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Synchronization of generalized Henon map via backstepping design

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Abstract

This paper proposes a backstepping method to resolve the synchronization of discrete-time chaotic systems. The proposed scheme offers systematic design method for the synchronization of a class of discrete-time hyper-chaotic systems, which implies much complicated high-order chaotic systems can be used to improve the security in chaos communications. A well-known chaotic systems: generalized Henon map is considered as illustrative example to demonstrate the general applicability of backstepping design. Numerical simulations verify the effectiveness of the approach.

1. Introduction

During the last decades, the problem of the synchronization of chaotic systems has gained the attention of an increasing number of researchers, especially in the light of its potential application in secure communications. The synchronization of chaotic systems is based on the drive-response conception: the trajectories of the drive system and the response system are identical after the transition time notwithstanding starting from different initial conditions. Many effective methods have already been successfully applied to the problem since Pecora and Carroll's original research work [1].

Recently, with the development of nonlinear control theory, backstepping design becomes an effective method to resolve the synchronization of chaotic systems [2,3,5,6]. Backstepping design represents a powerful and systematic technique that recursively interlaces the choice of a Lyapunov function with the design of feedback control. A major advantage is that backstepping can be applied for stabilization of several well-known chaotic or hyper-chaotic circuits and systems, which can be continuous-time or discrete-time chaotic systems [4]. In this paper the synchronization of chaotic system via backstepping approach is proposed for a class of discrete-time chaotic dynamical systems.

2. Problem statement

The backstepping strategy is characterized by a step-by-step procedure interlacing, at each step, a coordinate transformation and the design of a virtual control via a classical Lyapunov technique, with the definition of a tuning function, obtaining, as a result, at the last step, the true control expression.

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Consider a discrete-time nonlinear system in the strict-feedback form:

$$\begin{cases} x_i(k+1) = x_{i+1}(k) + \Phi_i(x(k)), \quad i = 1, \ldots, n-1, \\ x_n(k+1) = \Phi_n(x(k)) + \beta_0(x(k))u(k), \end{cases} \tag{1}$$

where $x(k) = [x_1(k), \ldots, x_n(k)]^{\mathrm{T}} \in R^n$ is the state vector, $u(k)$ is the control input, $\Phi_i(x(k)) \in R$ and $\beta_0(x(k)) \in R$ are known nonlinear functions. Note that the dependence of $\Phi(\cdot)$ on $x(k)$ is to be intended in the sense that Φ_i depends only on $x_1(k), \ldots, x_n(k)$, $i = 1, \ldots, n$.

Backstepping design can be applied to strict-feedback and parameteric-strict-feedback form systems. The strategy starts by considering the variable $x_2(k)$ as a virtual control input to stabilize the first equation. When $x_2(k)$ has been designed using control Lyapunov function, it goes on by considering the variable has been designed using control Lyapunov function. And it goes on by considering the variable $x_3(k)$ as the virtual control for the second equation, and so on. Therefore the design of the actual input $u(k)$ is systematically achieved in n steps. Note that the synchronization of chaotic systems is the same as a tracking problem, which controls the response system to track the drive system. So the controller u based on the backstepping design can be used to synchronize the drive-response system.

3. Synchronization of generalized Henon map

In order to show how backstepping design works, the discrete-time generalized hyper-chaotic Henon map is presented for synchronization via backstepping design.

Consider the discrete-time generalized hyper-chaotic Henon map

$$\begin{cases} y_1(k+1) = -by_3(k) + u(k), \\ y_2(k+1) = by_3(k) + y_1(k), \\ y_3(k+1) = 1 + y_2(k) - ay_3^2(k), \end{cases} \tag{2}$$

$$\begin{cases} y_{1d}(k+1) = -by_{3d}(k), \\ y_{2d}(k+1) = by_{3d}(k) + y_{1d}(k), \\ y_{3d}(k+1) = 1 + y_{2d}(k) - ay_{3d}^2(k), \end{cases} \tag{3}$$

where $u(k)$ is the control input and when $a = 1.07$, $b = 0.3$, the uncontrolled generalized Henon map displays a chaotic attractor when $u(k) = 0$.

The control goal is to find $u(k)$ so that the state vector $y_i(k)$ of the generalized Henon system (2) track the state vector $y_{id}(k)$ of the system (3), i.e. $\lim_{t\to\infty} \|y_i - y_{id}\| = 0$.

Subtracting Eq. (3) from Eq. (2), we get the error dynamics as follows:

$$\begin{cases} e_1(k+1) = -be_3(k) + u(k), \\ e_2(k+1) = be_3(k) + e_1(k), \\ e_3(k+1) = e_2(k) - ay_3^2(k) + ay_{3d}^2(k), \end{cases} \tag{4}$$

where $e_i(k) = y_i(k) - y_{id}(k)$, $i = 1, 2, 3$, is the synchronization error vector. Then the aim of synchronization is to design $u(k)$ so that the system (4) is stabilized to the origin. According to backstepping design, the following steps must be done:

Step 1. Let $z_1(k) = e_3(k)$. From (4) we have

$$z_1(k+1) = e_2(k) - ay_3^2(k) + ay_{3d}^2(k), \tag{5}$$

then, we use the error state e_2 as the virtual control in (5) and introduce the error variable $z_2(k) = e_2(k) - \alpha_1(k)$, where $\alpha_1(k)$ is a tuning function to be designed later. Let us choose the Lyapunov function as $V_1(k) = |z_1(k)|$, then the derivative of $V_1(k)$:

$$\Delta V_1(k) = V_1(k+1) - V_1(k) = \left|z_2(k) + \alpha_1(k) - ay_3^2(k) + ay_{3d}^2(k)\right| - |z_1(k)|. \tag{6}$$

We choose

$$\alpha_1(k) = c_1 z_1(k) + ay_3^2(k) - ay_{3d}^2(k), \tag{7}$$

where c_1 is a design constant to be chosen later, we obtain

$$\Delta V_1(k) = |c_1 z_1(k) + z_2(k)| - |z_1(k)| \leqslant (|c_1| - 1)|z_1(k)| + |z_2(k)| \tag{8}$$

and (5) becomes

$$z_1(k+1) = c_1 z_1(k) + z_2(k). \tag{9}$$

Step 2. From (4), the error variable $z_2(k)$ is

$$z_2(k+1) = be_3(k) + e_1(k) - c_1 z_1(k+1) + ay_3^2(k+1) - ay_{3d}^2(k+1), \tag{10}$$

when we introduce the error variable $z_3(k) = e_1(k) - \alpha_2(k)$, (10) becomes

$$z_2(k+1) = be_3(k) + z_3(k) + \alpha_2(k) - c_1 z_1(k+1) + ay_3^2(k+1) - ay_{3d}^2(k+1). \tag{11}$$

We choose

$$\alpha_2(k) = c_1 z_1(k) + c_2 z_2(k) - be_3(k) + c_1 z_1(k+1) - ay_3^2(k+1) + ay_{3d}^2(k+1) \tag{12}$$

and (11) becomes

$$z_2(k+1) = c_1 z_1(k) + c_2 z_2(k) + z_3(k), \tag{13}$$

where c_2 is a design constant to be chosen later. Choose the Lyapunov function as $V_2(k) = V_1(k) + d_1|z_2(k)|$, where we take the positive constant $d_1 > 1$, then the derivative of $V_2(k)$:

$$\Delta V_2(k) = \Delta V_1(k) + d_1|z_2(k+1)| - d_1|z_2(k)| \leqslant [(d_1+1)|c_1| - 1]|z_1(k)| + (d_1|c_2| + 1 - d_1)|z_2(k)| + d_1|z_3(k)|. \tag{14}$$

Step 3. By iterating the previous steps, the error variable $z_3(k)$ is

$$z_3(k+1) = e_1(k+1) - \alpha_2(k+1) = -be_3(k) + u(k) - c_1 z_1(k) + c_2 z_2(k) + \psi(k), \tag{15}$$

where $\psi(k) = -be_3(k+1) + c_1 z_1(k+2) - ay_3^2(k+2) + ay_{3d}^2(k+2)$.

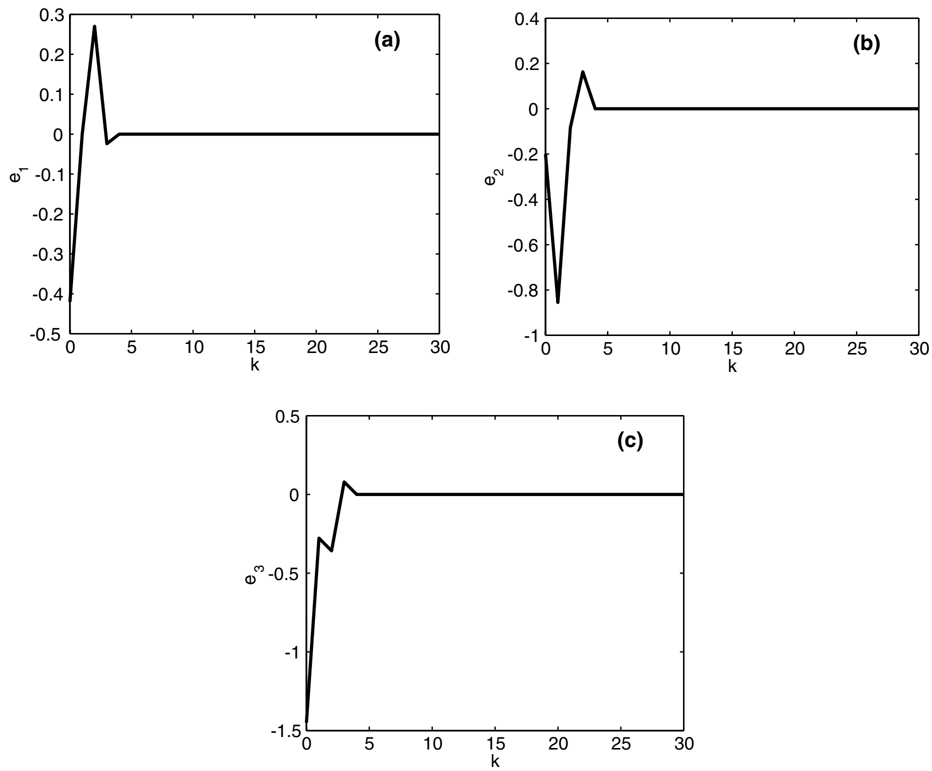

Fig. 1. Dynamics of synchronization errors for two generalized Henon systems (drive system and response system): (a) $e_1 = y_1 - y_{1d}$, (b) $e_2 = y_2 - y_{2d}$ and (c) $e_3 = y_3 - y_{3d}$.

At this step, we can determine the control $u(k)$ as

$$u(k) = be_3(k) + c_3z_3(k) + c_1z_1(k+1) - c_2z_2(k+1) - \psi(k). \tag{16}$$

So we have

$$z_3(k+1) = c_3z_3(k). \tag{17}$$

The third Lyapunov function is chosen as follows:

$$V_3(k) = V_1(k) + V_2(k) + d_2|z_3(k)|,$$

where d_2 is a positive constant and $d_2 > d_1 > 1$, then the derivative of $V_3(k)$:

$$\begin{aligned}\Delta V_3(k) &= \Delta V_1(k) + \Delta V_2(k) + d_2|z_3(k+1)| - d_2|z_3(k)| \leqslant [(d_1+2)|c_1| - 2]|z_1(k)| + (d_1|c_2| + 2 - d_1)|z_2(k)| \\ &\quad + (d_2|c_3(k)| + d_1 - d_2)|z_3(k)|.\end{aligned} \tag{18}$$

In order to make the $\Delta V_3(k)$ negative definite, we can choose c_1, c_2, c_3 such that $|c_1| < \frac{2}{d_1+2}$, $|c_2| < \frac{d_1-2}{d_1}$, $|c_3| < \frac{d_2-d_1}{d_2}$, where $d_2 > d_1 > 1$.

Based on Lyapunov stability theory, the error system (4) is globally stable about the origin. Therefore, the system (2) is synchronized with the system (1) via backstepping design.

4. Simulation studies

In this section, numerical simulations are given to verify the method proposed. In these numerical simulations, the fourth-order Runge-kutta method is used to solve generalized hyper-chaotic Henon map system, with k as 0.001. The parameters are selected as follows $a = 1.07$, $b = 0.3$, with initial values $y_1(0) = -0.25$, $y_2(0) = 0.3$, $y_3(0) = -0.75$; $y_{1d}(0) = 0.2$, $y_{2d}(0) = 0.5$, $y_{3d}(0) = 0.7$ and we choose the value of c_1, c_2, c_3, d_1, d_2 as $c_1 = 0$, $c_2 = 0$, $c_3 = 0$, $d_1 = 4$, $d_2 = 5$. The simulation results are illustrated in Fig. 1. Fig. 1(a) shows $e_1 = y_1 - y_{1d}$, Fig. 1(b) shows $e_2 = y_2 - y_{2d}$ and Fig. 1(c) shows $e_3 = y_3 - y_{3d}$. From the figures, we can see that the synchronization error will converge to zero finally and two generalized Henon map systems from different initial values are indeed achieving chaos synchronization.

5. Conclusion

Backstepping design represents a powerful and systematic technique that recursively interlaces the choice of a Lyapunov function with the design of feedback control. In this paper, based on the backstepping scheme, a method is proposed to synchronize a class of discrete-time hyper-chaotic systems, such as generalized Henon map system, which has much complicated high-order chaotic systems can be used to improve the security in chaos communications. The simulation studies show the effectiveness of the method.

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