considered the energy use in manufacturing systems, when the production rate varies, has been proposed by Zanoni et al. (2014). Finally, contracting agreements have been recently investigated by Oliveira et al. (2013) in the electricity industry.

A schematic classification of the reference literature is offered in the following table, where (1) purpose, (2) methodology and (3) application are resumed for each contribution.

As Table 1 shows, the present contribution gathers together different elements from the reviewed literature, presenting an original model for peak-power and energy usage, also considering supply contract aspects. Moreover, all the works above refer to systems for which historical data are available, while this work may be applied also to the *green-field* situation of a manufacturing system.

3.2. Queuing Theory

Queuing Theory makes available models (and consequent insights) useful to predict the behaviour of systems which provide services at randomly occurring demand; in addition, it considers statistical distributions for production operations (such as process times, processing cycles and production mix) which allow the description of complex environments. In fact, when modelling production systems, a main advantage of Queuing Theory is represented by its effectiveness and efficiency in offering a technique that easily describes and characterises the systems themselves by fitting performance indicators, such as mean time in the system, mean service time, mean number of customers in the system, probability of having a certain number of customers in the system, and so on.

According to this perspective, Queuing Theory allows the quick modelling of production systems, even when their environment is subject to a certain degree of uncertainty. Such an uncertainty is

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Table 1
Literature review classification.

	Purpose	*Methodology*	*Application*
Devoldere et al. (2007)	Energy usage determination for production and non-production activities.	LCA-type methodology based on an inventory of energy usage (production and non-production activities) combining a time study and an energy study, with an impact assessment and result interpretation phases.	Machining of discrete parts.
Hesselbach et al. (2008)	Energy efficient design and management of production facilities and technical buildings.	Simulation approach that involves four integrated discrete-event simulation tools for: (1) technical building service, (2) building climate, (3) production machines and material flow, (4) production management.	Production facilities (machines) and technical building services
Dietmair and Verl (2009)	Determination of the energy consumption of production machines.	Optimisation model based on a forecasting technique for power profile definition. As an extension, the authors propose the introduction of stochastic production for the optimisation of usage scenarios.	Tool machines.
Avram and Xirouchakis (2011)	Evaluation of use phase energy requirements to reduce energy consumption.	Analytical approach to estimate variable energy requirements for both steady-state and transient regimes, based on CAD/CAM and technological parameters of the machine (e.g. cutting forces). Specific energy consumption is determined.	Machine tool system.
Diaz and Dornfeld (2011)	Energy consumption characterisation and reduction.	Analytical model based on technological parameters of both machine (e.g. material removal rate) and process.	Mori Seiki milling machine.
Bruzzone et al. (2012)	Production scheduling optimisation for the minimisation of the shop floor power peak and energy consumption.	Integration of EAS (Energy-Aware-Scheduling) and APS (Advanced Planning and Scheduling) modules in a MIP model for production scheduling, without complete knowledge on power profile requirements.	Flow shop.
He et al. (2012)	Determination of task-oriented energy consumption.	Discrete-event simulation model that considers, for each machine, its related “working” and “idle” task times and power levels.	Machining manufacturing system.
Prabhu et al. (2012)	Reduction of wasted energy in manufacturing systems.	An analytical model based on Queuing Theory (M/M/1) is developed to design the manufacturing system without any detailed information about the system. Result robustness is tested by simulation experiments.	Serial production lines.
Jeon and Prabhu (2012)	Prediction of energy consumption.	Analytical model based on Queuing Theory. Results robustness is tested by simulation experiments.	Semiconductor fabrication process.
Fernandez et al. (2013)	Reduction of peak-power demand.	NILP model that considers in the objective function (1) holding costs of materials in the buffers and (2) electricity costs minimisation. As a main constraint, constant throughput is considered.	Typical manufacturing system with multiple machines and buffers.
Mouzon et al. (2007)	Minimisation of energy consumption of manufacturing equipment.	MILP model based on scheduling of production and non-production activities, as well as on power required for the same activities.	Single machine.
Zanoni et al. (2014)	Minimisation of costs of a production-inventory system including energy costs.	Analytical model that considers (1) inventory costs and (2) energy costs, related to power requirements during production activities.	Manufacturing system involving two finite-production rate machines and three buffers (stock points).
Oliveira et al. (2013)	Modelling of the electricity supply chain, considering the interaction between market structures and supply contracts.	Analytical model for the determination of a Nash equilibrium that gives information on optimal energy supply and tariffs.	General two-stage supply chain for electricity market.
This work	Energy demand and supply contract definition.	Analytical model based on Queuing Theory.	Production systems, such as machining and manufacturing systems.

well managed by the statistical distributions of the input parameters, as the arrival and service rates of the queuing model.

An example of Queuing Theory application to manufacturing environments may be found in Zavanella and Bugini (1992), where a model is proposed to solve the problem of dimensioning tool and fixture resources in a flexible productive environment. In a similar approach, it is shown how, in a queuing system, entities (data, parts, jobs, etc.) arrive at the system itself and require some form of service (operations, machining, assembly processes, tooling, etc.). When more demand for service occurs, e.g. at a level larger than resource available for service, a queue is formed and its optimisation may be carried out thanks to the queuing model.

The idea proposed in the study is to model the variability in electricity usage by a queuing system, where the entities are represented by the power devices switched on in the system considered. Thus, the arrival distribution of the queuing model describes the activation of the devices, while the service times correspond to the times required by the related manufacturing operations: arrival and service rates are expressed as power (i.e., kW) per unit of time. In such a queuing system, the meaning of the average number of entities is the average power required.

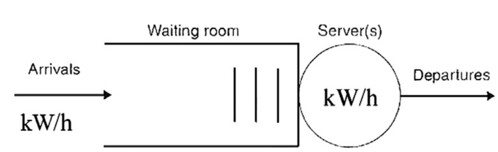

Fig. 4. The system considered in the analysis.

The *power-queuing* system is shown in Fig. 4. In order to obtain tractable models, it is assumed that the arrival process and the service process are stationary.

When a monitoring tool for the power level and energy requirements is not available or historical data are not given, the assumptions by Mouzon et al. (2007) may be applied. However, differently from Mouzon et al. contribution (which involves artificial neural networks to forecast the arrival process), this paper suggests to determine inter-arrival and service times from the available production data, i.e. production volumes and mix, production processes and related resources involved.

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Table 2
Notation of the queuing models.

Job-queueing model			*Power-queueing model*		
Symbol	*Units*	*Description*	*Symbol*	*Units*	*Description*
λ_i	Job/h	Mean arrival rate of *jobs* at the *i*-th department.	$\lambda_{p,i}$	kW/h	Mean inter-arrival rate of "production start" events in the *i*-th department (*power* customer).
λ	Job/h	Mean arrival rate of *jobs* at the system.	λ_p	kW/h	Mean inter-arrival rate of "production start" events considering all departments (*power* customer).
$1/\mu_i$	h	Mean service time of *jobs* at the *i*-th department.	$1/\mu_{p,i}$	h	Mean service time in the *i*-th department (*power* customer): it is the same service time of *job* customers.
$1/\mu$	h	Mean service time of *jobs* in the system.	$1/\mu_p$	h	Mean service time considering all departments (*power* customer).
ρ_i	%	Utilisation coefficient of the *i*-th department.	$\rho_{p,i}$	%	Utilisation coefficient of the *i*-th department (*power* customer).
L_i	Job	Average number of *jobs* in the *i*-th department.	L_p	kW	Average number of *power* customers in the system
W_i	h	Average time spent by a *job* in the *i*-th department.	W_p	h	Average time spent by *power* customers in the system
p_j	%	Probability of a number *j* of *jobs* in the system.	$p_{p,j}$	%	Probability of a number *j* of *power* customers in the system.
$p_{nq,i}$	%	Probability of a *job* to arrive at the *i*-th department and to find a *non-empty queue*.			

3.3. Modelling the production system by Queuing Theory

The arrival of a *job* to a machine is an input to the production, which requires to switch-on the machine from a low power level (equal to zero, if the idle machine is switched-on at the arrival of the *job* to be processed, or equal to a minimum power level, if the idle machine is in a stand-by state) to a higher power level (i.e. the power level required for production). Considering the whole set of the jobs to be processed on the machine and its inter-arrival time distributions, it is possible to obtain the distribution of the start-times at the machine itself, which corresponds to what can be defined as the arrival process of the "*power* customers". This concept will be further detailed and it can be extended to the whole production system: the start-time distribution of the whole set of resources (modelled as a single production system) can be represented as a general inter-arrival times distribution **A**, where $E(\mathbf{A})=1/\lambda$, so that λ is the arrival rate of the power customers [kW/h].

In a manufacturing environment, machine loading (scheduling) is defined on the basis of information on products (such as bills of materials, production mix and volumes), production times and system features (such as the machines involved, equipment and tools). The production times of the product–resource pairs can be extracted from the information system (as well as bills of materials and other information). Following the approach adopted above for the arrival time distributions, it can be assumed that the processing times of the system (i.e. considering all the production resources as a whole) are described by a general service-time distribution; in particular, the processing time of each *job* (i.e., *customer*) pertains to a distribution **S**, with $E(\mathbf{S})=1/\mu$, so that μ [kW/h] is the service frequency.

According to the traditional approach, when the power and energy requirements of a manufacturing system are designed, two aspects are mainly considered: (1) the load factor and (2) the coincidence factor, both depending on the resource typology and, in the case of the coincidence factor, also on the number of resources for each type. The load factor represents the mean power used by a machine with respect to its nominal power, expressed as a percentage. The coincidence factor takes into account the possible processing overlaps of resources (of the same type) in the given production system, also expressed as a percentage. Usually, design handbooks offer these factor values: in the present study, they have been empirically set according to the practical experience of experts in the design of manufacturing system.

Considering a production system consisting of different departments (i.e., the system has a job-shop structure), each department can be regarded as a queuing model, as well as the production system as a whole. Each department (or the whole system) is described by two statistical distributions: the inter-arrival times of *customers* and the service times at the department itself. Based on these distributions, a set of indices may be assessed to appreciate the system performance by the queuing model. In the energy-based perspective, *customers* represent the power levels reached at the department (system), so that the mean number of customers in the system corresponds to the mean power level required by the production activities [kW]: this value enables the computation of the average energy used [kW h], when multiplied by the mean service time [h] (i.e., the mean duration of production activities). Moreover, each power level is linked to its occurrence probability, thus the contractual power may be properly assessed considering the profile of these probabilities and weighing the service level required to the electrical supply.

4. Notation and assumptions

According to the Kendall-Lee notation, the queuing model proposed for the *power customers* is a GI/G/∞/GD/∞/∞ (an infinite-server system, in which inter-arrival and service times follow general probability distributions, and the maximum allowed number of customers in the system is infinite). The Queuing Theory approach allows the calculation of the probability that a given number of customers will be in the system, thus offering the possibility to calculate the service level obtained when setting a value to the contractual power level (i.e., the number of customers).

4.1. The system considered: Configuration and assumptions

The Queuing Theory approach is proposed for a job-shop environment and considering the power viewpoint. The features of the job-shop and production (number of resources and production rates, *job* cycles and production mix) allow the modelling of the manufacturing system by the statistical distribution of the job inter-arrival and their processing times. As the purpose of this contribution is to introduce the perspective of power needs, each arrival represents the requirement of a specific power level (expressed in kW) related to the resources involved. In other terms, each *job* arriving at the manufacturing resource requires the engaged resource to switch on and to reach the "manufacturing power level". According to this perspective, the system is modelled as a unique resource characterised by inter-arrival and service distributions which consider power requirements (i.e., kW) as customers.

4.2. The adopted notation

Table 2 offers a complete overview of the notation adopted, together with the description of the parameters involved, distinguishing those used for the *job-queueing* model from those related to the *power-queueing* model.

5. Model formulation

5.1. The job-queuing model

The demand of products must be satisfied by a production system according to its department organisation and to the route sheets of the different products: in this case, it is possible to study the system behaviour by applying a standard *job-queuing model*.

The arrival rate of *jobs* at the *i*-th department can be defined as λ_i, while the processing time of each *job* at the *i*-th department is μ_i. The utilisation coefficient ρ_i (equal to λ_i/μ_i in the case of a single-server model) may be also computed. The average number of *jobs* in the *i*-th manufacturing department is:

$$L_i = \frac{\lambda_i}{(\mu_i - \lambda_i)}, \tag{1}$$

and the corresponding average time spent by each *job* in each manufacturing department is W_i as Little's law is assumed to hold:

$$W_i = \frac{L_i}{\lambda_i}. \tag{2}$$

5.2. The power-queuing model

From the energy-use point of view, the arrival rate $\lambda_{p,i}$ at each department i represents the parameter characterising the statistical distribution of the production starts at the *i*-th department. In other terms, when a physical job is going to be processed at the *i*-th department, (1) it can immediately start its production, or (2) it must wait in the queue in front of the department, until it is available.

The arrival rate of the *power-queuing* model can be calculated as:

$$\lambda_{p,i} = P_{nom,i} \cdot f_{l,i} \cdot \lambda_i \tag{3}$$

where $P_{nom,i}$ and $f_{l,i}$ respectively represent the nominal power and the load factor of the *i*-th department and subscript "*p*" is used to represent the "*power*" nature of the customers in the system. For the power-queuing, a GI/G/∞/GD/∞/∞ model is proposed: it does not require any assumption on *exponential* distributions of arrivals and departures, which can be formulated as *general* distributions. When the whole productive system is considered, the following expressions, related to mean inter-arrival and mean service times, hold:

$$\lambda_p = \sum_{i=1}^{N} \lambda_{p,i}, \tag{4}$$

$$\frac{1}{\mu_p} = \frac{1}{\lambda_p} \sum_{i=1}^{N} \frac{\lambda_{p,i}}{\mu_i}. \tag{5}$$

It is interesting to highlight how the proposed GI/G/∞/GD/∞/∞ queuing model does not consider load or coincidence factors: it considers *power* customers, so that only statistical distributions of the overall inter-arrival process times, as well as of the overall service times of machines involved, are necessary. When the inter-arrival times of *power* customers are considered to be exponentially distributed, it is shown that, even for arbitrary service time distributions, the steady state probability of the *j*-th *power* customer (namely $p_{p,j}$) follows a Poisson distribution with mean λ_p/μ_p, thus implying that (Winston, 2003):

$$p_{p,j} = \frac{\left(\frac{\lambda_p}{\mu_p}\right)^j e^{-\left(\frac{\lambda_p}{\mu_p}\right)}}{j!}. \tag{6}$$

According to Eq. (6), it is possible to calculate the occurrence probability associated with each power level, as well as the mean power required. This information allows the calculation of the service level at each contractual power level $P_{contr,j}$: the service level represents the probability of a power request lower than a P_{contr} threshold, i.e. the summation of $p_{p,j}$ probability terms in Eq. (6) from $j=0$ to P_{contr}. The information provided is helpful while setting the economic trade-off represented by the power level agreed in a supply contract.

In Queuing Theory models, the average number of customers in the system and the mean service time are common performance indicators. The average number of customers in the system is computed as a weighted sum of the possible values of customers in the system, weighted by their specific occurrence probability. In the model proposed, the number of customers represents the power level reached at the production resource and its variability stands for the variable levels that the power requested for production may reach. As arriving customers are the events related to production starts, the average number of *power* customers in the system, L_p, is determined as follows:

$$L_p = \sum_{j=0}^{\infty} j \cdot p_{p,j}, \tag{7}$$

where j represents the power level of j [kW] and $p_{p,j}$ is the related occurrence probability. Therefore, L_p is the *average* power required by the system and the probability distribution of the number of *power* customers in the system may be useful to set the contractual power level. The contractual power set will be associated with its probability to be exceeded, too, thus bringing additional information to the system behaviour.

According to Fig. 4, the L_p value is the average power required by the system, thus offering an interesting viewpoint of Queuing Theory and Little's law, when applied to power utilisation in systems. In fact, as in (2), the λ_p arrival rate of power customers [kW/h], multiplied by their average time in system W_p [h], must give the average power required by the system L_p [kW].

In an industrial context, the supplier of electric power always fulfils the power request of a manufacturing department, unless the request exceeds the predetermined contractual power level: this threshold represents the limit above which the electricity supply may be interrupted or charged of penalties. Nevertheless, under the constraint of not exceeding the contractual power, a manufacturing department is generally motivated in monitoring its power needs, both in average, distribution and maximum level reached.

6. Numerical application

A numerical application is proposed to show the applicability of the model presented.

We consider the base case of a production system with a product demand equal to 15 [job/h] (i.e. the overall *system* arrival rate) and a mean service time equal to 0.05 [h] for each production stage (i.e., 0.05 h are spent on average by *jobs* at each visited department). The job-shop consists of four departments (D1, D2, D3 and D4), visited by *jobs* according to the following routings.

- 25% of *jobs* visiting department D1, then D3 and finally D4.
- 25% of *jobs* visiting department D1, then D2, D3 and finally D4.
- 25% of *jobs* visiting department D2, then D3 and finally D4.
- 25% of *jobs* visiting department D2, then D4.

Table 3
Numerical example data, job-queueing model.

Department	λ_i (job/h)	$1/\mu_i$ (h)	ρ_i (%)
D1	7.50	0.05	0.3750
D2	11.25	0.05	0.5625
D3	11.25	0.05	0.5625
D4	15.00	0.05	0.7500

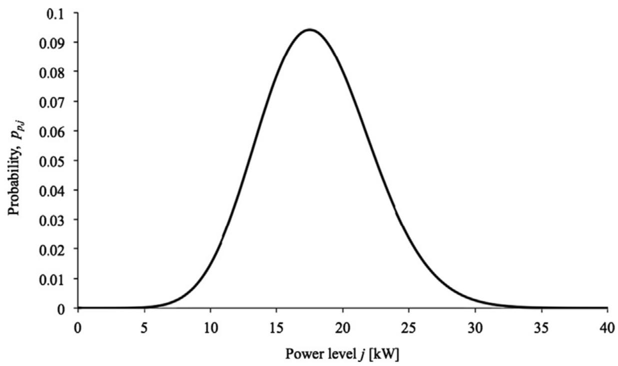

Fig. 5. Probability distribution of power levels.

Table 4
Parameters of the power-queueing model.

λ_p (kW/h)	$1/\mu_p$ (h)	L_p (kW)
360	0.05	18.00

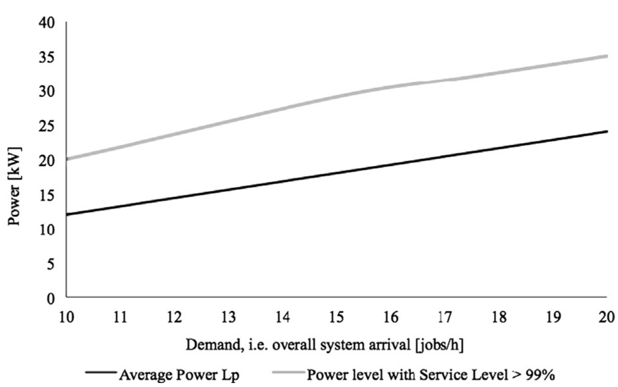

Fig. 6. Sensitivity analysis for different demand rates.

Considering the *job-queuing* model, the following parameter values have been computed for the involved departments, each one modelled as a M/M/1 queuing model.

Based on Table 3 values and assuming $P_{nom,i}=10$ [kW] and $f_{l,i}=0.8$ for each department, the parameters of the *power-queueing* model are reported in Table 4:

Moreover, the probabilities associated with each power level are reported in Fig. 5. As anticipated, the probability distribution leads to the appreciation of the service level associated with a given contractual power level.

The standard approach for the calculation of the contractual power level requires the joint estimation of the load and coincidence factors. In particular, the International Standard IEC 60439-1 (2004) recommends the use of a load factor equal to 0.8 for resources with nominal power of 10 [kW] and a coincidence factor, related to the four resources of the type considered (motors), equal to 0.8. Thus, the straight calculation leads to a contractual power level equal to 25.6 [kW]. Thanks to the results of Fig. 5, we can calculate the service level associated with this power level, i.e. 95.54%. Moreover, the power-queuing model allows the calculation of the contractual power level that guarantees a specific service level: for the numerical example considered, the 99% service level corresponds to a 29 [kW] power level.

6.1. Sensitivity analysis

So as to investigate the sensitiveness of the model to different levels of arrival rates (demand to be satisfied), the power level of the system has been computed for arrival rates in the range 10÷20 jobs/h. It should be noted that due to *job* routing, arrival rates and service times at the different manufacturing departments, it is not possible to increase the system arrival rate exceeding 20 [jobs/h], in order not to exceed any department utilisation coefficient. In fact, the *power-queuing* model depends on the *job-queueing* one, as shown in Fig. 1. Results of Average power Lp and Power level that guarantees the 99% of service level are shown in Fig. 5.

As expected, Fig. 6 shows how the larger the system arrival rate (demand increases), the larger the power level, as well as the energy requirement.

7. Discussion and conclusions

Nowadays, production systems require a careful appreciation of their energy requirements, so as to efficiently design their energy supply system and competitively manage production. However, when historical data are not available (as in cases of absence of energy-monitoring systems, green-field or revamping situations), the unavoidable approach is to model the expected power level and the consequent requirement of energy.

The present work describes a simple modelling tool, based on elementary production data, to support managers in their decision-making on energy issues, in particular by modelling the production system from the viewpoints of electrical power level and energy usage.

To this end, a simple model, derived from the Queuing Theory, has been proposed, modelling the variability of the electrical power use and the related probability of occurrence related to different power levels for the manufacturing system considered. The proposed queuing approach seems to be a promising and integrative/alternative method to the *traditional* one, based on load and coincidence factors. When compared to simulation-based tools, its main advantage lies in the prompt availability of results, based on analytical formulae, and the consequent possibility of foreseeing and exploring alternative scenarios. The reliable utilisation of the method should be supported by the analysis of multiple industrial/civil cases but, since now, it could efficiently integrate the traditional approach, being a critical comparison of it. In fact, the model enables the calculation of the occurrence probability associated with each power-peak demand, i.e. the probability to trespass the contractual power set by contract: this economic aspect particularly deserves further investigation, as it seems to be promising as a key-tool for proper contract design, due to its relevance in practice. In fact, also the time-duration of peak demand should be deeply investigated by further developments of the study. Additional research issues may include the integration of the *power-queuing* model by the idle state of machines and batch arrivals, which describe the cases of heavy power requests.

In conclusion, future works may encompass industrial cases differing in production and energy costs, as the model proposed interestingly offers the opportunity for quickly appreciating relevant costs, such as the contractual electrical power, the maximum electrical power and the energy used in a given period. Therefore, it seems to be a promising technique for the design of energy-supply systems in productive environments. To this end, it should be remarked that the traditional approach is a "static" one, while the present approach is based on a probabilistic view of the design problem and its design by a joint combination of *job* and *power-queuing* models. Finally, additional investigations could be usefully devoted to the modelling of other energy supplies of productive systems (such as heating, compressed air, etc…), that can be included into the proposed approach.

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