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A goal programming model for scheduling residents in an anesthesia and reanimation department

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ABSTRACT

Medicine residency is three to seven years of challenging graduate medical training that puts a lot of mental and physiological burden over the residents. Like other surgical branches, anesthesia and reanimation departments provide 24 h continuous service and the residents are the main providers of this service. The residents are assigned for on-call shifts during their training, as well as working during the regular day shifts. These schedules must address several considerations like preferences of the residents and coverage requirements of two different locations: the intensive care unit (ICU) and the surgery room (SR). In this study we develop a goal programming (GP) model for scheduling the shifts of the residents in the Anesthesia and Reanimation Department of Bezmialem Vakif University Medical School (BUMS). The rules that must be strictly met, like the number of on-duty shifts or preventing block shifts, are formulated as hard constraints. The preferences of the residents like increasing the number of weekends without shifts and assigning duties on the same night to the same social groups are formulated as soft constraints. The penalties for the deviation from the soft constraints are determined by the analytical hierarchy process (AHP). We are able to solve problems of realistic size to optimality in a few seconds. We showed that the proposed formulation, which the department uses currently, has yielded substantial improvements and much better schedules are created with less effort.

1. Introduction

The physicians in departments like brain surgery, general surgery, anesthesia and reanimation work under great stress, both mentally and physically. For example a brain operation may last more than ten hours and the physicians may not even have a chance to eat. A physician who works in an intensive care unit (ICU) deals with patients who are at the border of life and death. It is a frequent incidence for them to perform cardiopulmonary resuscitations, commonly known as heart massage, to the patients whose heart has stopped beating. This process takes about an hour and can happen more than once in a day. Moreover they have to inform the families of the those patients about the current situation and they need to address the irrational behavior of families/relatives resulting from anxiety. These departments also provide non-stop services for their patients, i.e., they continue their operations at nights and in the weekends. Residency is a stage of graduate medical training. A resident physician, or resident for short, is a person who has received a medical degree who practices medicine under the supervision of fully licensed physicians, usually in a hospital or clinic. Residents, who are students as well as being a physician, have to learn simultaneously while fulfilling all duties of a physician. On top of these, there are two shifts at medical schools and hospitals in Turkey: the day shift and the night shift. All of the residents are obliged to work in the regular daily shift[1] and are assigned for on-call shifts at nights and on the weekends as well. Although the scheduling of the shifts for the nights and the weekends can be easy when the number of residents is small, it is well known that the problem quickly gets complex when there is a large number of residents with different levels of experience and when there are several constraints like coverage requirements and resident preferences.

The residents must take a good rest in order to perform well in the physical tasks required by the job. More importantly, they must have a good mental rest in order to continue their educational process and to avoid job burnouts. Studies have shown that the scheduling of shifts has a significant impact on shift workers physiologically, psychologically and socially (Knauth, 1993, 1996) and better schedules prevent job burnout (Nelson, 2007). Moreover, it is demonstrated that the sleep deprivation that is seen in the residency training programs lead to medical errors (Grantcharov, Bardram, Peter, & Rosenberg, 2001) as well as serious

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[1] The only exception is the emergency service. They have a different shift structure.

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problems in daily life like traffic accidents (Marcus & Loughlin, 1996). We refer the reader to Owens (2001) for a review on effect of sleep deprivation on the residents. Following unfortunate statistics are relevant in this context: it is reported that the physicians working in anesthesia and reanimation departments rank third among all physicians in committing suicide (Roy, 2000). In Turkey, between 2008 and 2010, 10 physicians, who work in an anesthesia and reanimation department committed suicide (Cengiz, 2010).

Being established in 2010 in Istanbul, Turkey, Bezmialem Vakif University adopted a training and research hospital to establish a new school of medicine and started resident training programs at the beginning of 2011. Although a few of the residents are transferred to BUMS, most of the former residents continued their training at different hospitals. With two transferred residents and four new residents, the anesthesia and reanimation department started its medicine residency program in January 2011 with six residents. In 2012 September, there are 10 residents working in the department. The schedules of the residents in the anesthesia and reanimation department of BUMS are prepared manually by a physician who devote almost a week for preparing the schedule. In this paper we present a goal programming (GP) model for the scheduling problem of the residents in the department. The resulting formulation, which is currently in use, yields high-quality schedules in a few seconds. The mathematical model that we provide can easily be generalized for the use of any medical school hospitals in Turkey with small modifications since the shift structure of the other surgical departments is almost the same in the hospitals all over the country.

The rest of the paper is organized as follows. Section 2 gives some literature review. In Section 3, we give the current state of the department and define the problem. We give the GP model in Section 4. We comment on the benefits and give some computational results in Section 5 and finally conclude in Section 6.

2. Literature review

The major scheduling focus in health systems has been in nurse scheduling (Ernst, Jiang, Krishnamoorthy, & Sier, 2004). Nurse scheduling problem (NSP) is a type of resource-allocation problem, in which the workload needs to be assigned to nurses periodically, taking into account a number of constraints and requirements. Starting with the early studies Warner and Prawda (1972) and Miller, Pierskalla, and Rath (1976), the NSP has attracted attention from the operation researchers. There are various studies that use GP to solve the scheduling problem. For example, Ozkarahan and Bailey (1988), Berrada, Ferland, and Michelon (1996) and Azaiez and Sharif (2005) propose GP models which accounts for conflicting objectives of the hospital (continuous service with appropriate nursing skills and staffing size) and the preferences of the nurses (fairness for night and weekend shifts). Another approach is to use heuristic solutions. Aickelin and Dowsland (2004) and Tsai and Li (2009) propose genetic algorithm approaches, Bard and Purnomo (2005) uses column generation and Burke, Li, and Qu (2010) propose a hybrid model. For a survey on NSP, please refer to Cheang, Li, Lim, and Rodrigues (2003) and Ernst et al. (2004).

The differences in staffing requirements, worker preferences, specialties and policies distinguish the physician scheduling from the nurse scheduling. The literature on physician scheduling concentrates especially on the emergency room physician scheduling problem (ER-PSP). Beaulieu, Ferland, Gendron, and Michelon (2000) propose a mixed integer program to solve the ER-PSP. They formulate the problem over six months and solve the schedule of one month by considering the schedules of the previous five months. Carter and Lapierre (2001) classify the schedules of ER-PSP into three groups: the acyclic schedules, the cyclic schedules with rotation and the cyclic schedules without rotation. They develop general mathematical formulation of the physician scheduling problem and describe an application two different cases one of which has simple and the other having complex scheduling rules. In a recent work, Ferrand, Magazine, Rao, and Glass (2011) study the scheduling problem of the physicians at Cincinnati Children's Hospital Medical Center using integer programming to build cyclic schedules for eight weeks that can be repeated throughout the year. Based on their surveys with physicians, they showed that the new schedule provided well-balanced work patterns.

The resident scheduling problem (RSP) is different than the physician scheduling problems in the sense that the residents are students as well as being the providers of the medical services. The residents have seniority groups which determines the rule of thumbs of the scheduling rules. Ozkarahan (1994) provide the first attempts on RSP and proposes a GP model that solves weekly schedules and incorporates the preferences of the residents and the requirements of the residency program into the objective of the model. Sherali, Rahami, and Saifee (2002) formulates the problem as a mixed integer program and proposed heuristic solution procedures that are developed for different scenarios. Cohn, Root, Kymissis, Esses, and Westmoreland (2009) study the scheduling of residents of the psychiatry program at Boston University School of Medicine. They combine heuristic approaches and mathematical programming to come up with schedules that assigns 10 to 20 residents for the on call shifts at three different hospitals over 365 days period.

The Accreditation Council for Graduate Medical Education (ACGME) come up with certain rules in order to increase well-being of residents and the patient safety and reduce job burn-outs. The ACGME is a private, non-profit council that evaluates and accredits medical residency programs in the USA. Some of its rules include an upper limit of 80 hours per week and 24 h of continuous work. Day, Napoli, and Kuo (2006) provide an integer programming model for scheduling the weekly work hours of surgery residents considering the ACGME rules. Topaloglu and Ozkarahan (2011) develop a mixed integer programming model for scheduling residents shifts under the ACGME regulations. Their solution is based on the column generation which generates feasible schedules using constraint programming and then allocates residents to a subset of these schedules optimally considering demand coverage requirement.

Topaloglu (2006) develop a goal programming model for scheduling emergency medicine residents in Turkey. Her model accommodates both hard and soft constraints and the deviations from the soft constraints are penalized with the coefficients that are determined by the analytical hierarchy process (AHP). She showed that problems of realistic size can be solved efficiently. Topaloglu (2009) proposes a multi-objective programming model for scheduling residents with different seniority levels. The model considers the shift coverage requirements, seniority-based workload rules and preferences of the residents for their on and off days. For a recent review on RSP, we refer the reader to Rais and Viana (2011) and Kreeft (2012).

Although the model in Topaloglu and Ozkarahan (2011) is extensive and covers the ACGME rules, the residency programs in Turkey does not conform with the ACGME rules. For example, the residents may be assigned 15 shifts in a month which means 450 h of work in a month whereas ACGME proposes an upper bound of 320 h per month. Besides, the residents in Turkey work for 36 h if they are on-call for a night shift, however ACGME regulations has an upper bound of 24 h for continuous work. Similarly, the shift structure of the emergency room studied in Topaloglu (2006) is different than the other departments. They can take a day off after an on-duty shift. However, this is not the case in the surgical residency programs. Hence, our problem is different than the one that is given in Topaloglu and Ozkarahan (2011) and Topaloglu (2006). Topaloglu (2009) is the closest study to ours. The shift structure of the problem they formulate is the same as ours. Their main motivation for the formulation is to allocate the shifts by considering the strict seniority rules between the residents while maintaining the coverage requirements. In our case, BUMS is a newly established school and the managerial and practical issues concerning education and health services are still in progress. For example, the target level of residents in anesthesia and reanimation department is 15, however it was around 10 during 2012 summer. Hence, even if there are some regulations that are reflected by the seniority, incorporating the constraints in Topaloglu (2009) makes it impossible to generate a feasible solution for our problem. In our case, the department chair determines the number of shifts by taking seniorities into consideration. In fact, this is the case in many of the residency programs in Turkey. In our study, we focus on the *preferences of the residents* like keeping the friends together in a shift or matching Friday and Sunday shifts in order to free some of the weekends. To the best of our knowledge, we are the first to model these preferences in RSP. Another major difference is that, in all of the studies above, the departments are responsible for a *single* location. In our study, the department is responsible for two locations, the intensive care unit and the surgery room (SR) and has to assign at least one resident to these locations which adds another dimension to the classical RSP. A final interesting note is the following: despite the vast amount of literature on health care, it is reported that only 10% of the health care literature is implemented on a real life problem (Chambers & Williams, 2012). We develop a GP model which is currently in use in a medical school.

3. Problem statement

In this section, we explain the current system and state the problem. There are two shifts in Turkish hospitals: the day shift and the night shift. Although there can be some minor differences among different hospitals, the day shift is between 08:00 and 18:00 and the night shift is between 18:00 and 08:00. All of the physicians work in the day shift except the weekends. The brain surgery, general surgery, anesthesia and reanimation, pediatrics and neurology departments have to serve at nights and weekends as well. Both the specialists and the residents are responsible for maintaining the continuous service in these departments. In general, the resident who has the highest seniority or an assistant professor prepares a schedule in order to assign the resident and the specialists to the night shifts in weekdays and the weekend shifts. In our case an assistant professor, whom we call the scheduler from this point on, prepares the schedules. These schedules must satisfy the rules of the department and hospital. Moreover it should consider the preferences of the residents.

There are several issues which differentiates our problem from the ER-PSP and NSP literature. First, there are two shifts. Most of the related literature on NSP and ER-PSP deal with problems where there are three or four shifts. Second, all of the residents work during the day shift even if they have a duty at the previous night shift. Hence they do *not* have a rest after a duty in a shift. Third, the weekend day shifts include the whole day, i.e., a resident who is on duty on Saturday or Sunday works for two shifts (24 h). Fourth, the number of on-duty shifts are determined according to the seniority levels. The general tendency is to assign a resident to less number of shifts as they gain seniority. The main motivation is that, a resident who is on duty at a shift is not only serving the patients, but s/he is also fulfilling her/his training. Therefore the shifts are in fact part of their learning process.

The cyclic policies which are common in staff scheduling problems are not appropriate for the structure of this problem due to the following reasons. First, the number of residents changes with the graduations and newcomers. Second, the time of the updates in the seniority levels is different for every resident. Finally the scheduling is done in a monthly basis due to the regulations of the hospital management. Therefore an optimal schedule in a month may not be even feasible in the next month.

Consider the following example in order to understand the overall picture better. Let's look at an ordinary week of a resident. He regularly works during the day shift. He is on duty for the night shift on Monday and Thursday and have a weekend shift on Saturday. This resident starts his week at 08:00 on Monday. At 18:00, he starts to work for the night shift. On Tuesday 08:00 he starts his regular day shift. He leaves the hospital at 18:00 on Tuesday. He gets back to the hospital on Wednesday at 08:00, starts his regular shift and leaves the hospital at 18:00. He starts his regular day shift on Thursday at 08:00, leaves the hospital at 18:00 on Friday due to his night shift on Thursday. Finally he comes to the hospital at 08:00 on Saturday to take over the Saturday shift and leaves the hospital at 08:00 on Sunday. If he has a duty on the Sunday shift rather than Saturday, then he would start his duty on Sunday at 08:00 and leave the hospital at 18:00 on Monday. From this point on, when we say a shift, we mean the night shift if it is a weekday and we mean a weekend shift (a day and a night shift together) if it is a weekend day. Finally note that, a resident who is on-call on Fridays and Saturdays works for 24 h whereas he works for 34 h when he is on-call on other days' shift.

The anesthesia and reanimation department that we consider in this study has two responsibilities: taking care of the patients in the ICU and provide anesthesia for the patients in for the surgeries of several departments. Although there are several surgery units in the day shift (brain surgery, obstetrics and gynecology, general surgery, etc.), there is only one surgery room that operates at the night and the weekends. Hence the resident who is on duty is responsible only for this particular surgery room. The shifts in the ICU is harder and needs more experience, hence the residents who have high seniority are assigned for the ICU shifts and the residents with low seniority are assigned for the SR shifts. In this study we deal with scheduling the shifts of the residents for these two locations. The problem is formulated using a GP model. We deal with the mathematical model in detail in the next section.

4. The development of the GP model

In this section we give the mathematical formulation of the problem. We first give the regulations of the department and preferences of the residents. Then we formulate the problem using GP. We schedule the shifts in December 2011, January 2012 and February 2012 in order to illustrate the formulation throughout the section.

Before getting into the formulation, we give the definitions of the sets, parameters and the variables in Tables 1–3, respectively. The variable Y_{gt} will be used to assign the residents from the same social group for the shifts in the SR. Note that the l index is not used in the variable since there cannot be two resident in an ICU shift. However it can easily be generalized to more than one location. Finally, $d^{k}_{n_k it}$ is the variable that defines the deviation from a soft constraint. The objective of the GP is to satisfy all of the soft constraints which is achieved by minimizing the deviations from the soft constraints. The use of the deviation variables will be clarified during the formulation. We now give the constraint sets.

4.1. Locations and coverages

The anesthesia and reanimation department is responsible for two locations, the ICU and the SR. During the shifts, there must at least one resident at the ICU and at least one resident at the SR. According to the policy of the department there can be two

Table 1
The sets.

Sets	Definition
$\mathbb{W}$	The week days. 1 stands for Monday, 2 for Tuesday, etc. $\mathbb{W} = \{1, 2, \ldots, 7\}$
$\mathbb{T}$	The set of days in a period. $\mathbb{T} = \{1, \ldots, T\}$
$\mathbb{T}_w$	The wth week days in a period s.t. $w \in \mathbb{W}$, i.e., T_6 denotes Saturdays and T_7 denotes Sundays
$\mathbb{L}$	The set of the locations. $\mathbb{L} = \{1, \ldots, L\}$ In this study, 1 and 2 denote the ICU or the SR, respectively
$\mathbb{G}$	The set of the social groups. $\mathbb{G} = \{1, \ldots, G\}$ Currently there are 2 social groups. The set of the residents. $\mathbb{I} = \{1, \ldots, I\}$
$\widehat{\mathbb{I}}_g$	The set of resident at social group g
$\mathbb{I}^y$	The set of resident having only ICU shifts
$\mathbb{I}^+$	The set of senior residents who can be alone in the SR shift
$\mathbb{I}^T$	The set of residents who are in duty in the last day (Tth) of the previous month
$\mathbb{I}^{T-1}$	The set of residents who are in duty in the $(T-1)^{th}$ day of the previous month
$\mathbb{T}_i^-$	The set of the off days that are requested by A_i
$\mathbb{T}_i^+$	The set of the days (shifts) that A_i requests to be on duty
$\mathbb{K}$	The set of the type of deviation. $\mathbb{K} = \{1, 2+, 2-, 3\}$

Table 2
The parameters.

Parameter	Definition
S_i^T :	The total number of shifts that A_i is responsible for one month
S_i^y:	The number of ICU shifts for A_i
S_i^f:	The number of Friday shifts for A_i
S_i^s:	The number of Saturday shifts for A_i
S_i^p:	The number of Sunday shifts for A_i
S_i^w :	The number of weekend shifts for A_i
δ^T:	The parameter which is 1 is the last day (T^{th} day) of the previous month is Friday, 0 otherwise
δ^{T-1}:	The parameter which is 1 if the $(T-1)^{th}$ day of the previous month is Friday, 0 otherwise. Note that δ^T and δ^{T-1} are *not variables* but given parameters of the problem.
$c_{n_k it}^k$	The cost associated with the deviation variables $d_{n_k it}^k$

Table 3
The variables.

Variable	Definition
X_{it}^l	1 if resident i assigned to day t shift at location l, and 0 otherwise
Y_{gt}	1 if there is a resident of group g on day t, and 0 otherwise
$d_{n_k it}^k$	The deviations from the soft constraints

residents in the SR, and the residents who are at their first 6 months should be assigned to the SR shifts with a senior resident. However, this is not always possible since the number of residents is not enough. The constraints are given in the following:

$$\sum_{l=1}^{L} X_{it}^l \leqslant 1 \quad \forall i, t \tag{1}$$

$$\sum_{i=1}^{I} X_{it}^1 = 1 \quad \forall t \tag{2}$$

$$1 \leqslant \sum_{i=1}^{I} X_{it}^2 \leqslant 2 \quad \forall t \tag{3}$$

$$X_{i,t}^2 - d_{1it}^1 \leqslant \sum_{i' \in \mathbb{I}^+} X_{i',t}^2 \quad i = \mathbb{I} \setminus \mathbb{I}^+, \ \forall t \tag{4}$$

The constraint (1) states that a resident can be on duty at a single location on a given day. There can be only one resident in the ICU by the constraint (2) and there can be one or two residents in the SR by the constraint (3). The constraint (4) states that if an resident with low seniority is on duty in the SR, then a resident with high seniority should be on duty as well.

4.2. Number of shifts

The general rule of thumb in a month is that the number of shifts decreases with the seniority. However the number of shifts may change from month to month due to several practical and managerial issues. Let A_i denote the resident i. We will label them according to their seniorities, i.e., A_1 is the one with the highest seniority. The total number of shifts, including the weekends, are given in Table 4. A_9 has joined the department in 2012, hence there is no shift for A_9 in 2011.

In addition to Saturday and Sunday, the residents cannot make a plan covering the weekend if they have a shift on Friday since they take a rest after the Friday shift. Therefore Friday shifts are counted like a weekend shift and the number of Friday shifts is determined like Saturday and Sunday shifts. Table 5 gives the number of weekends shifts.

The related constraints are given in the following:

$$\sum_{t=1}^{T} \sum_{l=1}^{L} X_{it}^l = S_i^T \quad \forall i \tag{5}$$

$$\sum_{t=1}^{T} \sum_{l=1}^{L} X_{it}^l - d_{2i}^{2+} + d_{2i}^{2-} = S_i^f \quad \forall i \tag{6}$$

$$\sum_{t=1}^{T} \sum_{l=1}^{L} X_{it}^l - d_{3i}^{2+} + d_{3i}^{2-} = S_i^s \quad \forall i \tag{7}$$

$$\sum_{t=1}^{T} \sum_{l=1}^{L} X_{it}^l - d_{4i}^{2+} + d_{4i}^{2-} = S_i^p \quad \forall i \tag{8}$$

$$\sum_{t=1}^{T} \sum_{l=1}^{L} X_{it}^l = S_i^w \quad \forall i \tag{9}$$

The total number of shifts is given by the constraint (5). The department wants to set the exact number of Friday, Saturday and Sunday shifts, but several experiences show that this can give infeasible solutions. Hence the number of the shifts on these three days are given by the soft constraints (6)–(8), respectively. Still, the number of weekend shifts are given by the hard constraint (9) since the department is firm about the number of weekend shifts.

4.3. The ICU shifts

The department has a particular policy about the ICU shifts. The ICU has critical patients who are at the border of life and death.

Table 4
The total number of shifts.

Residents	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	Total
Dec. 2011	5	6	8	8	8	8	9	10	0	62
Jan. 2012	5	7	9	9	9	9	10	10	10	78
Feb. 2012	5	6	7	7	8	7	8	8	9	65

Table 5
The total number of weekend shifts.

		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Dec. 2011	Friday	1	1	1	1	1	1	2	2	0
	Saturday	2	1	1	1	1	0	2	2	0
	Sunday	0	1	1	1	1	2	1	1	0
Jan. 2011	Friday	0	1	1	1	1	1	2	1	2
	Saturday	0	1	1	0	2	1	1	2	1
	Sunday	2	1	1	2	0	1	2	1	2
Feb. 2011	Friday	1	1	1	1	1	1	1	1	1
	Saturday	0	1	1	1	1	1	1	1	2
	Sunday	1	1	1	1	1	1	1	1	1

First of all, the residents that has the highest seniority always have their shifts at the ICU. This due to the fact that the residents at the ICU must be experienced. All of the shifts of the residents A_1 and A_2 is in the ICU. Besides A_1 and A_2, the resident who is in the ICU rotation has shifts in the ICU. A resident starts his education at the anesthesia and reanimation department in the ICU or in one of the surgery rooms of the departments, say in the brain surgery, and continue working in the operations of that department for some period of time. After that period, the resident is assigned to another surgery department. This process is called a *rotation*. The resident who is working in the ICU is called to be in the ICU rotation. The number of the ICU shifts are given in Table 6.

The related constraints are:

$$\sum_{t=1}^{T} X_{it}^{1} = S_i^y \quad i \in \mathbb{I}^y \tag{10}$$

$$\sum_{t=1}^{T} X_{it}^{1} - d_{1i}^{2+} + d_{1i}^{2-} = S_i^y \quad i \in \mathbb{I} \setminus \mathbb{I}^y \tag{11}$$

The number of shifts at the ICU for A_1, A_2 and the resident that is in the ICU rotation is given by the constraint (10). For the remaining resident s, the numbers are assigned with the soft constraint (11).

4.4. Block shifts

Let us make the following definition. The shifts of two consecutive days are called *block shifts*. For example shifts of day 1 and day 2 are block shifts. Block shifts are not allowed in the department and the related constraints are given in the following:

$$\sum_{l=1}^{2} \left(X_{it}^{l} + X_{i,t+1}^{l} \right) \leqslant 1 \quad \forall i,\ t = 1, \ldots, T-1 \tag{12}$$

$$\sum_{l=1}^{2} X_{i,1}^{l} = 0 \quad i \in \mathbb{I}^T \tag{13}$$

The constraint (13) does not allow to assign a shift for the resident who is on-duty on the last day of the previous month.

4.5. A_1 and his thesis

A_1 is writing his Ph.D. thesis. It is decided to assign A_1 to the shifts with two residents in the SR. So the second resident in the SR can help A_1 whenever the workload in SR is low. So the workload of A_1 can be reduced and he can concentrate on his thesis. Following constraint does not allow to assign a single resident to the SR shift if A_1 is on duty.

$$X_{1,t}^{1} - d_{1,t}^{3} \leqslant \sum_{i=1}^{I} 2X_{it}^{2} \quad \forall t \tag{14}$$

4.6. Social groups

There are two social groups among the residents. Their request is to be assigned to the same shift in the SR so that they can share their spare time (if any) during the shift. The related constraints are given as:

$$\sum_{\widehat{i \in \mathbb{I}_g}} X_{it}^{2} - \|\mathbb{I}\| Y_{gt} \leqslant 0 \quad \forall t,\ g \in \mathbb{G} \tag{15}$$

Table 6
The number of the ICU shifts.

Assistants	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	Total
Dec. 2011	5	6	4	4	8	4	0	0	0	31
Jan. 2012	5	7	3	4	9	3	0	0	0	31
Feb. 2012	5	6	3	4	8	3	0	0	0	29

$$Y_{gt} \leqslant \sum_{\widehat{i \in \mathbb{I}_g}} X_{it}^{2} \quad \forall t,\ g \in \mathbb{G} \tag{16}$$

$$\sum_{g=1}^{G} Y_{gt} - d_{2,t}^{3} \leqslant 1 \quad \forall t \tag{17}$$

The set of soft constraints (15)–(17) assigns only a single group to the shifts in the SR room. Note that $\|\mathbb{I}\|$ stands for the total number of residents.

4.7. Tandem shifts

The shifts of every other day are called *tandem shifts*. For example the shifts of day 1 and day 3 are tandem shifts. Although tandem shifts are possible, they are not preferred by the residents. In fact, this constraint is one of the major motivations of this study. Three or more tandem shifts in row make the residents exhausted. They need at least two days between two consecutive shifts. The related constraints are given in the following:

$$\sum_{l=1}^{2} \left(X_{it}^{l} + X_{i,t+1}^{l} + X_{i,t+2}^{l} \right) - d_{3,i,t+2}^{1} \leqslant 1 \quad \forall i,\ t = 1, \ldots, T-2 \tag{18}$$

$$\sum_{l=1}^{2} X_{i,1}^{l} - d_{3,i,1}^{1} \leqslant 0 \quad i \in \mathbb{I}^{T-1} \tag{19}$$

$$\sum_{l=1}^{2} X_{i,2}^{l} - d_{3,i,2}^{1} \leqslant 0 \quad i \in \mathbb{I}^{T} \tag{20}$$

While the constraint (18) prevents tandem shifts in the current month, (19) and (20) prevent tandem shifts of the residents who are on-duty on the last day or the day before the last day of the previous month.

4.8. Sundays with friday

The residents prefer to have shift on Friday if they have a shift on Sunday since a Friday shift already prevents to enjoy the weekend as a whole. This is like the complete weekends constraints in the NSP problem (Burke et al., 2010). A similar preference is reported in Ferrand et al. (2011). Although this constraint seems to conflict with the previous one, the residents prefer having a single tandem shift than having an extra weekend occupied by a shift. We call the Sunday shifts which follow the Friday shifts as *Sundays with Friday*. If there is a shift on Friday, Saturday or Sunday of a week, then say that the resident misses the weekend and call the weekend as *missed weekend*. The related constraints are given by the following:

$$\sum_{l=1}^{L} X_{it}^{l} - d_{2it}^{1} \leqslant \sum_{l=1}^{L} X_{i,t-2}^{l} \quad \forall i,\ t-2 \in T_5,\ t \in T_7 \tag{21}$$

$$\delta^{T-1} \sum_{l=1}^{L} X_{i,1}^{l} - d_{2,i,1}^{1} \leqslant 0 \quad i \in \mathbb{I} \setminus \mathbb{I}^{T-1} \tag{22}$$

$$\delta^{T} \sum_{l=1}^{L} X_{i,2}^{l} - d_{2,i,2}^{1} \leqslant 0 \quad i \in \mathbb{I} \setminus \mathbb{I}^{T} \tag{23}$$

While the constraint (21) assigns Friday shifts to the residents having Sunday shifts in the current month, (22) and (23) do the same thing if the last day or the day before the last day of the previous month is Friday. Note that δ_T and δ_{T-1} are not variables, but parameters which show whether the Tth or $T-1$th day of the previous month is Friday, respectively.

4.9. On and off days

The residents may request to be on duty or *not* to be on duty for a particular shift due to their social constraints, i.e., they may have a wedding anniversary or there can be a derby match of their favorite football team or their spouse can be on duty as well. Let us define the *on days* and the *off days* as the days that the residents want to be and do not want to be on duty, respectively. Finally, no shifts are assigned if the resident are on vacation. The related constraints are given as:

$$\sum_{t\in\mathbb{T}_i^v}\sum_{l=1}^{L}X_{it}^l = 0 \quad \forall i \tag{24}$$

$$\sum_{t\in\mathbb{T}_i^-}\sum_{l=1}^{L}X_{it}^l - d_{5,i}^{2+} + d_{5,i}^{2-} = 0 \quad \forall i \tag{25}$$

$$\sum_{t\in\mathbb{T}_i^+}\sum_{l=1}^{L}X_{it}^l - d_{6,i}^{2+} + d_{6,i}^{2-} = \|\mathbb{T}_i^+\| \quad \forall i \tag{26}$$

The constraints (24)–(26) are for the vacation, off days and on days, respectively.

Finally we give the objective function which minimizes the deviations from the soft constraints:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{n_1=1}^{3} c_{n_1it}^1 d_{n_1it}^1 + \sum_{i=1}^{I}\sum_{n_2^+=1}^{6} c_{n_2^+i}^{2+} d_{n_2^+i}^{2+} + \sum_{i=1}^{I}\sum_{n_2^-=1}^{6} c_{n_2^-i}^{2-} d_{n_2^-i}^{2-} + \sum_{t=1}^{T}\sum_{n_3=1}^{3} c_{n_3t}^3 d_{n_3t}^3 \tag{27}$$

Since the number of the shifts per resident is predetermined, the cost of the shifts due the wages of the residents is constant and hence minimization of the cost of the shifts is not considered as a part of the objective function.

The constraints (4), (6)–(8), (11), (14), (17)–(21), (25) and (26) are soft constraints. Note that if these constraints are defined as hard constraints, then the problem may (probably will) be infeasible. Hence we prefer these constraints to be soft. We want the slack variables in these constraints to be zero. This is achieved by assigning fictitious costs to these variables in the objective function (27) so that the system incurs penalty costs when the soft constraints are violated. Therefore, the objective is simply to minimize the deviations defined for the soft constraints. Determination of these costs are given in details in the next section. If all of the deviations are zero, then all of the soft constraints are satisfied. One can check the positive deviations in order to see which constraints are violated. For example, if $d_{5,2}^{2+} = 1$, then this means that one of the off day requests of A_2 is violated.

5. The results and the performance of the model

In this section we give the schedules given by the proposed GP model. In particular we give the computational results for ten months (December 2011 to September of 2012) and comment on the performance of the proposed GP. We also give the schedules of December 2011, January 2012 and February 2012 and we discuss the quality of the solutions and compare them with the current schedules.

The proposed GP has hard constraints that must be met and soft constraints that can be violated at some costs. The objective of the GP is to satisfy all soft constraints which is achieved by minimizing the deviations from the soft constraints. The soft constraints have different levels of importance and this is reflected by the costs of the deviations. The AHP (Saaty, 1990) provides a methodology to establish the levels of importance for the soft constraints. The procedure of the AHP can be summarized as follows. First of all, the decision maker makes a pairwise comparison between the soft constraints. The comparison can be made in the scale of 1/9 to 9. A rating 9 indicates that one constraint is extremely more important than the other. Similarly ratings of 7, 5 and 3 respectively indicate that one constraint is very strongly important, strongly important and important than the other. A rating of 1 shows equal importance and even ratings (2,4,6,8) indicate intermediary values. If the importance of one constraint with respect to the other is given, then the importance of the other one with respect to the first constraint is simply the reciprocal. Table 7 gives the pairwise comparison matrix which is made by the scheduler. The relative importance of the constraints is calculated as follows: First we find the overall sum of the columns and normalize the values in each columns with this sum. The sum of the rows simply gives the relative importance. Topaloglu (2006) proposes a similar methodology and determines the costs of the deviations with the AHP.

The resulting costs are summarized in Table 8. The labels in the rows corresponds to the following constraints.

- *SR*-1: Leaving a three-month or older, unexperienced resident in the SR alone: d_{1it}^1.
- *SR*-2: Leaving a completely unexperienced resident in the SR alone: d_{1it}^1.
- *SwF*: Missing a Sunday with Friday, i.e., having a free Friday which is followed by a shift on Sunday: d_{2it}^1.
- *Tn*: Assigning a tandem shift: d_{3it}^1.
- *ICU*: Missing the target level of ICU shift: d_{1i}^{2+}, d_{1i}^{2-}.
- *Fr*: Missing the target level of Friday shift: d_{2i}^{2+}, d_{2i}^{2-}.
- *Sa*: Missing the target level of Saturday shift: d_{3i}^{2+}, d_{3i}^{2-}.
- *Su*: Missing the target level of Sunday shift: d_{4i}^{2+}, d_{4i}^{2-}.
- *Off*: Writing a shift for requested off-day: d_{5i}^{2+}, d_{5i}^{2-}
- *On*: Not writing a shift for requested on-day: d_{6i}^{2+}, d_{6i}^{2-}.
- A_1: Assigning a single resident to the SR when A_1 is on-duty at ICU shift: d_{1t}^3.
- *SG*: Assigning two different groups in an SR shift: d_{2t}^3.

The off days of the residents turn out to be the most important soft constraint and deserves a cost of 1.904. The Saturday and Sunday shifts and the number of ICU shifts of the residents except A_1, A_2 and the resident who is in the ICU rotation, has a penalty cost of 1.735 for unit deviation. Any deviation from the number of Friday shifts costs 1.547 and not matching a Sunday shift with a Friday shift costs 0.694. The cost of having a single unexperienced resident alone in the SR is 0.810 for the newcomers and 0.810 for the residents who have a seniority of 6 months. The cost of deviation from the requests of willing to have a shift on a particular day is 0.374. Having a tandem shift costs 0.343, a social group violation costs 0.295 and A_1 being on-duty in a shift on which there is a single resident in the SR costs 0.295. Note that the residents can be associated with different constraints at each month since they gain seniority as time passes. For example, A_9 is accounted in SR-2 when at start of his residency training, then he is accounted in SR-1 constraints after 3 months. Similarly, after 6 months they start for the ICU shifts and after a year they are ready to be assigned for the ICU rotation.

We model the problem in GAMS and use CPLEX 12.3.0.0 as the solver. The summary of the solutions are given in Table 9. It can be observed that, except March, the optimal schedules are found in less than a minute. The schedule of March is given in about 9 min. However the gap reduces to 1% in a minute, hence the program runs 8 min in order to close 1% gap to find the optimal solution.

Until March-2012, the scheduling problem of the residents was manually solved by the scheduler. He has been doing the scheduling of the residents for more than ten years, hence he has quite a lot of experience in the scheduling process. The main duty of the scheduler is to determine the number of total shifts, Friday shifts, weekend shifts and ICU shifts under the control of the department chair. The scheduler then generates the schedule by considering the off and on days and trying to match the Fridays with Sundays, but not paying enough attention for the social groups, the tandem shifts or for A_1. It turns out that the first draft of the schedule lasts about three days on the average based on the experiences of the scheduler. The draft is sent to the residents to get feedback. In general there exists some violations that are overlooked by the sched-

Table 7
Pairwise comparison of the soft constraints.

	SR-1	SR-2	SwF	Tn	ICU	Fr	Sa	Su	Off	On	A_1	SG	Weight
SR-1	1	1	1	2	1/3	1/2	1/3	1/3	1/3	2	5	2	0.670
SR-2	1	1	1	2	1/2	1/2	1/2	1/2	1/3	3	6	3	0.810
SwF	1	1	1	2	1/3	1/2	1/3	1/3	1/3	2	5	3	0.694
Tn	1/2	1/2	1/2	1	1/5	1/4	1/5	1/5	1/6	1/2	3	1	0.343
ICU	3	2	3	5	1	1	1	1	1	5	9	6	1.735
Fr	2	2	2	4	1	1	1	1	1	4	9	5	1.547
Sat	3	2	3	5	1	1	1	1	1	5	9	6	1.735
Sun	3	2	3	5	1	1	1	1	1	5	9	6	1.735
Off	3	3	3	6	1	1	1	1	1	6	9	8	1.904
On	1/2	1/3	1/2	2	1/5	1/4	1/5	1/5	1/6	1	3	1	0.374
A_1	1/5	1/6	1/5	1/3	1/9	1/9	1/9	1/9	1/9	1/3	1	1/2	0.158
SG	1/2	1/3	1/3	1	1/6	1/5	1/6	1/6	1/8	1	2	1	0.295

Table 8
The number of tandem shifts.

	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6
$d^1_{n,i,t}$	0.670–0.810	0.694	0.343	–	–	–
$d^2_{n,i}$	1.735	1.547	1.735	1.735	1.904	0.374
$d^3_{n,t}$	0.158	0.295	–	–	–	–

uler, hence it may last two or three more days to finalize the schedule.

From this point on, *current schedule* refers to the schedule in use which is created by the scheduler. The *new schedule* refers to the schedule created by our formulation. The current schedules and the new schedules of December 2011, January 2012 and February 2012 are respectively given in Table 11 and in Table 12 in Appendix A. In order to quantify the quality of the shift schedules, we check four measures. First, we check the number of tandem shifts which is the main motivation of this study. Second, we check the number of missed Sundays with Friday. Note that this number also shows extra missed weekends that could not be missed with a better schedule. Third, we check the number of social group violations, i.e., check the number of the SR shifts on which there are two residents from different social groups. Finally we check the days on which A_1 is on duty at the ICU shift and there is a single resident in the SR shift.

The number of tandem shifts are given in Table 10. As expected, the number of tandem shifts reduced dramatically. The residents had total number of 47 tandem shifts during the three months period. The new schedule on the other hand, yields only four tandem shifts. As it is mentioned in the previous sections, our main aim is to reduce the number of tandem shifts. This result shows that the proposed GP is able to handle this. Four tandem shifts is due to the fact that there are other objective like number of missed weekends and the mathematical program faces a trade-off between the objectives.

There are some points that have to be clarified when reporting the number of Sundays with Friday. First, December ends on Saturday and hence the weekend at the end of December continues on January. Since December starts with Thursday and February ends on Wednesday, there are no partial weekends at the beginning or at the end of this *three months* period. Therefore, we give the number of missed weekends on aggregate and report for total of three months. Second, the number of Friday, Saturday and Sunday shifts are given as parameters (Table 5). Hence one can calculate in advance how many Sundays can be matched with Fridays and therefore how many weekends are missed in the best scenario. Similarly, the total number of Friday, Saturday and Sunday shifts gives the worst scenario for the number of missed weekends. There are total of 86 Friday, Saturday and Sunday shifts. Hence the residents miss 86 weekends in the worst case. In December, A_2 has one Friday and one Sunday shift, hence one weekend of A_2 can be saved. In a similar manner, it can be observed that there are seven weekends in December and nine weekends in January that can be saved. Moreover, A_1 has a single Friday shift in December and two Sunday shifts on January. Hence the weekend of A_1 at the end of December can be saved as well. Finally, there are nine weekends in February that can be saved. Therefore, there are 26 weekends in which the resident can enjoy their weekends (no weekend shift) if the schedule is fine. This results in 60 weekends occupied with shifts in the best scenario. The current schedule yields 65 occupied weekends which shows that there are 5 extra weekends that are occupied with shifts. In fact this is a very good result for a schedule which is performed manually when compared with the worst scenario (86 weekends). This is due to the fact that the scheduler's main objective is to reduce the number of weekends that are occupied with shifts. The new schedule yields 60 weekends which shows that it gives the best scenario.

There are no social violations in December since there are no SR shifts with two residents. In January and February, there are 15 and 7 SR shifts with two resident s, respectively. Since the number of shifts is determined in advance, the number of SR shifts with two residents stays constant at all schedules. The current schedule has 10 violations in January and 2 violations in February while the new schedule yields no violations at all.

Table 9
Computational results.

Month	CPU		Binary		Num of rsdnt	Num of eqns	Obj value	Num of nodes	Num of iter
	Time	Var	Var	Days					
Dec 2011	2.62	1229	558	31	8	1269	6.86	480	28353
Jan 2012	16.63	1460	620	31	9	1461	74.72	511	128335
Feb 2012	58.37	1374	580	29	9	1371	79.80	7721	615056
Mar 2012	528.10	1460	620	31	9	1461	70.68	51781	6377735
Apr 2012	40.84	1551	660	30	10	1513	75.80	2970	358583
May 2012	16.82	1598	682	31	10	1562	69.22	1225	136567
Jun 2012	4.38	1655	720	30	11	1610	420.99	594	38230
Jul 2012	0.73	1429	620	31	9	1430	200.64	0	2695
Aug 2012	0.61	1429	620	31	9	1430	255.74	21	3456
Sep 2012	0.65	1253	540	30	8	1095	529.79	0	2338

Table 10
The number of tandem shifts.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	Total
December, current	1	1	1	2	3	2	1	2	0	13
December, new	0	0	0	0	0	0	0	0	0	0
January, current	0	1	2	2	2	2	3	2	1	15
January, new	0	0	0	1	0	0	0	1	0	2
February, current	0	1	2	1	5	3	3	2	2	19
February, new	0	0	0	0	0	0	0	1	1	2
Total, current	1	3	5	5	10	7	7	6	3	47
Total, new	0	0	1	0	0	0	0	1	1	4

Finally we give the number of shifts at which there is single resident in the SR room when A_1 is on shift in the ICU. Recall that A_1 is writing his ph.d. thesis, therefore the department requested two resident in the SR room on the days on which A_1 is on duty in the ICU. There are only one resident in the SR room during December, hence we check January and February. It turns out that there is a single resident in the SR on two shifts in January and on four shifts on February during the shifts of A_1 in the current schedules. The new schedules yield only one such shift in January.

6. Conclusion and future work

In this study, we introduce a GP model that schedules the night and the weekend shifts of the residents in the anesthesia and reanimation department of BUMS. The department is responsible for non-stop service of the ICU and the SR. The number of the shifts are determined by the department. The objective is to provide schedules which satisfy the coverage requirements of the ICU and the SR and considers the preferences of the residents. Before March 2012, the schedules were prepared manually by an assistant professor. The objective of these schedules is to satisfy the off days and assign Friday shifts to the residents who have a Sunday shift. The proposed GP model considers additional preferences of the residents like reducing the tandem shifts, assigning the residents of the same social group to the same shift in the SR on the same day and assigning two residents when A_1 is on-duty at the ICU. As a result, the proposed GP model provides high quality solutions, i.e., reduces tandem shifts, increases number of free weekends and assigns the residents in the same social group to the same shift and it is able to do this in a very short time (less than a minute). Since the number of shifts are determined mainly by the heads of the departments in Turkey, our model can be applied to scheduling of any surgical residents all over the country.

Although it requires less than a minute to solve the schedules to optimality except March 2012, it takes about nine minutes to solve the schedule of March 2012 to optimality. This reveals the fact that a solution procedure can be developed to solve the problem in a shorter time. Another future work is to develop a computer package embedded in a spreadsheet application for the scheduler. In the current setting, the scheduler collects the preferences of the residents and the scheduler send these information to the operation researcher who run the model in GAMS with the CPLEX. With this package, the scheduler can do the schedule by himself and the risks regarding the unavailability of the operation researcher can be reduced.

Appendix A. Schedules

Table 11
The current schedule.

Days	December			January				February			
We								1	A_2	A_7	
Th	1	A_1	A_7					2	A_6	A_4	
Fr	2	A_5	A_3					3	A_5	A_8	
Sa	3	A_2	A_8					4	A_3	A_7	
Su	4	A_5	A_3	1	A_1	A_4	A_7	5	A_5	A_8	
Mo	5	A_6	A_7	2	A_3	A_8		6	A_2	A_9	
Tu	6	A_4	A_8	3	A_5	A_6	A_9	7	A_4	A_7	
We	7	A_5	A_3	4	A_2	A_7		8	A_5	A_8	
Th	8	A_2	A_6	5	A_3	A_4	A_8	9	A_3	A_9	
Fr	9	A_4	A_8	6	A_5	A_6	A_9	1	A_4	A_7	
Sa	1	A_1	A_7	7	A_2	A_7		11	A_5	A_9	
Su	11	A_4	A_6	8	A_6	A_9		12	A_4	A_7	
Mo	12	A_5	A_8	9	A_5	A_4	A_8	13	A_1	A_3	A_8
Tu	13	A_2	A_7	1	A_2	A_3	A_7	14	A_2	A_9	
We	14	A_3	A_4	11	A_1	A_6	A_9	15	A_6	A_7	
Th	15	A_5	A_8	12	A_5	A_4		16	A_5	A_8	
Fr	16	A_6	A_7	13	A_3	A_7	A_8	17	A_2	A_3	
Sa	17	A_1	A_4	14	A_6	A_9		18	A_6	A_9	
Su	18	A_6	A_7	15	A_1	A_3	A_8	19	A_2	A_3	
Mo	19	A_3	A_8	16	A_5	A_7		2	A_5	A_8	
Tu	20	A_2	A_4	17	A_4	A_6	A_9	21	A_1	A_6	A_9
We	21	A_5	A_6	18	A_1	A_3	A_8	22	A_4	A_7	
Th	22	A_1	A_3	19	A_2	A_7		23	A_5	A_3	
Fr	23	A_2	A_8	2	A_4	A_9		24	A_1	A_6	A_9
Sa	24	A_5	A_7	21	A_5	A_8		25	A_2	A_4	A_8
Su	25	A_2	A_8	22	A_4	A_9		26	A_1	A_6	A_9
Mo	26	A_4	A_6	23	A_6	A_3		27	A_5	A_7	
Tu	27	A_5	A_7	24	A_2	A_7		28	A_3	A_4	A_8
We	28	A_3	A_8	25	A_5	A_4	A_8	29	A_1	A_6	A_9
Th	29	A_6	A_4	26	A_1	A_6	A_9				
Fr	30	A_1	A_7	27	A_2	A_7					
Sa	31	A_3	A_8	28	A_5	A_3	A_8				
Su				29	A_2	A_7					
Mo				3	A_4	A_6	A_9				
Tu				31	A_5	A_3	A_8				

Table 12
The new schedule of december.

	December 2011				January 2012				February 2012			
		ICU	SR			ICU	SR			ICU	SR	
We									1	A_2	A_3	–
Th	1	A_2	A_7	–					2	A_6	A_8	–
Fri	2	A_5	A_3	–					3	A_5	A_7	–
Sa	3	A_1	A_8	–					4	A_3	A_9	–
Su	4	A_5	A_3	–	1	A_1	A_7	–	5	A_5	A_7	–
Mo	5	A_4	A_6	–	2	A_2	A_6	A_9	6	A_2	A_8	–
Tu	6	A_2	A_8	–	3	A_3	A_4	A_8	7	A_4	A_9	–
We	7	A_5	A_3	–	4	A_5	A_7	–	8	A_5	A_7	–
Th	8	A_6	A_4	–	5	A_2	A_6	A_9	9	A_1	A_3	A_8
Fri	9	A_2	A_8	–	6	A_3	A_4	A_8	10	A_2	A_4	–
Sa	10	A_3	A_7	–	7	A_5	A_7	–	11	A_5	A_7	–
Su	11	A_2	A_8	–	8	A_3	A_4	A_8	12	A_2	A_4	–
Mo	12	A_5	A_6	–	9	A_2	A_6	A_9	13	A_6	A_9	–
Tu	13	A_1	A_7	–	10	A_5	A_7	–	14	A_5	A_3	A_8
We	14	A_2	A_8	–	11	A_1	A_3	A_8	15	A_2	A_7	–
Th	15	A_4	A_3	–	12	A_6	A_4	–	16	A_4	A_9	–
Fri	16	A_6	A_7	–	13	A_5	A_7	A_9	17	A_3	A_8	–
Sa	17	A_1	A_8	–	14	A_2	A_8	–	18	A_6	A_9	–
Su	18	A_6	A_7	–	15	A_4	A_9	–	19	A_3	A_8	–
Mo	19	A_5	A_4	–	16	A_5	A_3	–	20	A_5	A_7	–
Tu	20	A_1	A_8	–	17	A_4	A_7	–	21	A_1	A_6	A_9
We	21	A_3	A_7	–	18	A_1	A_6	A_9	22	A_4	A_8	–
Th	22	A_5	A_6	–	19	A_5	A_3	A_8	23	A_5	A_7	–
Fri	23	A_4	A_8	–	20	A_2	A_7	–	24	A_1	A_6	A_9
Sa	24	A_2	A_7	–	21	A_6	A_9	–	25	A_2	A_4	A_8
Su	25	A_4	A_6	–	22	A_2	A_7	–	26	A_1	A_6	A_9
Mo	26	A_3	A_8	–	23	A_5	A_4	A_8	27	A_5	A_3	–
Tu	27	A_5	A_7	–	24	A_1	A_6	A_9	28	A_4	A_7	–
We	28	A_6	A_4	–	25	A_2	A_3	A_8	29	A_1	A_6	A_9
Th	29	A_3	A_8	–	26	A_4	A_7	–				
Fri	30	A_1	A_7	–	27	A_6	A_9	–				
Sa	31	A_5	A_4	–	28	A_5	A_3	A_8				
Su					29	A_1	A_6	A_9				
Mo					30	A_4	A_7	–				
Tu					31	A_5	A_3	A_8				

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