Coordinating Regulation and Demand Response in Electric Power Grids: Direct and Price-Based Tracking Using Multirate Economic Model Predictive Control*

Haitham Hindi, Daniel Greene, and Caitlin Laventall

Abstract We propose a framework for reducing demand-supply imbalances in the grid, by jointly controlling both the supply-side electric power regulation together with the demand-side energy consumption by residential and commercial consumers demand response. We focus on performance improvements that arise from the complementary dynamics: regulation allows for frequent control updates but suffers from slower dynamics; demand response has faster dynamics but does not allow as frequent control updates. We propose a multirate model predictive control (MPC) approach for coordinating the two services, and we refer to this coordinator as an aggregator. Multirate MPC captures the varying dynamics and update rates, and nonlinearities due to saturation and ramp rate limits, and a total variation constraint limits the switching of the demand response signal. Our approach can operate with both direct reference or indirect market-price based imbalance signal. Numerical examples are presented to show the efficacy of this joint control approach.

---

*Based on "Coordinating regulation and demand response in electric power grids using multirate model predictive control," by H. Hindi, D. Greene, C. Laventall, which appeared in the IEEE Innovative Smart Grid Technologies Conference ISGT 2011, © 2011 IEEE.

H. Hindi (✉) • D. Greene
(Xerox) PARC, Palo Alto, CA, USA
e-mail: hhindi; greene@parc.com

C. Laventall
AOL.com, Palo Alto, CA, USA
e-mail: klaventall@aol.com

A. Chakrabortty and M.D. Ilić (eds.), *Control and Optimization Methods for Electric Smart Grids*, Power Electronics and Power Systems 3,
DOI 10.1007/978-1-4614-1605-0_5, © Springer Science+Business Media, LLC 2012

1 Introduction

In this chapter, we explore the idea of reducing demand-supply imbalances in the grid, by jointly controlling both the supply-side electric power regulation together with the demand-side energy consumption by residential and commercial consumers [15, 16, 20]. Currently, the main instrument for regulating the supply demand imbalance is a set of supply-side generation reserves, known as *ancillary services*, that operate on various scales of time and frequency. The control of energy consumption by residential and commercial customers is known as *demand response* and has become a key idea in the Smart Grid vision.

Here, we focus specifically on the potential performance improvements that arise from the complementary nature of two sets of dynamics: *regulation* and *fast demand response*. Regulation is an ancillary service that operates on the second-to-minute timescale, and is traded in units of Mega Watts [19, 22–24]; while fast demand response refers to the kind of rapid demand cutbacks that can be achieved "within the flick of a switch", e.g. from turning off a home appliance, such as an airconditioner or dryer or light [15, 16, 20]. The complementary dynamics we wish to explore in this chapter are the following: regulation allows for frequent control updates but suffers from slower dynamics of large generation equipment; demand response has faster dynamics but does not allow as frequent control updates, due to potential wear and tear on appliances.

We propose *multirate model predictive control* (MPC) as an effective computational framework for coordinating regulation and demand response and for exploring the space of different design scenarios [3,4,7,10,18,21,25]. The multirate MPC approach captures the varying dynamics and update rates, as well as the nonlinearities due to saturation and ramp rate limits, and we use a total variation constraint to limit the switching of the demand response signal. The multirate MPC approach results in a quadratic program (QP) that must be solved at each time step [2, 4, 5, 10, 13] or a more complex optimization problem, e.g. when nonconvex costs are considered. We call this multirate MPC-based coordinator or controller an *aggregator*, because it combines and coordinates the two services into an effective joint ancillary service.

In addition, we show that our approach has the flexibility to be implemented in the two most likely deployment scenarios. In the first, a direct demand-supply imbalance reference tracking signal is available. In the second, an indirect market price-based tracking signal is available [1, 6, 9, 19, 22–25]. This market-based tracking approach is related to recent so-called economic MPC [8].

There are some practical applications, which would not have the resources to solve a QP at each time step. Therefore, we also present a much simpler heuristic controller, which delivers reasonably good performance in some operating regimes.

Numerical examples are presented to show the efficacy of this joint control approach. Specifically, it is shown that, under certain conditions, fast demand response can significantly enhance the quality of traditional supply-side regulation, to achieve better overall performance, in terms of minimizing demand-supply imbalance.

This chapter is a slightly expanded version of [12]; new material includes the derivation of the price-based MPC, which was only sketched briefly in the original paper, along with a numerical example to demonstrate its efficacy.

Notation: $||x||_p$ denotes the p-norm of x for $p > 1$. The *saturation* function with level $\alpha > 0$ and the *Kronecker delta* function are defined, respectively, as

$$\mathrm{sat}_\alpha(x) = \begin{cases} -\alpha & \text{if } x < -\alpha, \\ x & \text{if } |x| \leq \alpha, \\ \alpha & \text{if } x > \alpha, \end{cases} \qquad \delta(x) = \begin{cases} 1 & \text{if } x = 0, \\ 0 & \text{otherwise.} \end{cases}$$

For compactness, we use $k \in I_{n_1}^{n_2}$ to denote $k \in \{n_1, n_1 + 1, \ldots, n_2\}$.

2 Problem Statement

The objective of this chapter is to design a controller, termed the aggregator, which simultaneously manages demand response and a regulation service. More specifically, we would like to design control signals which will enable our plant to track a time varying reference signal (i.e., the energy imbalance). The plant of our model (see Fig. 1) is composed of two subsystems, one for the demand response and one for the regulation service. Evolution of each of these subsystems is subject to a number of constraints, see Sect. 3. Denote x_t^{rf} as the state of the reference (imbalance) signal, x_t^{dr} as the state of the demand response, x_t^{rg} as the state of the regulation service, and

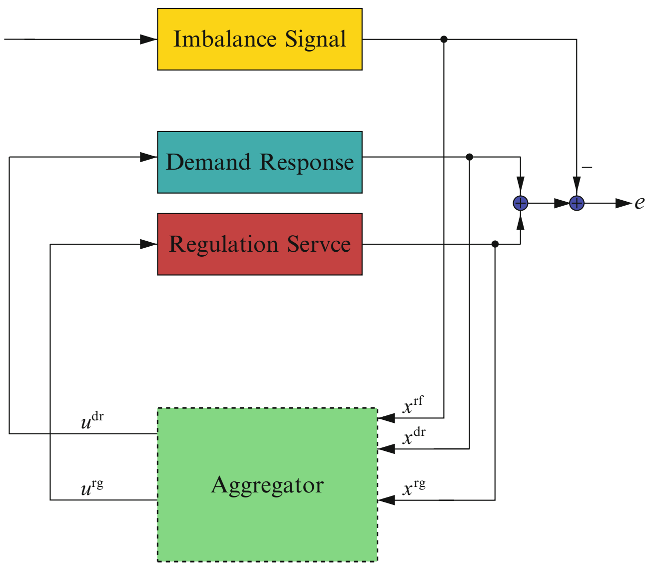

Fig. 1 Block diagram of the linked system

$e_t = x_t^{\mathrm{rg}} + x_t^{\mathrm{dr}} - x_t^{\mathrm{rf}}$ as the tracking error. The control signals u_t^{rg} and u_t^{dr} are produced by the aggregator, which is a time-varying state feedback mapping $f_{\mathrm{agg}} : \mathbf{R}^3 \to \mathbf{R}^2$. We model the closed loop nonlinear time-varying system as

$$\begin{aligned} x_{t+1}^{\mathrm{rf}} &= x_t^{\mathrm{rf}} + w_t^{\mathrm{rf}}, \\ x_{t+1}^{\mathrm{rg}} &= f_{\mathrm{rg}}\left(x_t^{\mathrm{rg}}, u_t^{\mathrm{rg}}, t\right), \\ x_{t+1}^{\mathrm{dr}} &= f_{\mathrm{dr}}\left(x_t^{\mathrm{dr}}, u_t^{\mathrm{dr}}, w_t^{\mathrm{dr}}, t\right), \\ e_t &= x_t^{\mathrm{rg}} + x_t^{\mathrm{dr}} - x_t^{\mathrm{rf}}, \\ \left(u_t^{\mathrm{rg}}, u_t^{\mathrm{dr}}\right) &= f_{\mathrm{agg}}\left(x_t^{\mathrm{rf}}, x_t^{\mathrm{rg}}, x_t^{\mathrm{dr}}, t\right), \end{aligned} \tag{1}$$

where f_{rg} and f_{dr} are decoupled, nonlinear functions, f_{agg} is the time varying state feedback control function, which will be computed using MPC or heuristic method. Details of these functions will be covered in Sect. 4.1. Precise modeling of the dynamics of imbalance reference signal neither is the focus of our chapter nor will affect our conclusions. This is because the multirate MPC framework is model based and does not place any restrictions on the reference dynamics. For simplicity, in this work we assume it evolves as a zero mean random walk, driven by a white noise w_t^{rf}. Historic data of this imbalance signal, and its associated price, are publicly available on numerous ISO and utility websites. We also include a noise term w_t^{dr} in the demand response dynamics, due to the expected higher uncertainty in the response of homes and small businesses.

3 Qualitative Description of Models and Specs

Numerous model constraints (e.g., capacity limitations, communication delays.) impede the performance of our system. In the section below, all model constraints we have incorporated into our system are outlined. A qualitative description of the constraints is discussed, followed by their mathematical representation. Once again, we note that the multirate MPC framework is capable of modeling a wide range of dynamical models, including: continuous, discrete, hybrid, and stochastic. Since our main goal here is to focus on the multirate dynamical aspects of regulation and demand response, we will use the standard abstractions of regulation and demand response dynamics as first-order processes with various capacity and ramp-rate limits [1, 6, 9, 19, 22–25].

3.1 Demand Response Constraints

- *Limited communication*: Because communication protocols have yet to be established for the demand response program, we assume limited communication between the aggregator and demand response. Part of this chapter's objective is to show that even when the performance of demand response is limited by infrequent control updates, it is still able to reduce the workload of a regulation service and contribute to grid stability.

- *Customer disutility*: One of the primary limitations of demand response is how much customers are willing to cut back before they become inconvenienced. From the aggregator's perspective, it must limit the total resource consumption by the demand response.
- *Mechanical wear-and-tear of appliances*: Excessive wear on appliances, specifically those with a duty cycle, should be avoided. This means the aggregator cannot send cutback signals to the demand response (homes and businesses) too frequently. We model this by limiting the total variation of the output by the demand response.
- *Uncertainty of response*: An essential component of the demand response program is the ability for a customer to override the aggregator's signal at any time. As the population of demand response participants increases, so does the variance of this uncertainty—hence the w_t^{dr} in (1).

3.2 Regulation Service Constraints

- *Maximum reserve capacity limitations*: Typically, a power plant has allocated a limited amount of its total capacity for regulation service. This means the peak value of regulation power output is limited.
- *Ramp rate*: The large inertia of plant generators limits how quickly they can ramp up or ramp down in response to the aggregator's input. This is modeled as a constraint on the peak value of the control input.

4 Quantitative Description of Models and Specs

4.1 System Model with Incorporated Constraints

Out next task is to put the constraints described in the previous section into a system theoretic framework. Each constraint adds nonlinear behavior to the dynamics. However, as we shall see in Sect. 5, our plant and controller can be reformulated as a linear system with linear constraints. We first define the following parameters.

- $\alpha_{\mathrm{max}}^{\mathrm{rg}}$, $\alpha_{\mathrm{max}}^{\mathrm{dr}}$: Maximum of the regulation service and demand response, respectively. Saturation point of maximum reserve capacity.
- $\alpha_{\mathrm{rmp}}^{\mathrm{rg}}$: Ramp rate constant for the regulation. Saturation point of the input u^{rg}. For completeness, we will also introduce $\alpha_{\mathrm{rmp}}^{\mathrm{dr}}$, but it will be set assumed infinite in this chapter.
- T_{rg}, T_{dr}: Input control update rates of the regulation service (demand response, respectively).

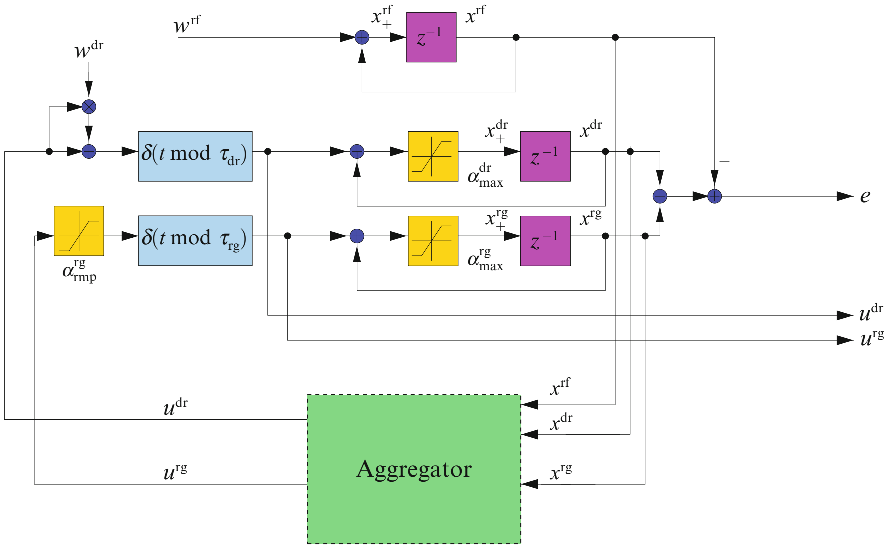

Fig. 2 Block diagram of the linked system with incorporated model constraints

With these parameters, we can describe the state space model (1) in more detail:

$$
\begin{aligned}
x^{rf}_{t+1} &= x^{rf}_t + w^{rf}_t, \\
x^{rg}_{t+1} &= \mathrm{sat}_{\alpha^{rg}_{max}}\left(x^{rg}_t + \delta(t \bmod T_{rg})\, \mathrm{sat}_{\alpha^{rg}_{rmp}}(u^{rg}_t)\right), \\
x^{dr}_{t+1} &= \mathrm{sat}_{\alpha^{dr}_{max}}\left(x^{dr}_t + \delta(t \bmod T_{dr})\left(u^{dr}_t + \sqrt{u^{dr}_t} w^{dr}_t\right)\right).
\end{aligned} \tag{2}
$$

See Fig. 2 for a block diagram representation of this system. Note the periodic time-varying characteristic of the control signal: $\delta(t \bmod T_{xx})u^{xx}_t \neq 0$ only if t is a multiple of T_{xx}. Thus when $T_{rg} \neq T_{dr}$ we effectively have control output which directs each sub-plant at different rates. For the purposes of our model, we assume $T_{rg} = cT_{dr}$ with $c < 1$. Thus, demand response receives information from the aggregator less frequently than the regulation service.

Also, note that we have modeled the uncertainty in the demand response as a noise w^{dr}_t, which enters in a *multiplicative* rather than additive way, scaled by the square-root of the input. This is in anticipation that ultimately, the demand response is likely to be aggregated from a large number of participants, e.g. homes and small businesses, thus the variance would scale as the sum of random variables.

We will not pursue the demand response uncertainty modeling any further in this chapter. Observe that in the absence of this multiplicative noise, the structure of the regulation and the demand response dynamics is identical, albeit with different values for their respective parameters, of course. In addition, the regulation will have a finite ramp-rate limit $\alpha^{rg}_{rmp} \leq \infty$, while the demand response will be assumed to have effectively infinite ramp rate $\alpha^{dr}_{rmp} = \infty$.

4.2 Performance Measures

The following performance metrics measure the costs associated with each model constraint:

- $||x_t^{\mathrm{rf}} - x_t^{\mathrm{rg}} - x_t^{\mathrm{dr}}||_2^2$: Euclidean norm of the tracking error. This is the primary measure we would like to keep small. A soft constraint.
- $||x_t^{\mathrm{rg}}||_1$, $||x_t^{\mathrm{dr}}||_1$: Total resource consumption by the regulation and demand response. This is the dominant cost deriving from regulation cost and consumer disutility. A soft constraint.
- $||x_t^{\mathrm{rg}}||_\infty$,$||x_t^{\mathrm{dr}}||_\infty$: Maximum peak of the regulator (demand response, respectively). Restricts state trajectories since they cannot operate beyond full capacity. A hard constraint.
- $||x_t^{\mathrm{dr}}||_{\mathrm{TV}}$: Total variation of demand response. A secondary cost related to mechanical wear-and-tear of the load. A hard constraint.
- $||u_t^{\mathrm{rg}}||_2$, $||u_t^{\mathrm{dr}}||_2$: Weighted input cost for regulation (demand response, respectively). A soft constraint.
- $||u_t^{\mathrm{rg}}||_\infty$,$||u_t^{\mathrm{dr}}||_\infty$: Ramp rate constraints. Limits the speed either service can ramp-up or ramp-down. A hard constraint.

Note that we represent the ramp rate constraints as slew-rate limits on the inputs; they could just as well be represented as direct rate limits on the state variables. Also, our choice of using the 1-norm and Euclidean norms to represent those costs are primarily for illustrative and computational convenience. Many other choices are possible for capturing the costs of regulation and demand response, depending on the generation resource being used; while many other choices are possible as metrics of tracking error. Although these other choices might change some of the computational properties of the optimization problem (e.g., convexity), they will still fall within the general framework here of optimizing an objective function with terms representing power costs of regulation and demand response, and terms representing an imbalance or tracking error metric (or price signal, as shown below).

Furthermore, the optimization framework presented here can also be used within a more general framework that considers other costs, prices, and constraints. These could include: fuel cost, startup/shutdown costs, regular operating costs; consumer and appliance utility and discomfort, minimum up/down time constraints; prices for other services, such as spinning and nonspinning reserve.

5 Reference Tracking Multirate MPC

We now design a MPC scheme where at each time step the aggregator solves a planning problem, which incorporates explicit knowledge of the plant model and feedback information into its formulation. For simplicity of notation, we consider a single regulation resource and a single demand response resource. However, the framework extends trivially to any number of either, by simply adding the obvious costs and constraints corresponding to each new agent.

Given the performance measures outlined in the previous section and a finite horizon N, the aggregator will solve the following planning problem at each time step $t = 0, 1, \ldots$:

$$\begin{aligned}
\underset{u^{\mathrm{rg}}, u^{\mathrm{dr}}, \hat{x}^{\mathrm{rg}}, \hat{x}^{\mathrm{dr}}}{\text{minimize}} & \begin{cases} \sum_{k=0}^{N} ||\hat{x}_k^{\mathrm{rf}} - \hat{x}_k^{\mathrm{rg}} - \hat{x}_k^{\mathrm{dr}}||_2^2 + \\ \sum_{k=0}^{N} \rho_0 ||\hat{x}_k^{\mathrm{rg}}||_1 + \rho_1 ||\hat{x}_k^{\mathrm{dr}}||_1 + \\ \sum_{k=0}^{N-1} \rho_2 ||\hat{u}_k^{\mathrm{rg}}||_2^2 + \rho_3 ||\hat{u}_k^{\mathrm{dr}}||_2^2 \end{cases} \\
\text{subject to } & \hat{x}_{k+1}^{\mathrm{rg}} = \hat{x}_k^{\mathrm{rg}} + \delta((t+k) \bmod T_{\mathrm{rg}}) \hat{u}_k^{\mathrm{rg}}, \; k \in I_0^{N-1} \\
& \hat{x}_{k+1}^{\mathrm{dr}} = \hat{x}_k^{\mathrm{dr}} + \delta((t+k) \bmod T_{\mathrm{dr}}) \hat{u}_k^{\mathrm{dr}}, \; k \in I_0^{N-1} \\
& \hat{x}_0^{\mathrm{rg}} = x_t^{\mathrm{rg}} \\
& \hat{x}_0^{\mathrm{dr}} = x_t^{\mathrm{dr}} \\
& ||\hat{x}_k^{\mathrm{rg}}||_\infty \le \alpha_{\max}^{\mathrm{rg}}, \; k \in I_0^N \\
& ||\hat{x}_k^{\mathrm{dr}}||_\infty \le \alpha_{\max}^{\mathrm{dr}}, \; k \in I_0^N \\
& ||\hat{u}_k^{\mathrm{rg}}||_\infty \le \alpha_{\mathrm{rmp}}^{\mathrm{rg}}, \; k \in I_0^{N-1} \\
& ||\hat{u}_k^{\mathrm{dr}}||_\infty \le \alpha_{\mathrm{rmp}}^{\mathrm{dr}}, \; k \in I_0^{N-1} \\
& \sum_{k=0}^{N-1} |\hat{x}_{k+1}^{\mathrm{dr}} - \hat{x}_k^{\mathrm{dr}}| \le \beta_{\mathrm{TV}},
\end{aligned} \tag{3}$$

where $\hat{x}_1^{\mathrm{rg}}, \ldots, \hat{x}_N^{\mathrm{rg}}, \hat{x}_1^{\mathrm{dr}}, \ldots, \hat{x}_N^{\mathrm{dr}}, \hat{u}_0^{\mathrm{rg}}, \ldots, \hat{u}_{N-1}^{\mathrm{rg}}, \hat{u}_0^{\mathrm{dr}}, \ldots, \hat{u}_{N-1}^{\mathrm{dr}}$ are our variables and x_t^{rf}, $T_{\mathrm{rg}}, T_{\mathrm{dr}}$, $\alpha_{\max}^{\mathrm{rg}}, \alpha_{\max}^{\mathrm{dr}}, \alpha_{\mathrm{rmp}}^{\mathrm{rg}}, \alpha_{\mathrm{rmp}}^{\mathrm{dr}}$, and the initial states x_t^{rg}, x_t^{dr} are given data (see Sect. 3); the constants $\rho_0, \rho_1, \rho_2, \rho_3$ allow us to weight the different terms in the cost function.

The reference imbalance signal $\hat{x}_k^{\mathrm{rf}}, k = 0, \ldots, N$, with $\hat{x}_0^{\mathrm{rf}} = x_t^{\mathrm{rf}}$, is assumed to be given. In practice, it could come from an internal or external forecast, previously agreed upon contracts, day-ahead / hour-ahead markets, or other mechanisms. For the purposes of this chapter, we will use a naive certainty equivalent estimate. We will not view the reference signal as a state of the system, but instead as a zero mean random walk that we are trying to track. At any time t we only know the current value x_t^{rf}, but not the future values. So for each planning step we will track $\mathbf{E}[x_k^{\mathrm{rf}} | x_t^{\mathrm{rf}}] = x_t^{\mathrm{rf}}$ for $k = t, t+1, \ldots$. In other words, at each time step t, the planning problem will track a *constant* $\hat{x}_k^{\mathrm{rf}} \equiv x_t^{\mathrm{rf}}, k = 0, \ldots, N$, that constant being our best estimate of the average value of the future values of x^{rf}, which is the current value x_t^{rf}, since it is a zero mean random walk.

The first summation in the objective function of the optimization problem above penalizes tracking error, while the second and third summations are meant to capture the input and output costs of the regulation and the fast demand response. Note that this MPC planning problem can be cast as a convex QP with linear constraints, which can be solved very efficiently, and to global optimality.

Thus at each time step t, given $(x_t^{\mathrm{rf}}, x_t^{\mathrm{rg}}, x_t^{\mathrm{dr}})$, the aggregator solves the planning problem (3) and selects control signals using

$$\begin{aligned} f_{\mathrm{agg}}\left(x_t^{\mathrm{rf}}, x_t^{\mathrm{rg}}, x_t^{\mathrm{dr}}, t\right) &= \left(u_t^{\mathrm{rg,mpc}}, u_t^{\mathrm{dr,mpc}}\right) \\ &= \left(\hat{u}_0^{\mathrm{rg}}, \hat{u}_0^{\mathrm{dr}}\right). \end{aligned}$$

The closed loop system with the MPC inputs will evolve as in (2). And because the MPC respects all the system saturation and ramp rate limits, in the absence of noise, (2) reduces to

$$\begin{aligned} x_{t+1}^{\mathrm{rf}} &= x_t^{\mathrm{rf}} + w_t \\ x_{t+1}^{\mathrm{rg}} &= x_t^{\mathrm{rg}} + \delta\left(t \bmod T_{\mathrm{rg}}\right) u_t^{\mathrm{rg,mpc}} \\ x_{t+1}^{\mathrm{dr}} &= x_t^{\mathrm{dr}} + \delta\left(t \bmod T_{\mathrm{dr}}\right) u_t^{\mathrm{dr,mpc}} \\ e_t &= x_t^{\mathrm{rf}} - x_t^{\mathrm{rg}} - x_t^{\mathrm{dr}}. \end{aligned}$$

Note again the time-varying characteristic of the control signal: $\delta(t \bmod T_{\mathrm{xx}})$ $u_t^{\mathrm{xx,mpc}} \neq 0$ only if t is a multiple of T_{xx}.

6 Market Price Based Multirate MPC

In this section, we model the scenario where the aggregator is operating off an indirect imbalance signal, a market price signal λ_t, rather than a direct reference signal x_t^{rf}. Again, for simplicity of notation, we consider a single regulation resource and a single demand response resource.

This indirect market price formulation can be rigorously derived from the direct reference tracking formulation above: one applies the standard economics method, of appending the market clearing constraint (demand=supply) to the objective in the first formulation, then duality is used to obtain a decomposition into the usual producer and consumer subproblems. The objective in our second formulation below is equivalent to producer subproblem, namely that of profit maximization; the constraints remain unchanged.

Toward this end, let us re-write our direct reference tracking problem (3) in an equivalent way, with an extra variable y:

$$\begin{aligned} \underset{u^{\mathrm{rg}},u^{\mathrm{dr}},\hat{x}^{\mathrm{rg}},\hat{x}^{\mathrm{dr}},y}{\text{minimize}} & \begin{cases} \sum_{k=0}^{N} ||\hat{x}_k^{\mathrm{rf}} - y_k||_2^2 + \\ \sum_{k=0}^{N} \rho_0 ||\hat{x}_k^{\mathrm{rg}}||_1 + \rho_1 ||\hat{x}_k^{\mathrm{dr}}||_1 + \\ \sum_{k=0}^{N-1} \rho_2 ||\hat{u}_k^{\mathrm{rg}}||_2^2 + \rho_3 ||\hat{u}_k^{\mathrm{dr}}||_2^2 \end{cases} \\ \text{subject to } & y_k = \hat{x}_k^{\mathrm{rg}} + \hat{x}_k^{\mathrm{dr}}, \quad k \in I_0^N \\ & \text{constraints of (3).} \end{aligned} \tag{4}$$

Clearly, this formulation is equivalent to (3), as the new equality could be used to eliminate y from (4) and recover (3). The new variable, y, represents the amount of load actually fulfilled, as compared to x^{rf}, which is to be interpreted as the amount of load power "desired." The expression $||\hat{x}_k^{\mathrm{rf}} - y_k||_2^2$ can be interpreted as demand disutility, or a penalty on unmet demand. The first line in the objective of (4) can be interpreted as the consumer benefit function, which models demand. The second and third lines of the objective can be interpreted as the production cost, as they measure the amount of inputs used and the amount of power produced. Hence, they model supply. The new constraint is simply enforcing that actual fulfilled demand y_k must equal supply $\hat{x}_k^{\mathrm{rg}} + \hat{x}_k^{\mathrm{dr}}$.

Forming the partial Lagrangian with the demand=supply constraint we obtain

$$\begin{aligned} L(u, x, y, \lambda) = & \sum_{k=0}^{N} ||\hat{x}_k^{\mathrm{rf}} - y_k||_2^2 + \lambda_k (y_k - \hat{x}_k^{\mathrm{rg}} - \hat{x}_k^{\mathrm{dr}}) \\ & + \sum_{k=0}^{N} \rho_0 ||\hat{x}_k^{\mathrm{rg}}||_1 + \rho_1 ||\hat{x}_k^{\mathrm{dr}}||_1 \\ & + \sum_{k=0}^{N-1} \rho_2 ||\hat{u}_k^{\mathrm{rg}}||_2^2 + \rho_3 ||\hat{u}_k^{\mathrm{dr}}||_2^2 \\ = & \left\{ \sum_{k=0}^{N} ||\hat{x}_k^{\mathrm{rf}} - y_k||_2^2 + \lambda_k \, y_k \right\} \\ & + \left\{ \sum_{k=0}^{N} \rho_0 ||\hat{x}_k^{\mathrm{rg}}||_1 + \rho_1 ||\hat{x}_k^{\mathrm{dr}}||_1 - \lambda_k \left(\hat{x}_k^{\mathrm{rg}} + \hat{x}_k^{\mathrm{dr}} \right) \right. \\ & \quad \left. + \sum_{k=0}^{N-1} \rho_2 ||\hat{u}_k^{\mathrm{rg}}||_2^2 + \rho_3 ||\hat{u}_k^{\mathrm{dr}}||_2^2 \right\}. \end{aligned} \tag{5}$$

The first line in the objective of (5) can be interpreted as the consumer surplus function, which trades off deviation penalty with payment $\lambda_k \, y_k$. The second and third lines of the objective can be interpreted as the (negative of) producer profit, as they measure the amount of inputs and power produced minus revenue $\lambda_k(\hat{x}_k^{\mathrm{rg}}+\hat{x}_k^{\mathrm{dr}})$. Note that, for a fixed λ, the consumer surplus is just a function of y, while the production cost is just a function of $\hat{x}$ and u. Therefore, the partial Lagrangian is seperable.

The dual function can now be defined as

$$
\begin{aligned}
q(\lambda) = \underset{y}{\text{minimize}} & \left\{ \sum_{k=0}^{N} ||\hat{x}_k^{\mathrm{rf}} - y_k||_2^2 + \lambda_k\, y_k \right\} \\
& + \underset{u,\hat{x},(3)}{\text{minimize}} \left\{ \sum_{k=0}^{N} \rho_0 ||\hat{x}_k^{\mathrm{rg}}||_1 + \rho_1 ||\hat{x}_k^{\mathrm{dr}}||_1 - \lambda_k \left(\hat{x}_k^{\mathrm{rg}} + \hat{x}_k^{\mathrm{dr}}\right) \right. \\
& \qquad \left. + \sum_{k=0}^{N-1} \rho_2 ||\hat{u}_k^{\mathrm{rg}}||_2^2 + \rho_3 ||\hat{u}_k^{\mathrm{dr}}||_2^2 \right\}. \qquad (6)
\end{aligned}
$$

Thus for a given price λ, computing the dual function amounts to solving two separate optimization problems: a consumer surplus maximization and a producer profit maximization.

The role of the market maker, aggregator, or ISO, would be to compute the optimal price $\lambda^\star$, which solves the dual optimization problem

$$
\underset{\lambda}{\text{maximize}} \quad q(\lambda). \qquad (7)
$$

Then, under suitable conditions (e.g. convexity of costs, and polytopic constraints) strong duality will hold, so if the consumers and producers solve their individual optimizations using the optimal prices $\lambda^\star$, they will also solve (4), with the demand=supply constraint intact, and hence the equivalent (3). Therefore, $\lambda^\star$ can be used for price-based tracking.

Hence, in the price-based market scenario, the aggregator would solve the following QP planning problem at each time step $t = 0, 1, \ldots$:

$$
\begin{aligned}
\underset{u^{\mathrm{rg}},u^{\mathrm{dr}},\hat{x}^{\mathrm{rg}},\hat{x}^{\mathrm{dr}}}{\text{minimize}} & \left\{ \begin{array}{l} \sum_{k=0}^{N} -\hat{\lambda}_k \cdot \left(\hat{x}_k^{\mathrm{rg}} + \hat{x}_k^{\mathrm{dr}}\right) + \\ \sum_{k=0}^{N} \rho_0 ||\hat{x}_k^{\mathrm{rg}}||_1 + \rho_1 ||\hat{x}_k^{\mathrm{dr}}||_1 + \\ \sum_{k=0}^{N-1} \rho_2 ||\hat{u}_k^{\mathrm{rg}}||_2^2 + \rho_3 ||\hat{u}_k^{\mathrm{dr}}||_2^2 \end{array} \right. \\
\text{subject to} \;\; & \hat{x}_{k+1}^{\mathrm{rg}} = \hat{x}_k^{\mathrm{rg}} + \delta((t+k) \bmod T_{\mathrm{rg}})\hat{u}_k^{\mathrm{rg}}, \; k \in I_0^{N-1} \\
& \hat{x}_{k+1}^{\mathrm{dr}} = \hat{x}_k^{\mathrm{dr}} + \delta((t+k) \bmod T_{\mathrm{dr}})\hat{u}_k^{\mathrm{dr}}, \; k \in I_0^{N-1} \\
& \hat{x}_0^{\mathrm{rg}} = x_t^{\mathrm{rg}} \\
& \hat{x}_0^{\mathrm{dr}} = x_t^{\mathrm{dr}}
\end{aligned}
$$

$$
\begin{aligned}
&||\hat{x}_k^{\mathrm{rg}}||_\infty \le \alpha_{\max}^{\mathrm{rg}},\ k \in I_0^N \\
&||\hat{x}_k^{\mathrm{dr}}||_\infty \le \alpha_{\max}^{\mathrm{dr}},\ k \in I_0^N \\
&||\hat{u}_k^{\mathrm{rg}}||_\infty \le \alpha_{\mathrm{rmp}}^{\mathrm{rg}},\ k \in I_0^{N-1} \\
&||\hat{u}_k^{\mathrm{dr}}||_\infty \le \alpha_{\mathrm{rmp}}^{\mathrm{dr}},\ k \in I_0^{N-1} \\
&\sum_{k=0}^{N-1} \left|\hat{x}_{k+1}^{\mathrm{dr}} - \hat{x}_k^{\mathrm{dr}}\right| \le \beta_{\mathrm{TV}},
\end{aligned}
\tag{8}
$$

where $\hat{\lambda}_k, k = 0, \ldots, N$ is the market price signal, with $\hat{\lambda}_0 = \lambda_t$; it is a surrogate for the market's best estimate of $\lambda^\star$. The other constants and variables are as defined earlier. The price signal $\hat{\lambda}_t$ could come from market clearing price, internal or external price forecasts, previously agreed upon contracts, day-ahead / hour-ahead markets, or other market mechanisms.

In this formulation, the objective function is minimizing the difference between the cost of regulation and demand response, captured in the second two summations, and the revenue, captured in the first summation. Minimizing this difference between cost and revenue is equivalent to maximizing profit, which is defined as revenue minus cost.

Thus at each time step t, given $(\lambda_t, x_t^{\mathrm{rg}}, x_t^{\mathrm{dr}})$, the aggregator solves the planning problem (8) and selects control signals using

$$
\begin{aligned}
f_{\mathrm{agg}}\left(\lambda_t, x_t^{\mathrm{rg}}, x_t^{\mathrm{dr}}, t\right) &= \left(u_t^{\mathrm{rg,mpc}}, u_t^{\mathrm{dr,mpc}}\right) \\
&= \left(\hat{u}_0^{\mathrm{rg}}, \hat{u}_0^{\mathrm{dr}}\right).
\end{aligned}
$$

As in the direct reference tracking case above, the resulting closed loop system with the MPC inputs will evolve according to (2).

A couple of comments regarding the practical implementation of this price-based controller are in order. We note that the seperability technique described here is applicable to any number of generators (producers) and loads (consumers). Under convexity and strong duality, the same optimal price vector will clear the entire market with multiple consumers and producers. On the other hand, it is well known that real-world power system market optimizations contain nonconvexities, e.g. due to generation constraints such as startup/shutdown, minimum up/down time, and general unit commitment issues. Regarding this issue we have two comments. First, it can be shown that as the number of participants becomes large, the effect of these nonconvexities diminishes, to the point where the duality gap can be negligible. Second, even if this is not the case, one can view our method as being instantiated after the unit commitment is done, i.e., during the economic dispatch phase, where the integer variables associated with the nonconvexities have been predetermined. Finally, the problem of estimating or forecasting the surrogate optimal market price schedule $\hat{\lambda}$ would need be adequately addressed, before a method such as the one we propose here could be deployed in practice.

7 Heuristic Control

In scenarios where it is not possible to solve a QP at each time step, we developed a heuristic controller which can still deliver good performance, in terms of linking demand response with regulation. This was achieved by designing the following control signals

$$\begin{aligned} u_t^{\mathrm{rg,heu}} &= x_t^{\mathrm{rf}} - x_t^{\mathrm{dr}} - x_t^{\mathrm{rg}}, \\ u_t^{\mathrm{dr,heu}} &= x_t^{\mathrm{rf}} - x_t^{\mathrm{dr}} - \mathrm{sat}_{\alpha_{\mathrm{res}}}\left(x_t^{\mathrm{rg}}\right), \end{aligned} \tag{9}$$

where α_{res} is a parameter which adjusts how much of the imbalance is taken on by the regulation service. The closed loop system will then evolve following (2).

Roughly speaking, in the absence of saturation, this controller feeds back the tracking error between the reference and the sum of the regulation and demand response. Thus, the closed loop system essentially acts like a multirate integral controller for rejecting the reference disturbance. The more frequently acting regulation service handles small variations below α_{res}, while the less frequent but potentially larger demand response handles larger variations that could create ramping problems for the regulation service. Beyond the level α_{res}, the regulation control input is saturated explicitly, so that more of the control effort must come from the demand response.

Section 8 provides simulation and performance evaluation of this system. We will see that in certain operating regimes, this controller performs surprisingly well, considering its simplicity.

8 Numerical Examples

Simulations of our model using both the heuristic and MPC control schemes are generated below. Using the MPC framework, a Pareto optimal trade-off curve between tracking error and total resource consumption by the demand response is also generated. Note that our examples are all implemented with dynamics on the timescale of seconds to emphasize the fast demand response aspects of this study. However, our multirate MPC framework does not depend on this in any way. Energy markets are continually evolving, so it is important to maintain generality.

8.1 Multirate MPC Versus Heuristic Controller

Figure 3 shows a simulation of (2) with the multirate MPC with the following parameters:

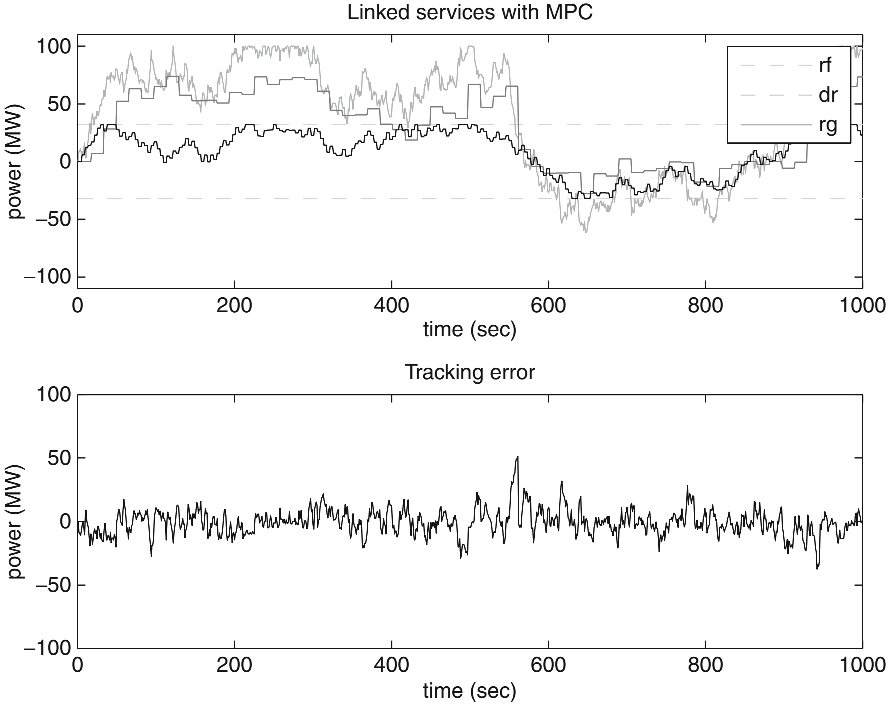

Fig. 3 State trajectories with the MPC controller for $T = 1{,}000\,\mathrm{s}$. The reference has the fastest update rate, followed by regulation, followed by demand response, with the slowest update rate

Parameter	Value
$\alpha_{\max}^{\mathrm{rg}}$	32 mw
$\alpha_{\max}^{\mathrm{dr}}$	100 mw
$\alpha_{\mathrm{rmp}}^{\mathrm{rg}}$	6 mw
$\alpha_{\mathrm{rmp}}^{\mathrm{dr}}$	∞
α_{res}	N/A
β_{TV}	∞
T_{rg}	4 s
T_{dr}	16 s

Note the reference signal is shown as having the fastest update rate (1 s). Next is the regulation, with slightly "blockier" looking response (4 s). Slowest in terms of update rate is the demand response, which has the "blockiest" response (16 s).

In order to compare the performance of the heuristic model with the MPC model, we matched as many parameters as possible. The MPC model enables us to control more constraints than the heuristic model (e.g., total variation, total

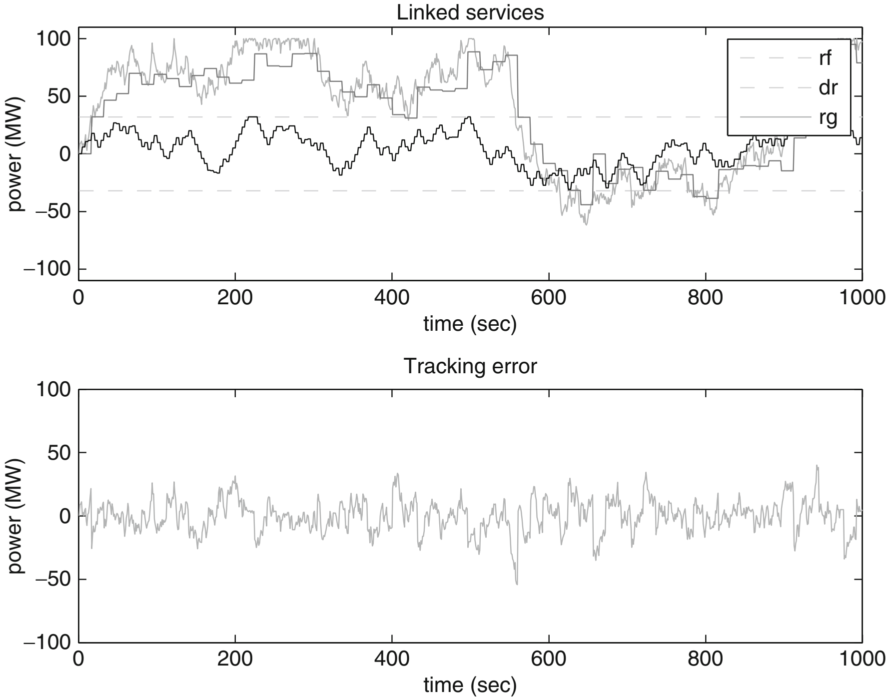

Fig. 4 State trajectories with the heuristic controller for $T = 1{,}000$ s

resource consumption.), so parameters not available to the heuristic model were left unbounded. Figure 4 shows a simulation of (9) and (2) with the following parameters:

parameter	value
$\alpha^{\mathrm{rg}}_{\max}$	32 mw
$\alpha^{\mathrm{dr}}_{\max}$	100 mw
$\alpha^{\mathrm{rg}}_{\mathrm{rmp}}$	6 mw
$\alpha^{\mathrm{dr}}_{\mathrm{rmp}}$	N/A
α_{res}	12 mw
β_{TV}	N/A
T_{rg}	4 s
T_{dr}	16 s

State trajectories are plotted in Fig. 4 over $T = 1{,}000$ s.

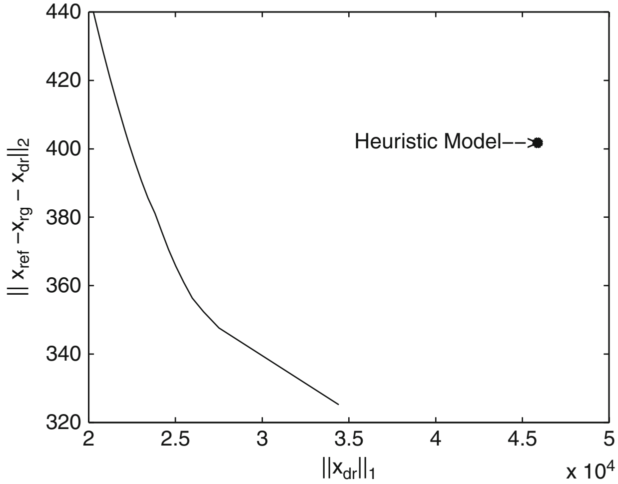

Fig. 5 Trade-off curve between the tracking error and total resource consumption by the demand response for $\rho_1 \in [1 \times 10^{-8}, 10]$, and $\rho_0 = \rho_2 = \rho_3 = 0$

8.2 *Pareto Optimal Performance Curve*

The following chart compares the performance of the heuristic model versus MPC with $\rho_1 = 1 \times 10^{-8}$, 2.5 and 9.5 and $\rho_0 = \rho_2 = \rho_3 = 0$. All other measures were adjusted to be approximately equal. Note that for $\rho_1 = 9.5$ both controllers have comparable tracking error cost, but the MPC controller reduces total resource consumption by more than 50%.

	Heuristic	$\rho_1 = 10^{-8}$	$\rho_1 = 2.5$	$\rho_1 = 9.5$
Tracking error	401.25	325.21	365.70	439.01
$\|\|x^{dr}\|\|_1$	45,870	34,415	25,020	20,322

A Pareto-optimal curve modeling the trade-off in cost between tracking error and resource consumption by the demand response is generated in Fig. 5. This curve defines the limits of performance of our system and can be used as a benchmark for measuring the performance of other controllers. This curve was generated via the scalarized multi-criterion optimization problem defined in Sect. 5 with $\rho_1 \in [10^{-8}, 10]$ and $\rho_0 = \rho_2 = \rho_3 = 10^{-8}$. More specifically, a full MPC simulation was run for 20 samples of $\rho_1 \in [10^{-8}, 10]$. For each sample, the metrics $||x^{rf} - x^{rg} - x^{dr}||_2^2$ and $||x^{dr}||_1$ were computed. A graph of these values generates the curve.

One might wonder if the performance of the heuristic controller could be improved, somewhat, by further tuning, or using a different heuristic controller. The Pareto curve gives us the answer, by showing us potentially how much more there would be to gain from such tuning or from another controller—quite a bit in this case.

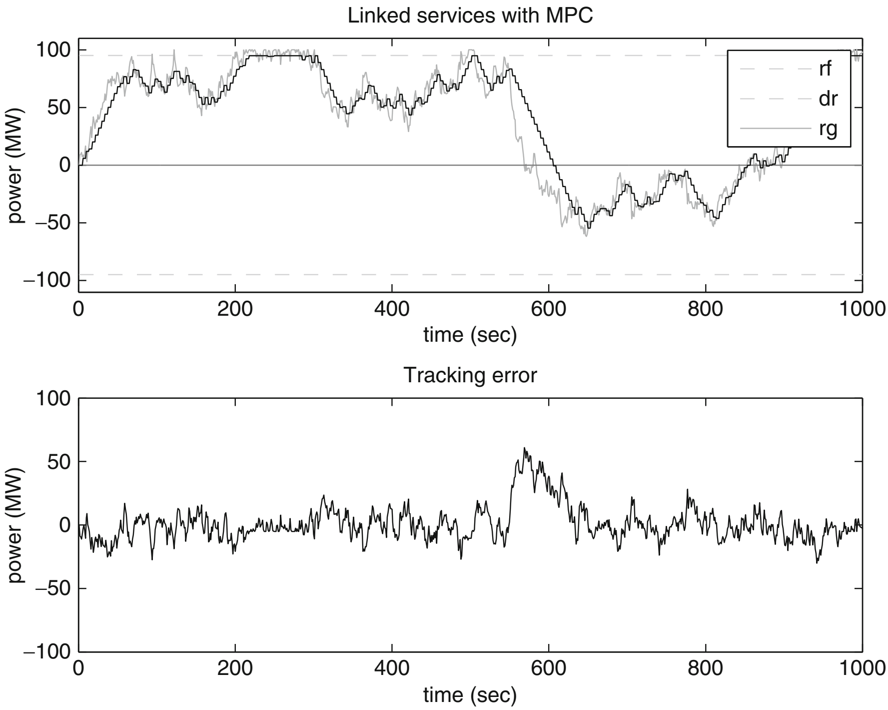

Fig. 6 MPC controller with regulation only; no demand response

8.3 Regulation Versus Demand Response

Figures 6 and 7 compare the performance of pure regulation versus combined regulation plus demand response. The results highlight how, in spite of the slower update rate, the speed with which demand response can react significantly reduces the error in the region around $t = 600$, where the imbalance reference transitions quickly from positive to negative. This is because the demand response has no limitation on its ramp rate: $\alpha^{\mathrm{dr}}_{\mathrm{rmp}} = \infty$, i.e., demand response can react almost instantly.

8.4 Price-Based Tracking via Economic MPC

Figure 8 illustrated tracking via the economic MPC method. The bottom plot shows the price signal, which is used as an input to the economic MPC of (8). Note that the price can go negative, e.g. around time 560 s. This happens when supply (DR+regulation) exceeds demand (load), in which case ancillary services are paid to spend more energy. Similar requirements exist already in today's ancillary markets, known as "reg-up"/"reg-down". The middle plot shows the multirate

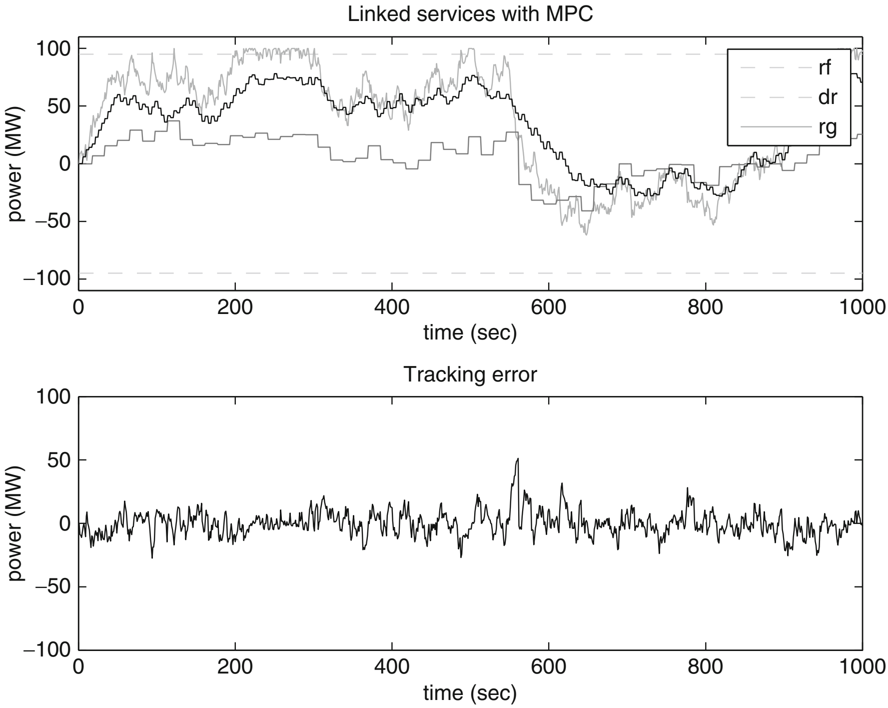

Fig. 7 MPC controller with equal regulation and demand response

control, with the larger less frequent stems being demand response, while the more frequent smaller stems are the regulation. The top plot demonstrates that effective tracking is possible using this price as a reference signal.

9 Conclusion

In this chapter, we have explored the question of whether it is possible to reduce demand-supply imbalances in the grid, by jointly controlling both the supply-side electric power regulation together with the demand-side energy consumption by residential and commercial consumers. Specifically, we focused on the potential performance improvements that arise from the complementary nature of the dynamics of the two: regulation allows for frequent control updates but suffers from slower dynamics; demand response has faster dynamics but does not allow as frequent control updates.

We proposed a multirate MPC approach. This captures the varying dynamics and update rates, as well as the nonlinearities due to saturation and ramp rate limits, and we use a total variation constraint to limit the switching of the demand response

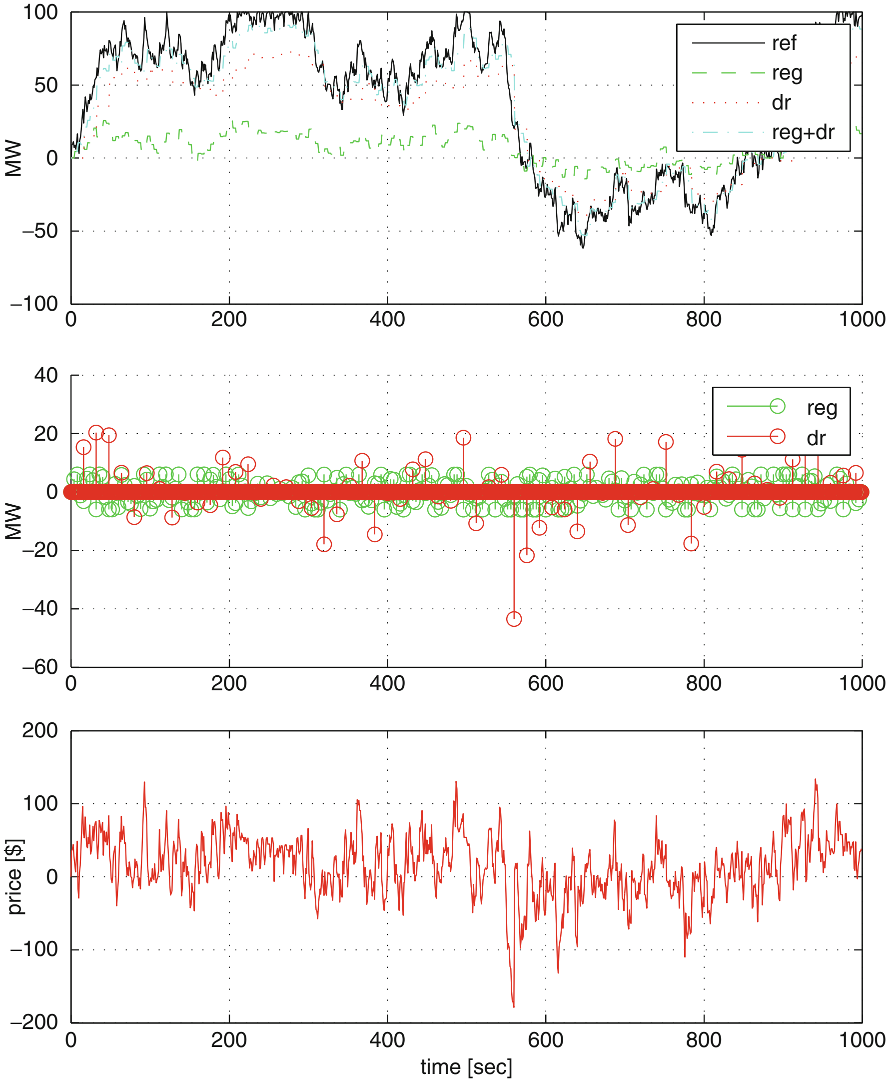

Fig. 8 Economic MPC: tracking using the price as the reference input. (*Top*) Imbalance signal along with regulation, DR, and regulation+DR. (Mid) Multirate control: regulation and DR. (*Bottom*) Price signal

signal. The multirate MPC approach results in a QP that must be solved at each time step. We also presented a much simpler heuristic controller which delivers reasonably good performance. In addition, we showed that our approach has the flexibility to be implemented in the two most likely deployment scenarios: where a direct demand-supply imbalance reference tracking signal is available; or where an indirect market price based imbalance signal is available.

Numerical examples were presented to show the efficacy of this joint control approach. Specifically, it was shown that there are indeed conditions under which fast demand response can significantly enhance the quality of traditional supply-side regulation, to achieve better overall performance.

In closing, we also point out that the multirate MPC framework presented here is not limited to modeling only regulation and demand response. It can just as well model any combination of loads and power sources, with differing dynamics, control update rates, and sample rates.

Acknowledgments We are grateful to Dr. Anna Osepayshvili for very helpful discussions about power system economics and markets. Our computations were done in Matlab, using the CPLEX [14] and Gurobi [11] solvers. We used the Yalmip [17] and Tomlab [13] solver interfaces to code up our optimization problems. We are grateful to Johan Löfberg for his help with our Yalmip implementations, and Per Rutquist for his help with our Tomlab implementations.

References

1. Arroyo J, Conejo A (2000) Optimal response of a thermal unit to an electricity spot market. IEEE Trans Power Syst 15(3):1098–1104
2. Bertsekas D (1999) Nonlinear programming. Athena Scientific, Belmont
3. Bittanti S, Colaneri P (2008) Periodic systems: filtering and control. Springer, Hiedelberg
4. Borrelli F, Bemporad A, Morari M (2010) Model predictive control. Unpublished book manuscript
5. Boyd S, Vandenberghe L (2004) Convex optimization. Cambridge Press, Cambridge
6. Carrion M, Arroyo J (2006) A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. IEEE Trans Power Syst 21(3):1371–1378
7. Constantinescu E, Zavala V, Rocklin M, Lee S, Anitescu M (2009) Unit commitment with wind power generation: Integrating wind forecast uncertainty and stochastic programming. Argonne National Laboratory, Technical Report
8. Diehl M, Amrit R, Rawlings J (2011) A Lyapunov function for economic optimizing model predictive control. IEEE Trans Autom Contr 56(3):703–707
9. Galiana FD, Motto AL, Conejo AJ, Huneault M (2001) Decentralized nodal-price self-dispatch and unit commitment. In: Hobbs B, Rothkopf M, O'Neill R, Chao H (eds) The next generation of electric power unit commitment models, vol 15. Springer, Heidelberg
10. Gondhalekar R, Jones C (2009) A unified framework for multirate and multiplexed model predictive control via periodic systems. In: IEEE conference on decision and control, Shanghai, China
11. Gurobi Optimization, Houston, TX, USA, http://gurobi.com
12. Hindi H, Greene D, Laventall C (2011) Coordinating regulation and demand response in electric power grids using multirate model predictive control. In: IEEE conference on innovative smart grid technologies ISGT, Anaheim, California, USA
13. Holmstrom K, Goran A, Edvall M (2005) TOMLAB/CPLEX Optimization Software for Matlab, TOMLAB Optimization, http://tomopt.com, 2005
14. IBM ILOG CPLEX Optimizer, IBM, CA, USA, http://www.ibm.com/CPLEX
15. Kirby B, Hirst E (1999) Load as a resource in providing ancillary services. Oak Ridge National Labortory, Technical Report
16. Kirby B, Milligan M (2008) An examination of capacity and ramping impacts of wind energy on power systems. Electricity J 21(7):30–42. [Online]. Available: http://www.sciencedirect.com/science/article/B6VSS-4TG8KCD-1/2/d4b9b37f9a250605a790c46c8d07c63f