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Minimum Cost Broadcast in Multi-radio Multi-channel Wireless Mesh Networks

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***Abstract*—A vast number of broadcasting protocols have been developed for wireless networks. However, most of these protocols assume a single-radio single-channel network model. Employing multiple channels can effectively improve the network capacity in wireless mesh networks. This paper considers *minimum cost broadcast* (MCB) problem in multi-radio multi-channel wireless mesh networks. We first present the multi-radio multi-channel network model, and then formulate the MCB problem using an integer linear programming model. Our model considers two cases of MCB. In the first case, there already exists a channel assignment in the network, and the formulation minimizes the broadcast cost and reduces interference amongst the adjacent neighbors. In the second case, each node has a set of available channels to be selected. We jointly consider channel assignment and the MCB problem. The joint channel assignment and MCB formulation fully exploits the channel diversity, and also further reduces interference in the network. We propose corresponding centralized and distributed heuristic algorithms to minimize the number of broadcast transmissions with full reliability. In our heuristic algorithms, each node participates in the broadcasting if chosen to maintain the network connectivity or to achieve maximum coverage. Extensive numerical results are presented to demonstrate the performance.**

***Keywords*-wireless mesh networks, broadcast, multi-radio, multi-channel**

I. Introduction

Wireless mesh networks (WMNs) have received much attention in recent years. Typical deployments of mesh networks utilize mesh routers equipped with only one IEEE 802.11 radio. If all nodes communicate with a single channel, the number of simultaneous transmissions is limited by interference. Research has indicated that single-radio single-channel mesh networks suffer from serious capacity degradation due to the multi-hop nature of WMNs [1]. An effective approach to improve the capacity of mesh networks is to provide each node with multiple-radio multi-channel capabilities and permit MAC protocols to adjust the transmission rate [2].

Broadcasting in wireless networks is fundamentally different to that in wired networks due to the well-known *Wireless Broadcast Advantage* (WBA) [3]. In wireless networks, as long as an omni-directional antenna is used, the transmission power corresponds to the coverage range in all directions. It is thus sufficient to transmit once to deliver a message to all devices within the range. A vast number of broadcasting protocols have been developed for wireless ad hoc networks with different focuses, such as minimum transmission power, minimum number of transmissions, reliability, maximum throughput, and minimum broadcast latency. Most of the work perform broadcasting through a virtual backbone or a broadcasting tree. By using local topological information or the entire network topological information, this approach can achieve a deterministic performance. However, a nontrivial overhead is involved to construct the virtual backbone or tree regardless of whether it is constructed in a centralized or a distributed way.

This paper addresses the problem of *minimum cost broadcast* (MCB) in multi-radio multi-channel (MRMC) WMNs. Assume that every node broadcasts at a fixed transmission range, so all transmission costs are identical. Thus, the MCB problem in a wireless network is equivalent to the problem of minimum number of transmissions. This problem has been studied for single radio single channel scenario. However, it has not been investigated much in MRMC WMNs. Such a problem is very different from that in the single radio single channel scenario. In MRMC WMNs, the presence of multi-radio allows a node to send and receive at the same time; the availability of multi-channel allows channels to be reused across the network, which expands the available spectrum and reduces interference. The channel assignment in MRMC WMNs is used to assign multiple radios of every node to different channels. It determines the actual network connectivity since adjacent nodes have to be assigned to some common channel. Transmissions on different channels cause different groups of neighboring nodes, which leads to different interference and impacts on the number of radios needed for the broadcasting.

The contributions of this paper include both theoretical understanding and algorithm design. First, we formulate the MCB problem in MRMC WMNs as an integer linear programming (ILP) model. The ILP model has been used extensively for minimum power or maximum lifetime broadcast problems in wireless networks. However, many work in the literature focus on the energy saving or lifetime in power-controlled or power-constrained wireless networks

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such as mobile ad hoc networks and wireless sensor networks. There have been little efforts in understanding how multi-channel and the number of transmissions affect the broadcast performance in WMNs with MRMC. In this paper, we study the MCB problem in WMNs with MRMC. We consider two cases. In the first case, there already exists a preexisting channel assignment in the network, i.e., a channel assignment of the network has been predetermined. In the second case, we jointly consider the MCB problem and channel assignment. Our solution focuses to minimize the broadcast cost while determining the forward nodes and assigning channels to them. We also propose centralized and distributed heuristic algorithms to solve the MCB problem. The algorithm with preexisting channel assignment only considers to minimize the cost and to reduce interference amongst the adjacent nodes to some extent. The MCB algorithm with joint channel assignment considers to minimize the cost and assign the transmission channel such that the channel reuse is improved and the interference is further reduced. In the heuristic algorithms, a node participates in the broadcasting in order to maintain the network connectivity or achieve maximum coverage. Third, we analyze the time and message complexity of the proposed distributed algorithms are both $O(N^2)$.

The rest of the paper is organized as follows. Section II briefly surveys the related work. The network model and problem formulation are described in Section III. Section IV presents a set of heuristic algorithms for tree construction, and analyze the complexity of the algorithms. The performance analysis provided in Section V. Section VI concludes the paper.

II. Related Work

The broadcast problem has been studied extensively in wireless ad hoc networks. In [4], [5], the focus is reliability such that every node in the network is guaranteed to receive the broadcast message. In [6], [7], the focus is to achieve a minimum broadcast latency. The broadcast latency is defined as the time the last node in the network receives the broadcast message. In [7], [8],the focus is to alleviate the *Broadcast Storm Problem* [9] by reducing the redundant transmissions. The work in [10] presents a distributed algorithm to minimize transmission. It generates a Connected Dominating Set (CDS) as virtual backbone of wireless ad hoc networks by first constructing a Maximal Independent Set (MIS), and then connecting the nodes in the MIS. Various heuristic algorithms have been proposed for solving minimum power multicast/broadcast problem. such that the total transmission powers used by the source and the nodes involved in forwarding messages are minimized. The broadcast/multicast incremental power (BIP/MIP) algorithm [3] is well known among these heuristic algorithms. In BIP/MIP, new nodes are added to the tree on a minimum incremental cost basis, until all intended destination nodes are included. The work in [11], [12] solve the broadcast case, and others in [13], [14] deal with the more general case of multicast. However, most of these work adopt the assumption that each node in the network is equipped with only one radio.

The presence of multi-radio multi-channel can improve the capacity of WMNs. In MRMC WMNs, the channel assignment is used to assign multiple radios of every node to different channels. It determines the actual network connectivity since adjacent nodes have to be assigned to some common channel. MRMC WMNs require efficient algorithms for channel assignment in order to minimize interference or efficient routing. The problem of channel assignment in MRMC WMNs has been studied extensively for unicast communications [15], [16]. One of the channel assignment approaches is static channel assignment [15], [17], [18]. In [15], the authors propose a linear optimization model channel allocation and interface assignment model. In [17], the authors propose a distributed channel assignment algorithm for mesh nodes whose connectivity graph is a tree. In [18], the authors propose centralized and distributed algorithms for channel assignment problem, and also a linear program formulation with the objective of minimum interference to quantify the performance bounds. Another channel assignment approach, dynamic assignment approaches [16], [19], assume the radio is capable of fast switching on per-packet basis. It frequently switches the channel on the radio. In SSCH [16], nodes switch channels synchronously in a pseudo-random sequence such that the neighboring nodes meet periodically at a common channel to communicate. In [19], the authors study how the capacity of multi-channel wireless networks scale with respect to the number of radio interfaces and the number of channels as the number of nodes grow.

Although there have been research efforts on various aspects of MRMC WMNs, such as channel assignment, and throughput optimization, few have been done on broadcast problem. The authors in [20] design a set of centralized algorithms to achieve minimum broadcast latency in multi-radio multi-channel and multi-rate mesh networks, and compare three channel assignment schemes with different connectivity and interference. However, the centralized algorithms result in a nontrivial overhead to construct and maintain the broadcast tree. The problem of channel assignment for broadcast has also been studied recently [21], [22], [23]. The work in [21] proposes two flexible localized channel assignment algorithms based on s-disjunct superimposed codes. Both algorithms support the local broadcast and unicast, and achieve interference-free channel assignment under certain conditions. However, they did not consider the problem of minimizing broadcast redundancy in multi-radio WMNs. To reduce the broadcast redundancy, the work in [22] presents a routing and channel selection algorithm to build a broadcast tree with minimum Relaying Channel Redundancy in multi-radio wireless mesh networks. Relaying Channel Redundancy is defined as the sum of the number of different channels selected by each forward node in the broadcast tree. In [23], the authors propose an interference-aware broadcast algorithm in MRMC WMNs, and jointly consider multiple performance metrics. The objective is to achieve 100% reliability, less broadcast redundancy, low broadcast latency, and high goodput.

ILP has been used for multicommodity flow problem, channel assignment problem for unicast communications, and also for finding minimum power broadcast and multicast trees in wireless ad hoc networks. ILP is very useful for performance evaluation of heuristic algorithms. In [24], the authors propose a flow-based integer programming model for minimum power broadcast/multicast problem in wireless networks. In the flow-based model, flows to various destinations are indexed separately, and connectivity is ensured by network flow equations. The authors in [14] propose an integer programming model and a relaxation scheme, as well as heuristic algorithms. The continuous relaxation of the model leads to a very sharp lower bound of the optimum.

In this paper, we study the MCB problem in MRMC WMNs with preexisting channel assignment and with joint channel assignment, respectively. We use ILP to formulate the problem, and then propose centralized and distributed heuristic algorithms to solve the problem.

III. System Model and Problem Formulation

A. System Model

In an MRMC WMN, each node has multiple radios, and each radio is tuned to one of the available non-overlapping channels in the system. Assume that all radios have a common transmission range, r. There is a specified source node which has to broadcast a message to all other nodes in the network. Any node can be used as a forward node to reach neighbor nodes in the network. Nodes that transmit, including the source node, are called forward nodes. Nodes that receive a transmission but do not retransmit it are classified as leaf nodes. Nodes that have not yet received the transmission are called uncovered nodes.

The network is represented by an undirected graph $\mathbf{G} = (\mathbf{V}, \mathbf{E_c})$, where $\mathbf{V}$ is the set of vertices and $\mathbf{E_c}$ is the set of colored edges. WMNs are generally relatively dense, and only the initially connected nodes are studied. Therefore, the MRMC WMN is assumed to be connected, and $\mathbf{G}$ is referred to the connectivity graph of the network. Let N denote the total number of vertices in $\mathbf{V}$. The set of available non-overlapping orthogonal frequency channels in the system is denoted by $\mathbf{C}$. Each vertex in $\mathbf{V}$ represents a node in the network. A channel assignment scheme assigns a channel in $\mathbf{C}$ to each radio associated with a node. An undirected edge (ij, k), which corresponds to the link between node i and node j on channel k, is in $\mathbf{E_c}$ if and only if the following two conditions hold,

- The Euclidean distance between nodes i and j is no greater than the communication range;
- One radio of node i is tuned to channel k for transmission and one radio of node j is tuned to channel k for receiving.

In channel assignment, assigning a channel to a link between a pair of nodes actually assigns a common channel to a specific radio of each node. Suppose channel k is assigned to the lth radio of node i and the mth radio of node j for the undirected edge (ij, k). The undirected edge can be represented as two directed edges, (ij, lk) and (ji, mk), in opposite directions. Here, (ij, lk) corresponds to the directed edge from the lth radio of node i to node j on channel k, and (ji, mk) corresponds to the directed edge from the mth radio of node j to node i on channel k.

Given the network model defined above, the MCB problem is to construct a broadcast tree, $\mathbf{T} = (\mathbf{V(T)}, \mathbf{E(T)})$, to ensure that all nodes in the network receive the broadcast messages with a minimum number of transmissions. $\mathbf{V(T)} \subset \mathbf{V}$ and $\mathbf{E(T)} \subset \mathbf{E_c}$ represent the set of nodes and the set of links that participate in the broadcasting, respectively. Denote $\mathbf{V}(\mathbf{T}, k)$ as the set of nodes in $\mathbf{V(T)}$ broadcast on channel k.

Definition 1. *The cost on channel k in the broadcast tree $\mathbf{T}$ equals $|\mathbf{V}(\mathbf{T}, k)|$. The tree cost is defined as the sum of the number of transmissions on each channel in $\mathbf{T}$, i.e.,*

$$cost(\mathbf{T}) = \sum_{k \in \mathbf{C}} |\mathbf{V}(\mathbf{T}, k)|.$$

Definition 2. *The MCB problem is to find a broadcast tree $\mathbf{T}$ in $\mathbf{G}$ and spans all nodes in $\mathbf{G}$ with the least tree cost.*

B. ILP formulation with preexisting channel assignment

In many scenarios, the channel assignment has been determined on behalf of unicast traffic, i.e., to maximize the performance of unicast traffic transport, and the broadcast traffic has to be carried based on this preexisting channel assignment. Assuming that the existing channel assignment is static during the process of broadcasting and the network is connected, the ILP formulation with preexisting channel assignment is summarized in Figure 1.

The network topology and preexisting channel assignment are described by a set of binary constants $E_{ij,k}$. $E_{ij,k}$ equals 1 if there is an undirected edge (ij, k) exists in $\mathbf{E_c}$, and 0 otherwise (constraint 7). A resulting broadcast tree is represented by a set of binary variables $X_{ij,k}$. $X_{ij,k}$ equals 1 if the broadcast tree includes edge (ij, k), and 0 otherwise (constraint 8). The variables $X_{ij,k}$ are needed only for those edges that can be added to the broadcast tree. Clearly, constraint (1) indicates that if an undirected edge (ij, k) is included in the tree, it must exist in graph $\mathbf{G}$. The aggregate amount of supply going from node i to node j on any channel is denoted as a flow variables F_{ij}.

$$\text{minimize} \sum_{k\in C}\sum_{i\in V} Y_{i,k}$$

$$s.t. \quad X_{ij,k} \le E_{ij,k}; \quad \forall i,j \in V, i \ne j, \forall k \in C \quad (1)$$

$$\sum_{j\in\{V\setminus i\}} F_{ij} = D; \quad i = \text{source} \quad (2)$$

$$\sum_{j\in\{V\setminus i\}} F_{ji} = 0; \quad i = \text{source} \quad (3)$$

$$\sum_{j\in\{V\setminus i\}} (F_{ji} - F_{ij}) = 1; \quad i \in \{V \setminus \text{source}\} \quad (4)$$

$$F_{ij} \le D \sum_{k\in C} X_{ij,k}; \quad \forall i,j \in V, i \ne j \quad (5)$$

$$Y_{i,k} \ge X_{ij,k}; \quad \forall i,j \in V, i \ne j, \forall k \in C \quad (6)$$

$$E_{ij,k} \in \{0,1\}; \quad \forall i,j \in V, \forall k \in C \quad (7)$$

$$X_{ij,k} \in \{0,1\}; \quad \forall i,j \in V, \forall k \in C \quad (8)$$

$$F_{ij} \ge 0; \quad \forall i,j \in V \quad (9)$$

$$Y_{i,k} \in \{0,1\}; \quad \forall i \in V, \forall k \in C \quad (10)$$

Figure 1. The ILP formulation for MCB with preexisting channel assignment.

Based on the network flow model presented in [24], the ILP formulation ensures that the resulting broadcast tree reaches all nodes in $\mathbf{V}$. Flow conservation constraints (2)-(5) keep all nodes connected and ensure that there are no loops in the broadcast. Constraint (2) represents that the source node injects a flow with $D = N - 1$ units of supply into the network. The number of units of supply equals the total number of destinations in the network. Each destination node consumes one unit of supply when the flow goes through it. Constraint (3) indicates that there is no input flow to the source node. Constraint (4) indicates that each non-source node consumes one unit of supply. At each forward node, the flow is split into sub-flows to its children nodes. The amount of supply of each sub-flow equals the number of the nodes in the sub-tree. Therefore, each forward node receives the amount of supply that equals the number of nodes in its sub-tree, and each leaf node receives and consumes exactly one unit of supply. Thus, if node i is a forward node and there exists an edge from node i to node j in the tree, F_{ij} is positive, and 0 otherwise (constraint 9). Constraint (5) defines the relationship between two sets of variables, F_{ij} and $X_{ij,k}$. It represents that it is possible that $F_{ij} > 0$ only if the broadcast tree includes one edge from node i to j on any channel $k \in C$.

To obtain the objective function which minimizes the tree cost, a set of binary auxiliary variables $Y_{i,k}$ is introduced in the formulation. $Y_{i,k}$ equals 1 if node i is a forward node on channel k, and 0 otherwise (constraint 10). If an edge in the tree is incident from node i, and operates on channel k,

$$E_{ij,k} = \sum_{l\in I_i} E_{ij,lk}; \; \forall i,j \in V, i \ne j, \forall k \in C \quad (11)$$

$$X_{ij,k} = \sum_{l\in I_i} X_{ij,lk}; \; \forall i,j \in V, i \ne j, \forall k \in C \quad (12)$$

$$\sum_{l\in I_i} X_{i,lk} \le 1; \quad \forall i \in V, \forall k \in C \quad (13)$$

$$\sum_{k\in C} X_{i,lk} \le 1; \quad \forall i \in V, \forall l \in I_i \quad (14)$$

$$X_{i,lk} = \max_{\forall j \ne i} X_{ij,lk}; \quad \forall i \in V, \forall k \in C, \forall l \in I_i \quad (15)$$

$$E_{ij,lk} \in \{0,1\}; \forall i,j \in V, \forall k \in C, \forall l \in I_i \quad (16)$$

$$X_{ij,lk} \in \{0,1\}; \forall i,j \in V, \forall k \in C, \forall l \in I_i \quad (17)$$

$$X_{i,lk} \in \{0,1\}; \forall i \in V, \forall k \in C, \forall l \in I_i \quad (18)$$

Figure 2. The new constraints for joint MCB and channel assignment.

then $Y_{i,k} = 1$. Constraint (6) relates $Y_{i,k}$ and $X_{ij,k}$.

The cost on channel k in the broadcast tree is the sum of $Y_{i,k}$ over all nodes i in $\mathbf{V}$, $|\mathbf{V}(\mathbf{T},k)| = \sum_{i\in V} Y_{i,k}$. According to Definition 1, the objective function is:

$$\text{minimize} \sum_{k\in C}\sum_{i\in V} Y_{i,k}.$$

C. ILP formulation with joint channel assignment

The ILP formulation for joint channel assignment and MCB must take the number of radios and the number of available channels into account. Thus, additional radio constraints have to be added based on the formulation in Figure 1. The additional constraints are summarized in Figure 2. Denote I_i as the number of radios of node $i \in \mathbf{V}$. A channel assignment scheme A assigns node i at most I_i different channels. The channel assignment (i, lk) represents channel k is assigned to the lth radio interface of node i.

While considering the particular node radio that is assigned with a channel by channel assignment scheme A, an undirected edge $(ij, k) \in \mathbf{E_c}$ can be represented as two directed edges (ij, lk) and (ji, mk) that also exist in $\mathbf{E_c}$. A set of binary variables $E_{ij,lk}$ for channel assignment is defined to represent the directed edges between a pair of nodes. $E_{ij,lk}$ equals 1 if there is a directed edge (ij, lk) which exists in $\mathbf{E_c}$, and 0 otherwise (constraint 16). A resulting broadcast tree is represented by a set of binary variables $X_{ij,lk}$. $X_{ij,lk}$ equals 1 if the broadcast tree includes an edge (ij, lk), and 0 otherwise (constraint 17). Since there is no benefit to assign a channel to more than one radio on the same node, $E_{ij,k} = 1$ indicates that there exists exactly one radio, l, in I_i such that $E_{ij,lk}$ equals 1. Therefore, constraint (11) relates between $E_{ij,k}$ and $E_{ij,lk}$. Similarly, constraint (12) relates $X_{ij,k}$ and $X_{ij,lk}$.

A set of binary variables $X_{i,lk}$ is defined to represent the channel assignment. $X_{i,lk}$ equals 1 if channel k is assigned to the lth radio interface of node i in the broadcast tree, and 0 otherwise (constraint 18). For a dedicated channel k, since at most one radio will be assigned to k among all the radios on node i, $\sum_{l \in I_i} X_{i,lk} \leq 1$ is true for any $k \in C$. Also for a dedicated radio l of node i, channel assignment will only assign possibly one channel to radio l, thus $\sum_{k \in C} X_{i,lk} \leq 1$ is true for any $l \in I_i$. These two constraints are represented in constraints (13) and (14), respectively. In the resulting broadcast tree, if node i forwards broadcast messages to any node j on channel k of its lth radio, $X_{ij,lk}$ equals 1. Node i must be a forwarding node on channel k at the lth radio, thus $X_{i,lk}$ equals 1 as well. Constraint (15) relates $X_{i,lk}$ and $X_{ij,lk}$.

IV. Heuristic algorithms for MCB

The MCB in single radio single channel wireless networks is equivalent to the minimum connected dominating set (MCDS) problem, which has been proved to be an NP-hard problem for an arbitrary graph [25]. Thus, it is NP-hard to find MCB in MRMC WMNs. In the worst case, it may be needed to examine all possible combinations within the search space to find the optimal solution. For large-scale networks, it is not trivial to find the optimal solutions using our ILP formulations. This section presents two heuristic algorithms to solve MCB.

A. Centralized algorithm for MCB with preexisting channel assignment

The main idea is to construct a broadcast tree by choosing a forwarding node iteratively. A node participates in the broadcasting if chosen to maintain the network connectivity or to achieve maximum new coverage. The following principles are considered:

1) A node does not participate in broadcast if all neighbors have already been covered;
2) A node has only one receiving channel should be covered;
3) A node with only one available incoming link must be covered by that link;
4) A node may broadcast more than once using different channels on different radios.

First, a centralized algorithm for MCB with Preexisting Channel Assignment (CPCA) is presented in Algorithm 1. Let $\mathbf{u\_set}$ and $\mathbf{f\_set}$ denote the set of uncovered nodes and the set of forward nodes in $\mathbf{V}$, respectively. Initially, $\mathbf{f\_set}$ includes the source node, and $\mathbf{u\_set}$ includes all non-source nodes. CPCA iteratively selects forwarding nodes and channels, and updates $\mathbf{f\_set}$ and $\mathbf{u\_set}$ until all nodes are covered. The algorithm first checks the one-hop neighbor nodes of $\mathbf{f\_set}$ in $\mathbf{u\_set}$. If there exists any node without any incoming links from other nodes in $\mathbf{u\_set}$ and with only one incoming link from any node in $\mathbf{f\_set}$, this node must be covered to maintain the network connectivity. Thus a node with such a link and maximum new coverage should be selected as a forwarding node. For all other one-hop neighbor nodes of $\mathbf{f\_set}$ in $\mathbf{u\_set}$, the algorithm selects a forwarding node which covers the maximum number of to-be-covered nodes. If there exist several such nodes, the maximum new coverage of their adjacent nodes can be used to break the tie as the modified greedy algorithm in [26].

Algorithm 1 CPCA

Input: graph $\mathbf{G}(\mathbf{V}, \mathbf{E_c})$, source node s
Output: Forwarding and receiving channel $\overline{f}_i$ and $\overline{r}_i$

1: $\overline{f}_i = \overline{r}_i = 0$
2: $\mathbf{u\_set} = \mathbf{V} - \{s\}$;uncovered set
3: $\mathbf{f\_set} = \{s\}$;forward set
4: **while** $\mathbf{u\_set} \neq \emptyset$ **do**
5: **while** $\exists j \in \mathbf{u\_set}$ such that $\sum_{k \in C} \sum_{l \in \mathbf{u\_set}} E_{lj,k} = \emptyset$ **do**
6: **for all** $\sum_{k \in C} \sum_{i \in \mathbf{f\_set}} E_{ij,k} = 1$ **do**
7: select i with $\overline{f}_i = k$ to cover j
8: update $\mathbf{f\_set}$ and $\mathbf{u\_set}$
9: **end for**
10: **end while**
11: find $i \in \mathbf{f\_set}$ with $\overline{f}_i$ to maximize coverage
12: **if** $\exists$ multiple i **then**
13: select $i \in \mathbf{f\_set}$ with $\overline{f}_i$ and $j \in \mathbf{V}(i)$ with $\overline{f}_j$ to maximize coverage
14: **else**
15: select $i \in \mathbf{f\_set}$ with $\overline{f}_i$
16: **end if**
17: update f_set and $\mathbf{u\_set}$
18: **end while**

In CPCA, the channel selection is based on the existing channel assignment. The Centralized algorithm for MCB with Joint Channel Assignment (CJCA) follows the same procedure as CPCA. Denote id_i as the next available radio of node i and $\overline{f}_{i,id}$ as the selected forwarding channel for the id_ith radio. The difference is that the forward node has a set of available channels and the channel selection is constrained by the number of radios in CJCA, i.e., $id_i \leq I_i$. Thus, compared with Algorithm 1, the pseudocode of CPCA has two main changes in lines 6 and 11. Line 6 will be

for all $\sum_{k \in C} \sum_{i \in \mathbf{f\_set}} E_{ij,k} = 1 \cup id_i \leq I_i$ **do**

and line 11 will be

find $i \in \mathbf{f\_set}$ with $\overline{f}_{i,id}$ to maximize coverage.

B. Distributed algorithm for MCB with joint channel assignment

Without loss of generality, we assume that the radios are assigned from the first to the last, and the first radio of every non-source node is assigned for the receiving channel. Each non-source node has exactly one receiving radio. A Distributed algorithm for MCB with Joint Channel Assignment (DJCA) is described in Algorithm 2.

The basic idea of the DJCA algorithm is as follows. Initially, all nodes are idle, and then source node s is activated. The active source node will be assigned a forwarding channel, $\overline{f}_s$, based on the maximum coverage amongst all available channels. Every reachable neighbor node of s with channel $\overline{f}_s$ will be the child of s on $\overline{f}_s$. Then an ACTIVE message with the forward set information is sent to every child on channel $\overline{f}_s$. Every child node tunes its receiving channel to $\overline{f}_s$, and becomes active upon receiving the ACTIVE message. For any active node, including the source node, if it has available radio(s) and channel(s), it chooses a channel with maximum coverage and sends a TEST message to each neighbor on that channel. The TEST message includes the coverage information. If in a given period, a node receives more than one TEST message, it compares the coverage of all TEST messages. It then responds an ACK message to the one with maximum coverage, and responds a REJECT message to others. The REJECT message includes the coverage and channel information of the winner. In the case that a node receives multiple TEST messages with same maximum new coverage from different senders, a winner can be chosen either randomly or by considering the interference factor. If the neighbor receives only one TEST message, it sends back an ACK message. While a node receives an ACK message, it will be assigned with the forwarding channel, and sends an ACTIVE message to each child listening on that channel. After a channel assignment is determined, the forward set and uncovered set will be updated. The child node will be assigned the receiving channel as well. While a node receives a REJECT message, it knows that the neighbor node has been covered by other node. Thus it updates its uncovered set. To reduce the potential interference, it also decreases the priority of the channel piggyback from the REJECT message. This process is executed iteratively until all nodes in the network are covered.

The proof of the correctness and the analysis of the time and message complexity of the DJCA algorithm are given as follows.

Algorithm 2 DJCA

Input: graph $\mathbf{G}(\mathbf{V}, \mathbf{E_c})$, source node s
Output: Forwarding channel set $\overline{\mathbf{f}}_i$ and receiving channel $\overline{r}_i$ for $\forall i \in \mathbf{V}$

$\mathbf{u\_set} = \mathbf{V} - \{s\}$;uncovered set
$\mathbf{f\_set} = \{s\}$;forward set
<u>initialize the local branch of node i:</u>
reset $\overline{\mathbf{f}}_i$ and $\overline{r}_i$
$id = 1$; ID of the next available radio
$\mathbf{u\_set}_i = \mathbf{V}(i) - \{s\}$
$\mathbf{f\_set}_i = \emptyset$
if $i = s$ **then**
    Set s as active
    $\mathbf{f\_set}_s = s$
    assign maximum coverage channel k to $\overline{f}_{s,id}$, $\forall k \in C_s$
    sends an ACTIVE message to each j on k if $E_{sj,k} = 1$
    update $\mathbf{u\_set}_s$
end if
<u>upon receiving ACTIVE message:</u>
if $i \neq s \wedge i$ is not active **then**
    set $\overline{r}_i$ and set i as active
end if
$id = id + 1$
if $id \leq I_i$ **then**
    calculate maximum coverage channel k, $\forall k \in C_i$
    sends a TEST message to each j on k if $E_{ij,k} = 1$
    sends a COVERED message to each m, $\forall m \in \mathbf{u\_set}_i$
end if
<u>upon receiving TEST message:</u>
if only receive TEST message from v, $\forall v \in V$ **then**
    respond an ACK message to v
else
    select a node v with maximum coverage
    respond an ACK message to v
    respond a REJECT message to others
end if
<u>upon receiving ACK message:</u>
if $\forall E_{ij,k} = 1$, receiving ACK from j on k **then**
    $\mathbf{f\_set}_i = i$
    update $\mathbf{u\_set}_i$
    assign channel k to $\overline{f}_{i,id}$
    sent an ACTIVE message to each j on k if $E_{ij,k} = 1$
end if
<u>upon receiving COVERED message:</u>
update $\mathbf{u\_set}_i$

Lemma 1. *In each iteration, there is at least one node chosen as a forward node.*

Proof: Denote $\mathbf{G}_c(k)$ as the graph consisting of all covered nodes and corresponding links after the kth iteration to run the message passing protocol in DJCA. Thus $\mathbf{G}_c(k) \subset \mathbf{G}$. Initially, $\mathbf{G}_c(0)$ only contains the source node s. $\mathbf{G}_c(k)$ is partitioned as: $\mathbf{G}_c(k) = \bigcup_i \mathbf{P}_i(k)$, where $\mathbf{P}_0(k)$ is the set of nodes such that all of their neighbor nodes have already been covered, and for any $i > 0$, $\mathbf{P}_i(k)$ is the set of competition nodes. The nodes in $\mathbf{G}_c(k) - \mathbf{P}_0(k)$ are divided into competition sets based on two rules: none of nodes from different competition sets will compete with each other, and any pair of nodes in the same competition set will compete either explicitly or implicitly.

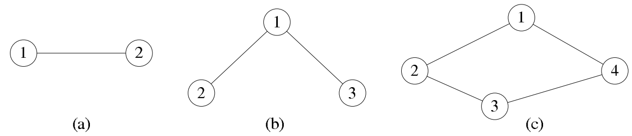

Figure 3. Three scenarios of node competition for the MCB problem with joint channel assignment.

Figure 3 demonstrates the node competition under three cases. In DJCA, an active node sends out a TEST message with its maximum new coverage to its neighbors, and wins the competition if it receives all ACK messages from the neighbors. If a node receives multiple TEST messages, it responds an ACK message to the sender with maximum new coverage, and responds one REJECT message to others. Nodes compete for TEST messages explicitly or implicitly. Two nodes are considered as explicit competition nodes if they have any common node in TEST messages. Two nodes are considered as implicit competition nodes if they have any common explicit competition node, or iteratively, at least one pair of their explicit competition nodes has a common node or a common implicit competition node. The link between any pair of nodes represents the explicit competition relationship, instead of the wireless communication.

In a pair of explicit competition nodes, the node with maximum new coverage wins the competition. As depicted in Figure 3(a), nodes 1 and 2 are a pair of explicit competition nodes, and the one with maximum new coverage will be potentially chosen as a forward node. In Figure 3(b), nodes 1 and 2, nodes 1 and 3 are two pairs of explicit competition nodes. Nodes 2 and 3 are a pair of implicit competition nodes as node 1 is their common explicit competition node. Node 1 will be potentially chosen as a forward node if it has the maximum new coverage. If node 1 has the minimum new coverage, both nodes 2 and 3 will be potentially chosen as forward nodes depending on the competition with their other explicit competition nodes. Otherwise, based on the transitivity of inequality, either node 2 or node 3, whichever has the maximum new coverage, will be potentially chosen as a forward node. In Figure 3(c), there are two pairs of implicit competition nodes, nodes 1 and 3, nodes 2 and 4. All other pairs are explicit competition nodes. Only if a node has the maximum new coverage and its implicit competition node has the second maximum new coverage, two nodes are chosen as forward nodes. For all other cases, there is only one node chosen as a forward node. Therefore, there is one winner between a pair of explicit competition nodes, and at least one winner between two implicit competition nodes. There is at least one partition $\mathbf{P}_i(k)$ such that $i > 0$ while $\mathbf{G}_c(k) \neq \mathbf{G}$. Thus, in each partition $\mathbf{P}_i(k)$ for $i > 0$, there is at least one node chosen as a forward node. Overall, there is at least one node chosen as a forward node in each iteration. ∎

Theorem 1. *The DJCA algorithm is solvable.*

Proof: It is sufficient to prove that the message passing protocol in the DJCA algorithm is run iteratively until all nodes in the network are covered. From Lemma 1, there is at least one winner in each $\mathbf{P}_i(k)$ for $i > 0$ in the $k+1$ iteration. Therefore, after the $k+1$ iteration, $\mathbf{G}_c(k+1)$ consists of $\mathbf{G}_c(k)$ and the new covered nodes. As long as $\mathbf{G}_c(k) \neq \mathbf{G}$, $\mathbf{P}_0(k) \neq \mathbf{G}$ since there exists some node with uncovered neighbor node(s). In each partition, excluding $\mathbf{P}_0(k)$, at least one node will be chosen as a forward node. Thus, the total number of chosen forward nodes in the k iteration at least equals the number of the partitions $\mathbf{P}_i(k)$ for all $i \geq 0$. Once $\mathbf{G} - \mathbf{G}_c(k)$ becomes empty, the construction of the broadcast tree is finished. Since the size of $\mathbf{G} - \mathbf{G}_c(k)$ is a bounded number and keeps decreasing until it is empty, the algorithm solves the problem in finite steps. ∎

Theorem 2. *The DJCA algorithm runs in $O(N^2)$.*

Proof: The heuristic involves solving a sequence of maximum selection problems. The maximum selection is to find the node with maximum new coverage, which runs in linear time. In each partition $\mathbf{P}_i(k)$ for $i > 0$, the selection problem can be solved independently and simultaneously. Therefore, in any iteration, the selection problem is bound by $O(N)$. Since there are at most N iterations, algorithm DJCA runs in $O(N^2)$. ∎

Theorem 3. *The DJCA algorithm has $O(N^2)$ message complexity in overall.*

Proof: Denote the maximum number of radios amongst all nodes in the network as $I = \max_{i \in \mathbf{V}} I_i$. In a MRMC WMN, a pair of nodes may have multiple links with different channels on different radios. The MRMC WMN can be considered as a single radio single channel WMN if ignoring the number of radios and the number of channels in the network. Denote $E_{\overline{C}}$ as the number of links in the corresponding single radio single channel WMN. Without consideration of the number of radios and the number of channels, the multiple links between a pair of nodes are only counted as one link in $E_{\overline{C}}$. The number of control messages that node i needs to send can be counted. First, the number of ACTIVE does not exceed the number of its neighbors multiplied by the number of radios, I_i, since node i only needs to send one ACTIVE message to each child for each radio. Second, node i sends at most I_i TEST message to each neighbor. Third, for each radio, node i sends no more than either one ACK message or one REJECT message to each neighbor. ACK and REJECT messages can be considered as the same type of message since they are response messages for TEST message. Notice that these three type of messages need to be transmitted no more than the number of radios between any pair of nodes since the channel assignment is relatively in a large time scale, and the channel assignment information is included in the message. Fourth, for all I_i radios, node i sends no more than one COVERED message to each neighbor since it is assigned with only one receiving channel. In summary, no more than three control messages traverse any pair of nodes for each radio, and at most one COVERED message traverses any pair of nodes. Since I is the maximum available number of radios, the total number of messages is bounded by $(3I+1)|E_{\overline{C}}|$. Equivalently, the message complexity of DJCA is $O(N^2)$, where N is the total number of nodes in the network. ■

The Distributed algorithm for MCB with Preexisting Channel Assignment (DPCA) works similarly to DJCA. The difference is that the group of neighbor nodes on a specific channel is fixed in DPCA because the channel assignment is predetermined. Thus, there will be no channel assignment in lines 11 and 37, and all lines with id need to be changed or removed correspondingly. The calculation and comparison of coverage become simpler. The time and message complexity of DPCA are $O(N^2)$.

V. Performance Evaluation

This section evaluates the performance of the ILP formulations and the heuristic algorithms. Nodes are randomly deployed within a $1000 \times 1000m$ square area, and the transmission range is set to $200m$ for every node. One node is randomly selected as the source node.

The first experiment considers the MCB with preexisting channel assignment, and compares the performance of ILP and the one of CPCA. Preexisting channel assignment can be considered as a special case of joint channel assignment, if I equals 1, I_i for any node i also equals 1. In this case, no matter how many available channels a node has, all nodes in the network must share a common channel to maintain the network connectivity. Therefore, both $I = 1$ and $C = 1$ are considered as the single channel scenario. Moreover, for a given number of radios I in joint channel assignment, there is no benefit for channel assignment and broadcast cost if I is larger than C. Denote (I, C) as the radio and channel configuration. For instance, two configurations, (2,2) and (3,2), have the same channel assignment and broadcast cost if, except the additional radio, the availability of the two channels of each node in the configuration (3,2) is the same as that in the configuration (2,2). In the first experiment, I_i radios at node i are randomly tuned to select distinct channels. The channel is assigned as follows. In the single channel scenario, one radio of each node must be tuned to a common channel to maintain the network connectivity. In the multi-radio multi-channel scenario, every node randomly determines whether its first free radio is tuned to the first available channel or not. The radio and the assigned channel are marked as busy and not available, respectively. The same process is carried out for the next available channel. The channel assignment is finished until all channels are iterated or there is no free radio. The network connectivity is checked up, and the channel selection is adjusted to maintain the network connectivity. To simplify the channel assignment, the first free radio can be always assigned to the first available channel. The algorithms then avoid to use the first channel as forward channel in the multi-radio multi-channel scenario if other available channel has comparable coverage. For each configuration, the average broadcast cost is calculated from 20 randomly generated instances, where N varies from 10 to 50.

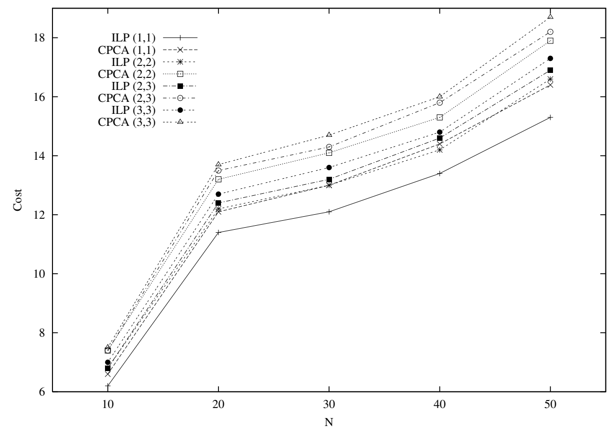

Figure 4. Broadcast cost of MCB with preexisting channel assignment for different configurations of (I, C) when I and C vary from 1 to 3 and N varies from 10 to 50.

Figure 4 shows the average cost in four different configurations of (I, C). These four configurations represent four different channel assignments, and result in different broadcast costs while I and C vary from 1 to 3. As can be seen from Figure 4, the ILP provides the optimal, and the CPCA performs quite reasonably well on average. In all cases, CPCA is less than 10% away from the optimal. Figure 4 demonstrates that the cost increases while the number of channels increases from 1 to 3. Compared to the single channel scenario while $C = 1$, the cost of ILP increases about 9% for $C = 2$, and about 13% for $C = 3$, respectively. Since MRMC probably reduces the number of adjacent neighbors on a specified channel, the number of broadcast transmissions increases due to the assigned multiple channels at the forward nodes.

The second experiment uses the same parameters as in the first experiment, and compares the performance of ILP and DJCA. In the case of joint channel assignment, each node can be tuned to a set of selected distinct channels, and the actual tuned channels are constrained by the number of radios. A similar conclusion can be made from Figure 5 as from Figures 4. In all cases, DJCA is less than 12% away from the optimal. Compared with $C = 1$, the cost of ILP increases at most 8% for $C = 2$, and less than 9% for $C = 3$, respectively.

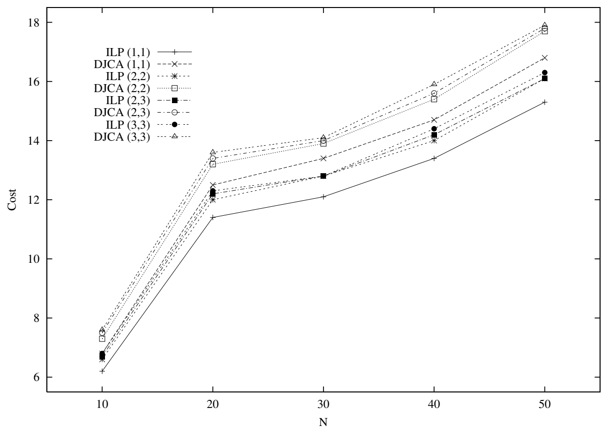

Figure 5. Broadcast cost of MCB with preexisting channel assignment for different configurations of (I, C) when I and C vary from 1 to 3 and N varies from 10 to 50.

Table I
PERFORMANCE DEVIATION (%) FROM ILP WITH JOINT CHANNEL ASSIGNMENT OF FOUR MCB ALGORITHMS WHEN $N = 30$.

(I, C)	CJCA	DJCA	CPCA	DPCA
(1,1)	7.44	10.74	7.44	10.74
(2,2)	7.14	9.52	11.90	14.29
(2,3)	7.87	10.24	12.60	15.75
(3,3)	8.59	11.72	14.84	18.75

Table I shows the performance deviation from the ILP with joint channel assignment of four MCB algorithms. The results with joint channel assignment are better than these with preexisting channel assignment in all cases. When I and C increase from 1 to 3, the cost of ILP with preexisting channel assignment increases in the range from 4% to 8%, while the cost of ILP with joint channel assignment increases in the range from 4% to 5%. When $I = 1$ or $C = 1$, CJCA and CPCA have the same performance, and DJCA and DPCA have the same performance, since the channel assignments in preexisting and scenarios are identical.

VI. CONCLUSION

This paper studied the MCB problem in MRMC WMNs, both with preexisting channel assignment and with joint channel assignment. Two ILP formulations have been developed. In the case with preexisting channel assignment, the ILP formulation minimizes the broadcast cost and reduces interference amongst the adjacent neighbors. In the second case, the MCB problem and channel assignment are jointly considered. The joint channel assignment can further reduce interference in the network. Two heuristic algorithms, with centralized and distributed versions, are presented to construct the broadcast tree rooted at the source node. In our heuristic algorithms, a node is chosen to participate in broadcasting to maintain the network connectivity or to achieve maximum coverage. Our distributed algorithms have $O(N^2)$ time complexity and message complexity. Numerical results demonstrate that the heuristic algorithms minimize the number of broadcast transmissions with full reliability and fully exploit the channel diversity.

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