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Analysis of the effect of voltage level requirements on an electricity market equilibrium model

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ABSTRACT

This paper presents a conjectural-variation-based equilibrium model of a single-price electricity market. The main characteristic of the model is that the market equilibrium equations incorporate the effect of the voltage constraints on the companies' strategic behavior. A two-stage optimization model is used to solve the market equilibrium. In the first stage, an equivalent optimization problem is used to compute the day-ahead market clearing process. In the second stage, some generation units have to modify their active and reactive power in order to meet the technical constraints of the transmission network. These generation changes are determined by computing an AC optimal power flow.

Introduction

Deregulation in electric power systems has been conducted using different processes in the past decades in several countries. Electric power systems have gone from being centralized and vertically integrated to systems with different degrees of competition in their different activities. In the generation activity, electricity markets were created to determine the amount of energy scheduled of the generation units, as well as the ancillary services that they should provide in order to maintain system stability.

Several models have been developed to study electricity markets. Usually, these models are based on game theory and they try to determine the outcome of the interaction between different companies under the hypothesis of rational behavior. The companies' behavior is modeled using a strategic game where companies take an action knowing that the rest of companies play in the same way. Among the game theory models are Perfect Competitive models [1,2], Cournot models where companies compete in quantities [3–8], Bertrand models where companies compete in prices [3], Supply Function Equilibrium models where strategic behavior is modeled by means of supply functions that combine price and quantity competition [9–16], and Conjetural Variation Based Equilibrium models where the supply functions are parametrized with a parameter known as the company's conjecture [11,17–25].

Most of these models have focused on solving the day-ahead electricity market and they disregard ancillary service markets and mechanisms used to clear the different technical constraints that may appear on the electric power system. Some models include the effect of network congestion on the companies' strategic behavior [1,3–6,12–16,19–23]. However, they only study the congestion caused by the thermal limits of the transmission lines. Therefore, they use a DC approximation of the power flow equations, and it is not possible to analyze other technical constraints such as voltage constraints or reactive power requirements.

Few models [2,7–11,17] study the effect of voltage constraints on the companies' strategic behavior. However, all of them are focused on nodal-price electricity markets, and none of them assess the effect on single-price electricity markets. Almeida and Senna [2] proposed a bilevel optimization problem that models the active and reactive power dispatch under competence. The first level corresponds to the active power market and the second level minimizes the opportunity cost of the reactive power which is defined in terms of the marginal price of the power active market. Bautista et al. [7,8] presented a Cournot model to study the influence of the reactive power requirements on the active power dispatch. These works argue that the DC approximation of the power flow is not accurate enough because it does not take into account the capability curve of the generation units that models the tradeoff between active and reactive power. Bautista et al. [9] was an extension of the previous approaches using a supply function equilibrium model. Soleymani [10] developed a supply function equilibrium model for optimal bidding strategy of generation companies in active and

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reactive power markets, where the companies have incomplete information about their rivals. Petoussis et al. [11] assessed different parametrization methods of the companies' supply functions in an active power market taking into account an AC representation of the network. Chitkara et al. [17] proposed a model to analyze the companies' strategic behavior in a reactive power market. This model assumes that the active power is already scheduled, thereby there is no feedback between the reactive and active power markets, i.e., reactive power requirements do not modify strategic behavior in the active power market.

This paper presents a conjectural-variation-based model of a single-price electricity market. The main characteristic of this model is that the companies' strategic behavior takes into account the effect of the voltage constraints. The market equilibrium equations are solved by means of a two-stage optimization problem. In the first stage, a mixed complementary problem models the day-ahead market clearing process. In the second stage, an optimal power flow is solved to determine the changes in active and reactive power needed to meet the voltage system requirements. Moreover, this paper presents an iterative algorithm to resolve the two-stage optimization problem. This model is based on the model proposed in [23]. The main difference between the two models is that the model in [23] only analyzes the effect of network congestion caused by the thermal limits of the transmission lines. Thus, the model in [23] uses a DC-OPF which assumes that there is enough reactive power compensation in all nodes to maintain voltage at the desired level, so the terms related to reactive power are discarded and the voltage levels are equal to 1 p.u. in all nodes. However, this DC approximation is not suitable to study the effect of the voltage level requirements because it is not possible to assume that voltage levels are constant in all nodes. Hence, the model presented in this paper uses an AC-OPF to properly model the voltage requirements at the transmission network. It is important to point out that in recent years optimal power flow has been used to assess the operation of the electricity systems not only in high voltage levels but also in medium and low voltage levels in the distribution grids, e.g., [26–28] studied the optimal operation of the system taking the integration of renewable generation, distributed generation and microgrids into account.

The remainder of this paper is organized as follows: Section 'Market equilibrium model' presents the market equilibrium model that includes the effect of the voltage constraints on the companies' strategic behavior. Section 'Numerical example' provides and analyzes a numerical example. Finally, Section 'Conclusions' draws the most relevant conclusions.

Market equilibrium model

This section generalizes the model presented in [23] in order to study the effect of voltage constraints on the companies' strategic behavior in a single-price electricity market. In the electricity market, the scheduled day-ahead generation is usually determined first. Then, a subsequent procedure is carried on if the day-ahead market solution does not meet the technical requirements necessary to maintain system stability. Different technical constraints are assessed and the power produced by units may change with respect to the scheduled day-ahead generation.

Market clearing conditions

The day-ahead market clearing process determines the active power P_j of each generation unit j as well as the market price λ. Since it is a single-price electricity market, the total generation and demand have to be balanced (1) and the market price λ is equal to the bid of the marginal unit:

$$\sum_{j\in J} P_j = \sum_{a\in A} DP_a + losses \tag{1}$$

Subsequently, the changes in production necessary to maintain system stability are determined using a mechanism to solve the technical constraints. There are different schemes to remunerate the power active changes as presented in [29–32]. In this paper, the Spanish mechanism [29] is modeled in which the power active increments X_j are paid at the price γ while the reductions W_j are charged at the day-ahead market price λ. In order to maintain the system active power balance, the total active power increment is equal to the total active power reduction:

$$\sum_{j\in J} X_j = \sum_{j\in J} W_j \tag{2}$$

The company's problem

A generation company i will try to maximize its profit by determining the production of its generation units, P_j, as well as the production changes, X_j and W_j, required to meet the technical system constraints. Moreover, since the generation company behaves strategically, it can change the electricity prices when the production of its units changes. This strategic behavior could be modeled by means of the parameters θ_i and β_i. θ_i corresponds to the conjectured-price response in the day-ahead market [18] and β_i to the conjectured-price response in the subsequent mechanism.

Since the reductions are charged at the day-ahead market price, it is possible to represent the quantity reduced W_j as a ratio of the day-ahead market production P_j, i.e., $W_j = m_j \cdot P_j$, where m_j represents the proportion of the active power generation that unit j has to reduce in order to meet the network constraints. Thus, the value of m_j has to be computed taking into account the power flow constraints.

Therefore, the profit maximization problem of company i is:

$$\max_{\lambda_i,\gamma_i,P_j,X_j} \lambda_i \cdot \sum_{j\in J_i}(1-m_j)\cdot P_j + \gamma_i \cdot \sum_{j\in J_i} X_j - \sum_{j\in J_i} C((1-m_j)\cdot P_j + X_j) \tag{3}$$

s.t.

$$\lambda_i = \lambda^* - \theta_i \cdot \left(\sum_{j\in J_i} P_j - \sum_{j\in J_i} P_j^*\right) \tag{4}$$

$$\gamma_i = \gamma^* - \beta_i \cdot \left(\sum_{j\in J_i} X_j - \sum_{j\in J_i} X_j^*\right) \tag{5}$$

$$\overline{P_j} - P_j \geqslant 0 : (\overline{\mu_j}) \quad \forall j \tag{6}$$

$$\overline{P_j} \cdot w_j - X_j \geqslant 0 : (\overline{\nu_j}) \quad \forall j \tag{7}$$

$$\overline{P_j} - P_j - X_j \geqslant 0 : (\overline{\xi_j}) \quad \forall j \tag{8}$$

$$P_j \geqslant 0, \quad X_j \geqslant 0 \quad \forall j \tag{9}$$

In the event that the scheduled active power determined in the day-ahead market does not meet the technical system constraints, the units' generation has to be modified in the subsequent mechanism. Assuming that these modifications happen on a regular basis, the companies can predict them, and may use this information to behave strategically. Thus, in the company's optimization problem, this information is modeled using the reduction factors, m_j, and the binary variables, w_j, that indicate which units have to increase generation. Both are determined in the subsequent mechanism as shown in Section 'Subsequent mechanism'. The Eq. (3) is the profit of the company i. Constraints (4) and (5) represent how the company conjectures that electricity prices will change if the company changes its production. Each company i has an estimation of the prices λ_i and γ_i. However, in the equilibrium these prices are equal to the day-ahead market price, λ^*, and the price of the active power increments, γ^*, respectively. Constraint (4) is the conjecture for the day-ahead market price, in which the company assumes that λ_i deviates from λ^* if the company's active power generations P_j change from their equilibrium values P_j^*. Constraint (5) is the conjecture for the subsequent mechanism, showing how γ_i changes from γ^* when the increments of active power of the generation units, X_j, are shifted from X_j^*. Constraints (6)–(9) are the boundaries of the variables.

Market equilibrium

By gathering together the first-order conditions for all companies and then adding the market-clearing conditions, the mixed complementarity model MCP (10)–(16) can be defined, and the market equilibrium corresponds to the solution of this MCP. An alternative way to compute this market equilibrium is by means of an equivalent quadratic optimization problem as shown in [23]. However, that methodology was not successful in solving this problem because the power balance constraints have to be modified in each iteration and the convergence of the procedure is not guaranteed.

$$\sum_{j\in J} P_j = DP \qquad \lambda \text{ unrestricted} \tag{10}$$

$$\sum_{j\in J} X_j = Y \qquad (\gamma \text{ unrestricted}) \tag{11}$$

$$0 \leqslant \overline{\mu_j} \perp \overline{P_j} - P_j^* \geqslant 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{12}$$

$$0 \leqslant \overline{\nu_j} \perp \overline{P_j} \cdot w_j - X_j^* \geqslant 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{13}$$

$$0 \leqslant \overline{\xi_j} \perp \overline{P_j} - P_j^* - X_j^* \geqslant 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{14}$$

$$0 \leqslant P_j^* \perp -(1-m_j)\cdot\lambda^* + \theta_i \cdot \sum_{k\in J_i}(1-m_k)\cdot P_k^* + (1-m_j)\cdot MC_j\left((1-m_j)\cdot P_j^* + X_j^*\right) + \overline{\mu_j} + \overline{\xi_j} \geqslant 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{15}$$

$$0 \leqslant X_j^* \perp -\gamma^* + \beta_i \cdot \sum_{k\in J_i} X_k^* + MC_j\left((1-m_j)\cdot P_j^* + X_j^*\right) + \overline{\nu_j} + \overline{\xi_j} \geqslant 0 \qquad \forall j \in J_i,\ \forall i \in I \tag{16}$$

Eqs. (10) and (11) are the market-clearing constraints. The values of *DP* and *Y* are the total active power demand and the total active power increments, and they are computed using an iterative procedure as presented in Section 'Solution methodology'. The generation company's behavior is modeled by means of Eqs. (12)–(16). These equations are the Karush–Kuhn–Tucker (KKT) conditions of the problem (3)–(9) for each company *i*. It is important to note that the variables λ_i and γ_i are substituted by constraints (4) and (5), respectively. Moreover, since the solution of the MCP corresponds to the equilibrium, the production variables P_j and X_j are replaced by the equilibrium variables P_j^* and X_j^*, respectively. The operator $\perp$ denotes the inner product of two vectors equal to zero, i.e., $0 \leqslant x \perp f(x) \geqslant 0$ corresponds to the system equations $x \geqslant 0, f(x) \geqslant 0$ and $x \cdot f(x) = 0$.

For units whose productions P_j^* and X_j^* are less than the maximum values, i.e., constraints (6)–(8) are not binding and the dual variables $\overline{\mu_j}$, $\overline{\nu_j}$ and $\overline{\xi_j}$ are equal to zero, the Eqs. (15) and (16) could be written as:

$$\lambda^* = MC_j\left((1-m_j)\cdot P_j^* + X_j^*\right) + \frac{\theta_i}{(1-m_j)} \cdot \sum_{k\in J_i}(1-m_k)\cdot P_k^* \tag{17}$$

$$\gamma^* = MC_j\left((1-m_j)\cdot P_j^* + X_j^*\right) + \beta_i \cdot \sum_{k\in J_i} X_k^*. \tag{18}$$

The right-hand side on (17) and (18) corresponds to the *apparent cost* of the unit in the day-ahead market and in the subsequent mechanism, respectively. The *apparent cost* is defined as the equivalent marginal cost perceived by the system when the unit produces a determined quantity in the market [33]. In the apparent cost perceived by the company in the day-ahead market, the conjectured-price response is modified by factor $1/(1-m_j)$ which is greater than 1 when $m_j > 0$. This means that the company perceives that this unit is more expensive in the day-ahead market because it knows that the active power of the unit has to be reduced in the subsequent mechanism in order to meet the technical system constraints.

Subsequent mechanism

In single-price electricity markets, a procedure is used to clear the technical system constraints when the day-ahead market solution is not technically feasible. This procedure is subsequent to the day-ahead market and determines the changes in active power as well as reactive power needed to maintain system stability. With those results, the companies can determine the reduction factors m_j and which units increase active power generation ($w_j = 1$). The optimal power flow (19)–(29) models this procedure. In [23], the OPF is solved using a DC approximation in which the voltage levels are fixed to 1 p.u. and the reactive power and system losses are disregarded. The DC approximation is valid to analyze the effect of congestion due to thermal limits of the lines. However, in order to study the effect of voltage requirements it is necessary to use an AC-OPF where the voltage levels are not fixed and the active and reactive power levels are taken into account.

$$\min_{\Xi} \sum_{j\in J} ACX_j \cdot X_j^{\Omega} + (K - ACW_j)\cdot W_j^{\Omega} \tag{19}$$

s.t.

$$\sum_{j\in J_a} P_j^{\Omega} - DP_a = \sum_{b\in A} V_a \cdot V_b \cdot (G_{ab}\cos(\delta_a - \delta_b) + B_{ab}\sin(\delta_a - \delta_b)) \ \forall a \in A \tag{20}$$

$$\sum_{j\in J_a} Q_j^{\Omega} - DQ_a = \sum_{b\in A} V_a \cdot V_b \cdot (G_{ab}\sin(\delta_a - \delta_b) - B_{ab}\cos(\delta_a - \delta_b)) \ \forall a \in A \tag{21}$$

$$P_j^{\Omega} = P_j + X_j^{\Omega} - W_j^{\Omega} \qquad \forall j \in J \tag{22}$$

$$\underline{V_a} \leqslant V_a \leqslant \overline{V_a} \qquad \forall a \in A \tag{23}$$

$$0 \leqslant X_j^{\Omega} \leqslant \overline{P_j} \qquad \forall j \in J \tag{24}$$

$$0 \leqslant W_j^{\Omega} \leqslant \overline{P_j} \qquad \forall j \in J \tag{25}$$

$$\underline{P_j} \cdot u_j \leqslant P_j^{\Omega} \leqslant \overline{P_j} \cdot u_j \qquad \forall j \in J \tag{26}$$

$$\underline{Q_j} \cdot u_j \leqslant Q_j^{\Omega} \leqslant \overline{Q_j} \cdot u_j \qquad \forall j \in J \tag{27}$$

$$Q_j^{\Omega} \leqslant Q_j^{0,max} \cdot u_j + n_j^{max} \cdot P_j^{\Omega} \qquad \forall j \in J \tag{28}$$

$$Q_j^{\Omega} \geqslant Q_j^{0,min} \cdot u_j + n_j^{min} \cdot P_j^{\Omega} \qquad \forall j \in J \tag{29}$$

$$u_j \in \{0,1\} \qquad \forall j \in J \tag{30}$$

where the decision variables are $\Xi = \left\{P_j^{\Omega}, X_j^{\Omega}, W_j^{\Omega}, V_a, \delta_a, u_j\right\}$. The objective function (19) minimizes the *apparent cost* of the changes in active power with respect to the day-ahead market solution. The apparent cost is used because it corresponds to an equivalent marginal cost perceived by the system. In the model, two different apparent costs ACX_j and ACW_j have been considered. The apparent cost ACX_j corresponds to the cost when the generation unit *j* has to increase its active power while ACW_j corresponds to the cost when it has to reduce its active power. Therefore, in the minimization problem, the units with lower apparent cost ACX_j increase their active power while the units with higher apparent cost ACW_j reduce their active power production. The term *K* is a constant higher than the maximum value of ACW_j. Constraints (20) and (21) are the power flow equations for active and reactive power, respectively. Constraints (22) relate the active power in the OPF with the active power in the day-ahead market. Constraints (23)–(29) establish the minimum and maximum bounds of the variables. A linear approximation of the P–Q capability curve of the generation units, known as *D-curve*, is modeled with constraints (26)–(29) where $Q_j^{0,max}, Q_j^{0,min}, n_j^{max}, n_j^{min}$ are parameters of the lineal approximation of this curve as illustrated in Fig. 1, where the shaded portion represents the feasible operating region for the unit. This curve models the trade-off between active and reactive power of the generation units, and therefore it determines the feasible operation region where it is not possible to produce the maximum active power and maximum reactive power at the same time. It is important to note that the binary variables u_j are necessary to meet the minimum and maximum requirements of the generation units, and to avoid solutions in which the active power of a unit is below the minimum to generate more reactive power. Finally, constraints (30) indicates that variables u_j are binary.

Solution methodology

An iterative algorithm similar to the one presented in [23] is used to determine the market equilibrium taking into account the power changes required to meet the technical constraints:

1. Initialize the variables $\kappa = 1$, $m_j^{(\kappa)} = 0$, $w_j^{(\kappa)} = 0$, $DP^{(\kappa)} = \sum_a DP_a$, $Y^{(\kappa)} = 0$. These values correspond to the case without network constraints.
2. Solve the MCP (10)–(16). This gives a solution for P_j^*, X_j^*, λ^*, γ^*.
3. Update the active power, prices and apparent cost values:

$$P_j^{(\kappa)} = \alpha \cdot P_j^* + (1-\alpha) \cdot P_j^{(\kappa-1)} \quad (31)$$

$$X_j^{(\kappa)} = \alpha \cdot X_j^* + (1-\alpha) \cdot X_j^{(\kappa-1)} \quad (32)$$

$$\lambda^{(\kappa)} = \alpha \cdot \lambda^* + (1-\alpha) \cdot \lambda^{(\kappa-1)} \quad (33)$$

$$\gamma^{(\kappa)} = \alpha \cdot \gamma^* + (1-\alpha) \cdot \gamma^{(\kappa-1)} \quad (34)$$

$$ACX_j^{(\kappa)} = MC\left(\left(1-m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)}\right) + \beta_i \cdot \sum_{k \in J_i} X_k^{(\kappa)} \quad (35)$$

$$ACW_j^{(\kappa)} = MC\left(\left(1-m_j^{(\kappa)}\right) \cdot P_j^{(\kappa)} + X_j^{(\kappa)}\right) + \frac{\theta_i}{\left(1-m_j^{(\kappa)}\right)} \cdot \sum_{k \in J_i} \left(1-m_k^{(\kappa)}\right) \cdot P_k^{(\kappa)} \quad (36)$$

   the learning rate α is used to achieve a smooth convergence in the value of the variables, and to prevent the solution from jumping between different values. A value of $\alpha = 1$ means that the variables are updated using only the information given in the last iteration while a value of $\alpha = 0$ means that only the information given in the first iteration is used.
4. Solve the AC-OPF (19)–(29). This gives a solution for $P_j^\Omega, X_j^\Omega, W_j^\Omega, V_a, \delta_a, u_j$.
5. Update the reduction factor, the units that increase their generation and the demand values:

$$P_j^{\Omega(\kappa)} = \alpha \cdot P_j^\Omega + (1-\alpha) \cdot P_j^{\Omega(\kappa-1)} \quad (37)$$

$$X_j^{\Omega(\kappa)} = \alpha \cdot X_j^\Omega + (1-\alpha) \cdot X_j^{\Omega(\kappa-1)} \quad (38)$$

$$W_j^{\Omega(\kappa)} = \alpha \cdot W_j^\Omega + (1-\alpha) \cdot W_j^{\Omega(\kappa-1)} \quad (39)$$

$$m_j^{(\kappa)} = \alpha \cdot \frac{W_j^\Omega}{P_j^\Omega} + (1-\alpha) \cdot m_j^{(\kappa-1)} \quad (40)$$

$$w_j^{(\kappa)} = \begin{cases} 1 \text{ if } X_j^{\Omega(\kappa)} > 0 \\ 0 \text{ if } X_j^{\Omega(\kappa)} = 0 \end{cases} \quad (41)$$

$$DP^{(\kappa)} = \sum_{j \in J} P_j^{\Omega(\kappa)} \quad (42)$$

$$Y^{(\kappa)} = \sum_{j \in J} W_j^{\Omega(\kappa)} \quad (43)$$

6. If the change of the variables is lower than an ϵ value, the algorithm stops; otherwise increase the iteration counter κ and go to 2.

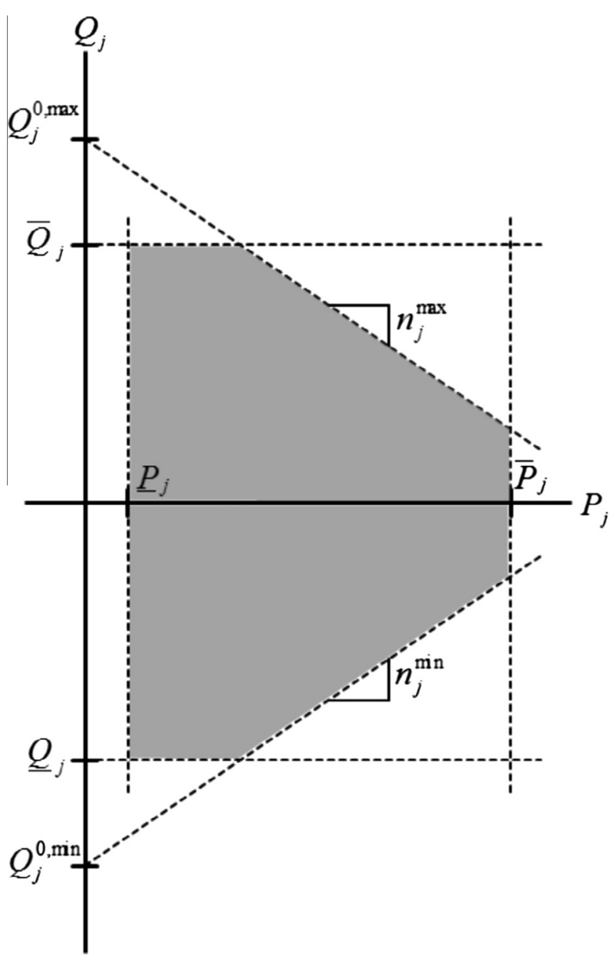

Fig. 1. P–Q capability curve.

Numerical example

This section presents a simple example to study the effect of voltage constraints on the companies' strategic behavior. The market equilibrium is solved using PATH [34] in GAMS [35] and the AC-OPF is solved using MATPOWER [36] in Matlab [37].

The power network has 3 nodes connected by 3 transmission lines as shown in Fig. 2 and Table 1. It is important to note that

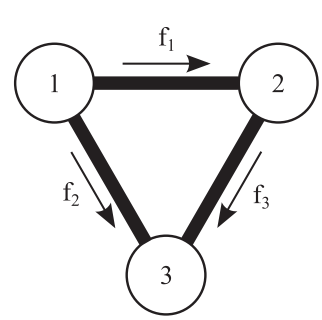

Fig. 2. Three-node system.

Table 1
Parameters of the lines.

From node	To node	Resistance (p.u.)	Reactance (p.u.)	Susceptance (p.u.)
1	2	0.12	0.35	0.01
1	3	0.24	0.70	0.01
2	3	0.24	0.70	0.01

The parameters are in the base of 100 MVA.

the values of the parameters of the transmission lines are significantly higher than the actual parameters, in order to highlight the effect of voltage requirements. The demand is equal to 100 MW and 35 Mvar and it is concentrated at node 3. The three nodes have generation units; however, the units located at node 3 are the most expensive. Thus, the day-ahead market solution is that units at nodes 1 and 2 have to supply the demand at node 3. If this was the final solution, there would be a significant voltage drop in the lines 1–3 and 2–3 caused by the impedance of these lines. In that case, the voltage level at node 3 would be lower than the specified minimum (0.95 p.u.).

Three different cases are analyzed. In case A, companies 1 and 2 own generation units at nodes 1 and 2, and there is only one unit at node 3 owned by company 3. Therefore, this unit is the only one that can solve the voltage drop at node 3. In case B, there are the same units as case A, but company 1 also has one generation unit at node 3, so there are now 2 units that can solve the voltage requirements. Finally in case C, the three companies have generation units at node 3. In the three cases, the strategic behavior of company 3 is studied modifying its conjectured-price response in the subsequent mechanism, β, from the case in which the company does not exercise market power, i.e., $\beta_3 = 0$, and increasing the market power to $\beta_3 = 0.1$ and $\beta_3 = 1$. The data of generation units and the conjectured-price response of the companies are shown in Tables 2,3, respectively.

In the AC-OPF (19)–(29) the binary variables u_j are the commitment variables of the generation units. Since MATPOWER cannot compute binary variables in the solution of the OPF, the approach taken is therefore to evaluate all the possible combinations of these binary variables and select the case with the lowest value in the objective function. In this numerical example, there are 7 generation units, and therefore $2^7 = 128$ possible combinations for the commitment variables u_j. This means that it is necessary to solve the AC-OPF 128 times in each iteration. In this numerical example, the computational time to solve the 128 AC-OPF is around 6 s. Although that approach could be valid in small-size power systems, it would be intractable in large-size power systems in which the number of generation units is considerably higher, what makes the number of combinations growth exponentially, and solving one AC-OPF may take several minutes. Another inconvenience is that the algorithm used to find the optimal solution could converge to a local optimum depending on the initial values of the variables, so different solutions could be found in the iterative procedure used in this model and there is no certainty about the convergence of the model. In practice, the solution methodology has achieved satisfactory results in terms of convergence as shown in [23]. In the iterative procedure, about 1000 iterations are necessary with a learning rate $\alpha = 0.01$ to achieve a convergence of the variables. The total computational time of the 1000 iterations is around 110 min.

Table 3
Conjectured-price response.

Company i	θ ((€/MW h)/MW)	β ((€/MW h)/MW)
1	0.05	0.1
2	0.05	0
3	0	0–0.1–1

Table 4
Voltage levels.

Node a	First iteration V (p.u)	Last iteration V (p.u)
1	1.11	1.02
2	1.11	1.04
3	0.82	0.95

Table 5
Power solution.

Unit	Day-ahead market		Subsequent mechanism	
	First iteration	Last iteration	Last iteration	
j	P (MW)	P (MW)	P (MW)	Q (Mvar)
1	50.3	50.3	36.3	1.1
2	–	–	–	–
3	–	–	–	–
4	60.4	60.4	60.4	4.1
5	–	–	14	58

Case A

If the generation companies do not take into account the voltage level requirements in their bids to the day-ahead market then the generation units 1 and 4 at areas 1 and 2 are dispatched. However, in that solution, the voltage level at area 3 is only 0.82 p.u. and this value is below the required minimum of 0.95 as shown in Table 4.

Companies 1 and 2 do not modify their strategic behavior in the day-ahead market because they do not own any unit at area 3 to meet the voltage requirements. Hence, the final solution in the day-ahead market is not modified. On the other hand, unit 5 owned by company 3 is the only unit that can resolve the voltage constraint at area 3. This unit has to generate the maximum reactive power in order to reach the voltage level of 0.95 at area 3, and

Table 2
Generation units.

Unit j	Case	Node a	Company i	Variable cost (€/MW h)	$\underline{P}$ (MW)	$\overline{P}$ (MW)	$\underline{Q}$ (Mvar)	$\overline{Q}$ (Mvar)	$Q^{0,max}$ (Mvar)	$Q^{0,min}$ (Mvar)	n^{max}	n^{min}
1	A, B, C	1	1	40.5	7	70	−40	40	56.4	−56.4	−0.643	0.643
2	A, B, C	2	1	42.0	7	70	−40	40	56.4	−56.4	−0.643	0.643
3	A, B, C	1	2	42.0	7	70	−40	40	56.4	−56.4	−0.643	0.643
4	A, B, C	2	2	40.0	7	70	−40	40	56.4	−56.4	−0.643	0.643
5	A	3	3	43.5	14	70	−58	58	72	−72	−1	1
5	B, C	3	3	43.5	7	35	−29	29	36	−36	−1	1
6	B, C	3	1	40.0	7	35	−29	29	36	−36	−1	1
7	C	3	2	44.0	7	35	−29	29	36	−36	−1	1

Table 6
Voltage levels.

Node a	First iteration V (p.u)	Last iteration V (p.u)
1	1.05	1.01
2	1.04	1.05
3	0.77	0.95

Table 7
Power solution.

	Day-ahead market		Subsequent mechanism	
Unit	First iteration	Last iteration	Last iteration	
j	P (MW)	P (MW)	P (MW)	Q (Mvar)
1	15.3	41.8	37.1	−8.1
2	–	–	–	–
3	–	–	–	–
4	60.4	59.6	59.6	13.5
5	–	–	7	29
6	35	9.3	7	29

its active power generation is equal to the minimum given its P–Q capability curve. The active power increased by this unit in the subsequent mechanism is compensated by a reduction in the active power of unit 1 as shown in Table 5.

Case B

In this case, unit 6 at area 3 is dispatched to the maximum of its active power in the initial day-ahead market. However, this unit cannot generate the reactive power necessary to maintain the voltage level at area 3 due to its capability curve (Table 6), and therefore, as in the previous case, unit 5 is necessary in the subsequent mechanism.

Unlike case A, in which company 1 does not modify its strategic behavior in the day-ahead market, company 1 foresees that the active power generation of unit 6 has to be at the minimum while the reactive power generation has to be at the maximum for maintaining the voltage level at area 3. This makes the reduction factor $m_6 > 0$, and therefore its *apparent cost* increases in the day-ahead market as explained in Section 'Market equilibrium'. Thus, a higher *apparent cost* of this unit results in a change in the strategic behavior of company 1 in the day-ahead market generating only 9.3 MW with unit 6 (Table 7). If the company does not change the active power generation of its units, the apparent cost of this units decreases, and therefore the day-ahead market price also decreases. However, that is not a good strategy because the company knows that the active power generation of unit 6 has to be reduced in the subsequent mechanism. On the other hand, unit 5 is dispatched in the subsequent mechanism to the minimum active power and the maximum reactive power to reach a voltage level equal to 0.95 p.u. at area 3.

Case C

The initial day-ahead market solution of this case is the same as the initial solution to case B. Thus, another generation unit at area 3 is required to maintain the voltage level (Table 8).

The final result in the day-ahead market is exactly the same as in case B. This means that the strategic behavior of company 1 in the day-ahead market is not altered by the new power unit at area 3. Nevertheless, the outcome of the subsequent mechanism is modified depending on the strategic behavior of company 3. In cases A and B, company 3 could exercise market power because its unit was the only one that could resolve the voltage constraint.

Table 8
Voltage levels.

Node a	First iteration V (p.u)	Last iteration V (p.u)
1	1.05	1.01
2	1.04	1.05
3	0.77	0.95

Table 9
Power solution.

		Day-ahead market		Subsequent mechanism	
	Unit	First iteration	Last iteration	Last iteration	
	j	P (MW)	P (MW)	P (MW)	Q (Mvar)
	1	15.3	41.8	37.1	−8.1
	2	–	–	–	–
	3	–	–	–	–
	4	60.4	59.6	59.6	13.5
	6	35	9.3	7	29
(a)	5	–	–	7	29
	7	–	–	–	–
(b)	5	–	–	–	–
	7	–	–	7	29

(a) Solution for $\beta_3 = 0$.
(b) Solution for $\beta_3 = 0.1$ and $\beta_3 = 1$.

Table 10
Prices.

	λ (€/MW h)						γ (€/MW h)		
	First iteration			Last iteration			Last iteration		
β_3	Case A	Case B	Case C	Case A	Case B	Case C	Case A	Case B	Case C
0	43.02	43.02	43.02	43.02	42.98	42.98	43.5	43.5	43.5
0.1	43.02	43.02	43.02	43.02	42.98	42.98	44.9	44.2	44.0
1	43.02	43.02	43.02	43.02	43.98	42.98	57.5	50.5	44.0

However, in case C, company 2 also has a unit at area 3. Thus, the market power of company 3 is mitigated, and the value of its conjectured-price response in the subsequent mechanism cannot be higher than 0.071 (€/MW h)/MW because a higher value would cause the apparent cost of unit 5 be greater than the apparent cost of unit 7. Table 9 shows how unit 5 is dispatched in the subsequent mechanism when $\beta_3 = 0$ while unit 7 is dispatched when $\beta_3 = 0.1$ and $\beta_3 = 1$.

Prices

In the results above, the day-ahead market generation is affected by the voltage constraints at area 3 in cases B and C. These changes occur because the *apparent cost* of the units is modified by the reduction factor m_j. However, the changes in the day-ahead market price, λ, are not significant, and they are only equal to 0.04 €/MW h between the initial and the final solution.

On the other hand, the price in the subsequent mechanism γ may be modified by the market power of the companies which have the generation units necessary to maintain the voltage levels. In cases A and B, unit 5 of company 3 is indispensable to resolve the voltage constraints, and therefore this company has a high market power in the subsequent mechanism. On the contrary, in case C, the market power of company 3 is limited by unit 7 of company 2. Hence, company 3 cannot make bid prices of unit 5 above the bids of unit 7 in order to be dispatched. Moreover, the price γ in case C when β_3 is higher than 0.071 (€/MW h)/MW is equal to the variable cost of unit 7 as shown in Table 10.

Conclusions

This paper has studied the effect of voltage requirements on companies' strategic behavior in a single-price electricity market. The market equilibrium equations take into account the solution of the mechanism used to clear the technical constraints which is modeled by means of an AC optimal power flow. One of the contributions of this paper is that the technical constraints are not limited only to congestion due to the thermal constraints of the transmission lines, but the model can also analyze other technical constraints such as voltage levels or reactive power requirements.

The results of the numerical example show how one company may exercise market power in the mechanism used to clear the technical requirements if it is the only company that can resolve the voltage constraints. Also, it has been shown how this market power is mitigated as more companies are able to resolve this technical constraint.

List of symbols

Indices

a	node index
b	node index
i	company index
j	production unit index
k	production unit index
κ	iteration counter index
Ω	optimal power flow

sets

A	set of indices of nodes
I	set of indices of companies
J	set of indices of production units
J_a	set of indices of production units connected to node a
J_i	set of indices of production units owned by company i
Ξ	set of optimization variables in the OPF

Constants

n_j^{max}	parameter of the capability curve of unit j
n_j^{min}	parameter of the capability curve of unit j
B_{ab}	element of the susceptance matrix (p.u.)
DP_a	active power demand at area a (MW)
DQ_a	reactive power demand at area a (Mvar)
G_{ab}	element of the conductance matrix (p.u.)
$\underline{P}_j$	minimum active power generation of unit j (MW)
$\overline{P}_j$	maximum active power generation of unit j (MW)
$\underline{Q}_j$	minimum reactive power generation of unit j (Mvar)
$\overline{Q}_j$	maximum reactive power generation of unit j (Mvar)
$Q_j^{0,max}$	parameter of the capability curve of unit j (Mvar)
$Q_j^{0,min}$	parameter of the capability curve of unit j (Mvar)
α	learning rate (can take values in the interval (0,1)
β_i	conjectured-price response of company i in the subsequent market ((€/MW h)/MW)
ϵ	level of solution accuracy
θ_i	conjectured-price response of company i in the day-ahead market ((€/MW h)/MW)

Variables

m_j	reduction factor of production unit j
u_j	commitment variable of unit j
w_j	binary variable that is equal to 1 if the unit j increments its production and 0 otherwise
ACW_j	apparent cost of unit j (€/MW h)
ACX_j	apparent cost of unit j (€/MW h)
C_j	production cost of unit j (€)
DP	total active power demand (MW)
MC_j	marginal cost of unit j (€/MW h)
P_j^*	equilibrium value of the active power generation of unit j (MW)
P_j^Ω	active power generation of unit j in the optimal power flow (MW)
Q_j^Ω	reactive power generation of unit j in the optimal power flow (MW)
V_a	voltage magnitude at node a (p.u.)
W_j^Ω	decrement in the production of unit j in the optimal power flow (MW)
X_j^*	equilibrium value of the increment in the production of unit j (MW)
X_j^Ω	increment in the production of unit j in the optimal power flow (MW)
Y	total active power increment (MW)
γ^*	equilibrium price of the active power increments (€/MW h)
γ_i	estimation of the price of the active power increments made by agent i (€/MW h)
δ_a	phase angle at area a (rad)
λ^*	day-ahead market equilibrium price (€/MW h)
λ_i	estimation of the day-ahead market price made by agent i (€/MW h)
μ	dual variable
ν	dual variable
ξ	dual variable

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