Inclusion of Short Duration Wind Variations in Economic Load Dispatch

Sanjoy Roy, *Senior Member, IEEE*

***Abstract*—Randomness of wind speed around a short-duration-stable mean value is commonly referred to as *short duration wind variation*. This paper investigates the effect of substantial wind-based capacity inclusion on optimal load dispatch, with the source wind susceptible to short duration variations. Analytical formulation of the economic load dispatch (ELD) problem inclusive of wind power generation is presented separately for cases with and without representation of transmission losses. In each formulation, the effect of short duration wind variations is included as an aggregate, thereby avoiding the complexity of stochastic models. Three-generator and 20-generator study cases are discussed to illustrate two distinct aspects of the ELD problem. First, the optimal cost, losses, and system-λ are presented across a range of short-duration-stable mean wind speed. Thereafter, the sensitivity of all three metrics is discussed with reference to different levels of short duration wind variations.**

***Index Terms*—Power system economics, sustainable energy, wind energy, wind power generation.**

I. Introduction

With significant wind-based capacity additions to power networks worldwide, operational economics continues to be a matter of prime concern to utilities [1], [2]. Among others, the classic problem of economic load dispatch (ELD) [3], [4] has evoked new interest with debate on how wind energy conversion systems (WECS) are to be taken into consideration within dispatch schedules. Questions are generally raised about the variability of wind at source, and the way the same is to be accounted for within the framework of an ELD.

The problem has been investigated for some time in the past [5]; while more recently, attempt has been made to focus on WECS units as independent sources, with appropriate cost components assigned to buy-back of power, reserve requirements, and failure to utilize available wind power [6]. In [7] and [8], probabilistic availability of wind power is used to define constraints to the ELD problem. Most works thus far [6]–[8] have used the well-known Weibull distribution [9] to represent variability of wind, which is known to be valid statistics at least across periods of long duration [10].

This paper is based on the premise that for many applications [11], the optima of an ELD is of interest across a short duration of time (referred to in the rest of the paper as the *validity interval* of ELD), within which the Weibull is not necessarily the best statistical model for wind speed variations [10]. Short duration wind variations primarily include two types of aerodynamic nonidealities, namely *turbulence* and *gusts*. Turbulence consists of random fluctuations superposed on a *short-duration-stable* mean value $\bar{u}$. The source of such fluctuations can be traced back to disturbed streamlines of wind flow. Gusts are distinct surges within turbulent wind fields, for which quantifiable features such as *amplitude*, *rise time*, *peak*, and *lapse time* can be identified. Effect of both nonidealities has been suitably modeled using simple Gaussian distributions around the short-duration-stable mean wind speed [12].

Further, as indicated by recent studies [12], [13], it is possible to aggregate the effect of short duration variations on the power output by a WECS. This allows the ideally expected output power of WECS units to be "corrected" for such variations prior to inclusion in the conventional ELD. A rigorous stochastic model for short duration wind variations may thereby be avoided within the ELD formulation.

Certain features of the conventional ELD problem almost immediately suggest the use of aggregates in preference to stochastic models. First, in practice the conventional units are not entirely free of minor dynamics over the set point of power, while the load can always undergo changes as decided by consumer behavior. The optimal generation levels that emerge when an ELD attempts to meet total power demand are, therefore, not precise instantaneous values but rather similar to aggregates. The concept of aggregate WECS-based generation, therefore, blends with conventional ELD concepts almost naturally.

Again, *loss coefficients* have been used in the conventional ELD to model aggregate system loss as a function of generation levels [3], [4]. Consequently, such representation is more compatible with the concept of aggregate generation from WECS-based units, rather than instantaneous power levels as required by stochastic methods. In fact, the conventional procedure for evaluating network loss formula in terms of loss coefficients [3] can be extended to systems with significant wind-based generation *only if* an acceptable aggregation of WECS output power is used. If this is not the case (say for example, in the presence of long duration wind variations), then the overall energy loss must be evaluated across time horizons.

Finally, questions can be raised about the appropriate use of stochastic objective or constraints when optimizing a problem with one or more random time series data as input. For an ELD problem including WECS-based generation, the primary wind speed data is random according to Weibull or Gaussian distribution. However, such a feature neither justifies nor requires that the *optimization process* should involve stochastic relations. On

Manuscript received March 25, 2011; revised July 01, 2011; accepted September 02, 2011. Date of current version March 21, 2012.

The author is with the Department of Electrical Engineering, Indian Institute of Technology, Ropar, Pb-140001, India (e-mail: roys@iitrpr.ac.in).

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Digital Object Identifier 10.1109/TSTE.2011.2179954

the contrary, representation of wind power as aggregates avoids any such doubts, since stochastic data or relations are avoided altogether.

Sections II and III examine the ELD problem (neglecting, and inclusive of network losses, respectively) with WECS units included as part of the utility generation portfolio. In each case, the effect of short duration variations on the ELD optima is taken into consideration without assuming any specific probability distribution.

A question that emerges almost naturally from the above discussion may be stated as: *how is the validity interval to be decided for an ELD that includes a significant share of wind-based generation?* In a broad sense, the choice of *validity interval* must permit *acceptable aggregation of wind power P_w in the presence of short duration wind variations* (say, at the generic wth WECS-based generation site) into *a crisp aggregate value* $\langle P_w \rangle$. If this applies to each wind-based generation site within the network, then applicability of conventional ELD concepts follows as a consequence.

Section IV provides a deeper treatment of the above statement leading to a definition of $\langle P_w \rangle$, which is suitable and convenient for inclusion in ELD problems. Sections V and VI present various aspects of such inclusion through three-generator and 20-generator ELD examples, respectively.

A disclaimer is perhaps in order before this section is closed. The focus of this paper is the inclusion of WECS units in the generation portfolio of utilities, its impact on optimal cost of generation, and the consequence of short duration wind variations. In order to study these aspects in an exclusive manner, some of the conventional challenges of the ELD problem have been ignored throughout the treatment as well as the examples of Sections V and VI. Among others, consideration of valve-point loading [14]–[16], and reserve margins [17], [18] are excluded; these classic problems having been exhaustively reported in the literature.

II. Economic Load Dispatch Neglecting Network Losses

Consider an interval within which a total demand of P_D is to be supplied by N conventional generating stations and W WECS stations, all of which are utility owned. In terms of P_n, the active power output of the nth conventional station, its cost of generation is given by [3], [4]

$$C_n(P_n) = c_{0,n} + c_{1,n} \cdot P_n + c_{2,n} \cdot P_n^2 \tag{1}$$

while for the wth WECS station, the cost expression is

$$C_w(P_w) = c_{1,w} \cdot P_w \tag{2}$$

where the power output P_w is subject to variations due to wind speed that need to be accounted for. It is, however, assumed that within the validity interval of ELD, whatever the values of P_w can be absorbed by the system without any congestion or reliability problems. In practice this would be the case for utilities that have WECS installed on the basis of proper planning, so that operational problems are not to be expected when dispatch follows the ELD optima. $\{P_w\}$ can then be considered as exogenous variables for the ELD problem.

If losses in the system are neglected, then the ELD can be defined as the following optimization problem:

$$\begin{aligned}
\text{Minimize :}\quad & C(\{P_n\},\{P_w\}) \triangleq \sum_n^N C_n(P_n) + \sum_w^W C_w(P_w)\\
\text{Subject to :}\quad & P_n^{\min} \le P_n \le P_n^{\max};\quad n = 1, \ldots N\\
& P_D = \sum_w^W P_w + \sum_n^N P_n.
\end{aligned} \tag{3}$$

The Karush–Kuhn–Tucker (KKT) conditions for the optima of (3) are

$$\begin{aligned}
&\frac{dC_{n1}(P_{n1})}{dP_{n1}} = \lambda;\quad \text{for } P_{n1}^{\min} < P_{n1} < P_{n1}^{\max}\\
&\frac{dC_{n2}(P_{n2})}{dP_{n2}} \le \lambda;\quad \text{for } P_{n2} = P_{n2}^{\max}\\
&\frac{dC_{n3}(P_{n3})}{dP_{n3}} \ge \lambda;\quad \text{for } P_{n3} = P_{n3}^{\min}\\
&\sum_{n1}^N P_{n1} = P_D - \sum_w^W P_w - \sum_{n2}^N P_{n2}^{\max} - \sum_{n3}^N P_{n3}^{\min};\\
&\qquad \text{where } n1, n2, n3 \in \{1, 2, \ldots N\};\quad n1 \ne n2 \ne n3.
\end{aligned} \tag{4}$$

In (4), the set of conventional units has been “split” between generators with inactive power limits (index $n1$), active maximum power limit (index $n2$), and active minimum power limit (index $n3$).

For each conventional generator with inactive power limits (index $n1$), the cost-derivative relation in (4) leads to the respective *optimal generations* $\{P_{n1}^*\}$ in terms of the optimal marginal cost λ^*. Substitution in the demand constraint gives

$$\begin{aligned}
&\sum_{n1}^N \frac{\lambda^* - c_{1,n1}}{2c_{2,n1}} = P_D - \sum_w^W P_w - \sum_{n2}^N P_{n2}^{\max} - \sum_{n3}^N P_{n3}^{\min}\\
&\Longrightarrow \lambda^* = \left[\sum_{n1}^N \frac{1}{2c_{2,n1}}\right]^{-1}\\
&\qquad \cdot \left[P_D - \sum_w^W P_w - \sum_{n2}^N P_{n2}^{\max} - \sum_{n3}^N P_{n3}^{\min} + \sum_{n1}^N \frac{c_{1,n1}}{2c_{2,n1}}\right]\\
&\Longrightarrow P_{n1}^* = \frac{\lambda^* - c_{1,n1}}{2c_{2,n1}}.
\end{aligned} \tag{5}$$

It follows that the optimal generation level at each conventional unit is a linear function of the WECS outputs $\{P_w\}$, the latter being dependent on wind speed variables.

Evaluation of the optimal generation levels $\{P_n^*\}$ may now be simplified in view of the following:

i) At WECS installation sites, *long duration variation* of wind speed (typically modeled as Weibull distributions [9], [10]) may be assumed to have negligible impact across the validity interval of an ELD. Rather for such time spans, a *short-duration-stable mean wind speed* ($\bar{u}_w$ at the WECS hub, wth site) may be assumed with *short duration variations* superposed [10], [12].

ii) Across the validity interval of ELD, the mean wind speed $\bar{u}_w$ at a WECS hub (distributed as per Weibull over long duration) is not proportionately reflected on the power output. For pitch angle controlled WECS, the output can be obtained in terms of the rated power P_w^{rat} and a *zero-turbulence output coefficient* $\mu_w(\bar{u}_w)$ as

$$P_w(\bar{u}_w) = \mu_w(\bar{u}_w) \cdot P_w^{rat} \tag{6}$$

where $\mu_w(\bar{u}_w)$ assumes values between zero and unity, according to the following description:

$$\mu_w(\bar{u}_w) = \left\{ \begin{array}{ll} 0; & \text{if } \bar{u}_w \le u_w^{in} \\ \left(\bar{u}_w - u_w^{in}\right) / \left(u_w^{out} - u_w^{in}\right); & \text{if } u_w^{in} < \bar{u}_w \le u_w^{rat} \\ 1; & \text{if } u_w^{rat} \le \bar{u}_w \le u_w^{out} \\ 0; & \text{if } \bar{u}_w > u_w^{out} \end{array} \right\} \tag{7}$$

with rated, cut-in, and cut-out speeds u_w^{rat}, u_w^{in}, and u_w^{out}, respectively, at the wth wind-based generating station [9]. Equation (7) is simply an analytical description for the standard output curve of pitch-angle controlled WECS [9].

The average of $\lambda^*(\langle\lambda^*\rangle)$ over the validity interval of an ELD can then be obtained from (5), while average values for generation at conventional units $(\{\langle P_{n1}^*\rangle\})$ would follow:

$$\begin{aligned} \langle\lambda^*\rangle &= \left[\sum_{n1}^{N} \frac{1}{2c_{2,n1}}\right]^{-1} \\ &\quad \cdot \left[P_D - \sum_{w}^{W}\langle P_w\rangle - \sum_{n2}^{N} P_{n2}^{\max} - \sum_{n3}^{N} P_{n3}^{\min} + \sum_{n1}^{N}\frac{c_{1,n1}}{2c_{2,n1}}\right] \\ \langle P_{n1}^*\rangle &= \frac{\langle\lambda^*\rangle - c_{1,n1}}{2c_{2,n1}}. \end{aligned} \tag{8}$$

The terms of importance in (8) are the *short duration average values* $\langle P_w\rangle$ of wind-based generation over the validity interval of ELD, to be estimated as described in Section IV.

Equation (8) is a somewhat conveniently "weak" requirement since it accommodates randomness of marginal cost λ^* at optima, as decided by the WECS power output $\{P_w\}$. Optimal output $\{P_{n1}^*\}$ at the conventional units follow accordingly, and have corresponding averages $\{\langle P_{n1}^*\rangle\}$.

A "stronger" requirement (which is preferable from the utility's viewpoint!) will be that of a firm deterministic value for λ^* at the optima regardless of the short duration variations in $\{P_w\}$, so that the conventional units may consequently dispatch definite levels of active power $\{P_n^*\}$. From (5), this is seen to be the case if the random generation $\{P_w\}$ sum up to a definite total power P_{wT} for the validity interval of ELD

$$\sum_{w}^{W} P_w = P_{wT}. \tag{9}$$

For the ideal zero-turbulence case, this would apply to validity intervals within which a short-duration-stable mean wind speed $\bar{u}_w$ is ensured at each WECS hub. Equations (6) and (7) show that WECS outputs $\{P_w\}$ would then assume firm values corresponding to $\{\bar{u}_w\}$.

III. Economic Load Dispatch Inclusive of Network Losses

The generic loss function suitable for conventional ELD problems has been defined in [3] as

$$P_L(\{P_n\}) = \sum_{n1}^{N}\sum_{n2}^{N} P_{n1} b_{n1,n2} P_{n2} + \sum_{n}^{N} b_{n,0} P_n + b_{0,0} \tag{10}$$

where the parameters $\{b_{n1,n2}\}$, $\{b_{n,0}\}$, and $b_{0,0}$ are coefficients known for the specific network. Installation of WECS units within the network would add three additional summation terms thereby leading to an augmented loss function

$$\begin{aligned} P_L(\{P_n\},\{P_w\}) &= \sum_{n1}^{N}\sum_{n2}^{N} P_{n1} b_{n1,n2} P_{n2} + \sum_{n}^{N} b_{n,0} P_n + b_{0,0} \\ &\quad + \sum_{n}^{N}\sum_{w}^{W} P_n b_{n,w} P_w + \sum_{w}^{W} b_{w,0} P_w \\ &\quad + \sum_{w1}^{W}\sum_{w2}^{W} P_{w1} b_{w1,w2} P_{w2}. \end{aligned} \tag{11}$$

With the WECS generations $\{P_w\}$ accommodated as exogenous variables, (11) can be restated as

$$\tilde{P}_L(\{P_n\},\{P_w\}) = \sum_{n1}^{N}\sum_{n2}^{N} P_{n1} b_{n1,n2} P_{n2} + \sum_{n}^{N} \tilde{b}_{n,0} P_n + \tilde{b}_{0,0} \tag{12}$$

where the derived loss coefficients in (12) are given by

$$\begin{aligned} \tilde{b}_{n,0} &\triangleq b_{n,0} + \sum_{w}^{W} b_{n,w} P_w \\ \tilde{b}_{0,0} &\triangleq b_{0,0} + \sum_{w}^{W} b_{w,0} P_w + \sum_{w1}^{W}\sum_{w2}^{W} P_{w1} b_{w1,w2} P_{w2} \end{aligned} \tag{13}$$

and have random components decided by $\{P_w\}$, as well as the original coefficients $\{b_{n,0}\}$ and $b_{0,0}$ of (10)–(11).

The ELD problem inclusive of the loss function (12) may then be defined as

$$\begin{aligned} &\text{Minimize}: && C(\{P_n\},\{P_w\}) \triangleq \sum_{n}^{N} C_n(P_n) + \sum_{w}^{W} C_w(P_w) \\ &\text{Subject to}: && P_n^{\min} \le P_n \le P_n^{\max}; \qquad n = 1,\ldots N \\ & && P_D + \tilde{P}_L(\{P_n\},\{P_w\}) = \sum_{w}^{W} P_w + \sum_{n}^{N} P_n. \end{aligned} \tag{14}$$

For specific power generation $\{P_w\}$ by the WECS units, the KKT conditions for (14) can be obtained as

$$\begin{aligned}
&\frac{dC_{n1}(P_{n1})}{dP_{n1}} = \lambda\left(1 - \frac{\partial \tilde{P}_L(\{P_n\},\{P_w\})}{\partial P_{n1}}\right); \\
&\qquad\qquad \text{for } P_{n1}^{\min} < P_{n1} < P_{n1}^{\max} \\
&\frac{dC_{n2}(P_{n2})}{dP_{n2}} \le \lambda\left(1 - \frac{\partial \tilde{P}_L(\{P_n\},\{P_w\})}{\partial P_{n2}}\right); \\
&\qquad\qquad \text{for } P_{n2} = P_{n2}^{\max} \\
&\frac{dC_{n3}(P_{n3})}{dP_{n3}} \ge \lambda\left(1 - \frac{\partial \tilde{P}_L(\{P_n\},\{P_w\})}{\partial P_{n3}}\right); \\
&\qquad\qquad \text{for } P_{n3} = P_{n3}^{\min} \\
&\sum_{n1}^{N} P_{n1} = P_D + \tilde{P}_L(\{P_n\},\{P_w\}) \\
&\qquad - \sum_{w}^{W} P_w - \sum_{n2}^{N} P_{n2}^{\max} - \sum_{n3}^{N} P_{n3}^{\min}; \\
&\text{where } n1, n2, n3 \in \{1, 2, \ldots N\}; \quad n1 \ne n2 \ne n3 \qquad (15)
\end{aligned}$$

where the set of conventional units has been "split" using indices $n1$, $n2$, and $n3$ as was done for (4).

For the $n1$th conventional unit with inactive limits, the KKT relation becomes

$$\begin{aligned}
&2c_{2,n1}P_{n1}^{*} + c_{1,n1} \\
&= \lambda^{*} \cdot \left(1 - 2b_{n1,n1}P_{n1}^{*} - \tilde{b}_{n1,0} - \sum_{n2 \ne n1}^{N} b_{n1,n2}P_{n2}^{*}\right). \qquad (16)
\end{aligned}$$

Unlike in the case of (8), for (16) it is practically impossible to claim a "weak" relation between the averages of λ^* and P_{n1}^*, essentially due to nonlinear dependence of variables. The "stronger" set of relations that requires a firm deterministic value of λ^* can hold if the derived loss coefficients $\{\tilde{b}_{n,0}\}$ are approximately constant within the validity interval of ELD.

Further, if $\tilde{b}_{0,0}$ too is approximately constant within the validity interval, then the demand constraint in (15) becomes

$$\begin{aligned}
\sum_{n1}^{N} P_{n1} &= P_D + \sum_{n1}^{N}\sum_{n2}^{N} P_{n1}b_{n1,n2}P_{n2} + \sum_{n}^{N} \tilde{b}_{n,0}P_n + \tilde{b}_{0,0} \\
&\quad - \textstyle\sum_{w}^{W} P_w - \sum_{n2}^{N} P_{n2}^{\max} - \sum_{n3}^{N} P_{n3}^{\min}; \\
&\text{where } n1, n2, n3 \in \{1, 2, \ldots N\}; \quad n1 \ne n2 \ne n3 \\
&\qquad (17)
\end{aligned}$$

and convergence to the optima may be expected for (14).

To summarize the inferences of Sections II and III, the possibility of firm dispatch values from the conventional units in the ELD depends on:

a) for the lossless case, approximate constancy of total wind power generation in the validity interval;
b) for the lossy case, approximate constancy of derived loss coefficients $\{\tilde{b}_{n,0}\}$ and $\tilde{b}_{0,0}$ in the validity interval; these to be evaluated from the basic network loss coefficients and wind power generations $\{P_w\}$.

As mentioned in the last section, though "a–b" above may seem to be too demanding in view of the variability of wind speed, they may hold approximately because of the crisply defined output curve (7) of pitch-angle controlled WECS under the ideal circumstances of zero turbulence.

An acceptable form of such approximations in the presence of short duration variations is examined in Section IV.

IV. WECS Output Within the Validity Interval of an ELD

The most accepted measure of short duration wind variations is the *turbulence intensity*, defined (for the wth installation site) as the ratio between the standard deviation σ_w and the mean $\bar{u}_w$ of wind speed—both evaluated typically over a duration of no more than an hour, with data sampled about once every second [10]. The *turbulence intensity* for WECS installations at a site typically assumes values in the range of 0.1–0.4 [10], though sites with values within 0.05–0.20 are preferred [13].

The exact effect of short duration variations (particularly turbulence) on the power output by WECS stations has been the subject of much investigation in recent times [12], [13]. Both empirical as well as analytical techniques have been employed for this, though currently there seems to be considerable debate regarding the latter. Some common conclusions, however, seem to emerge from most studies:

- Deep within the "constant parts" of the WECS output curve, short duration variations have little effect as speed oscillations do not reflect on power (7).
- Close to the cut-in speed, *swings below* ($u_w \swarrow u_w^{in}$ and $u_w \nearrow u_w^{in}$) have little effect, while *swings above* ($u_w \nwarrow u_w^{in}$ and $u_w \searrow u_w^{in}$) enhance the power output by *forcing into* the $u_w^{in} < \bar{u}_w \le u_w^{rat}$ range of operation in (7). Variations close to the cut-in speed, therefore, tend to *increase* the output power over values indicated by (7).
- Close to the rated speed, *swings above* ($u_w \nwarrow u_w^{rat}$ and $u_w \searrow u_w^{rat}$) have little effect, while *swings below* ($u_w \swarrow u_w^{rat}$ and $u_w \nearrow u_w^{rat}$) reduce the power output, once again by *forcing into* the $u_w^{in} < \bar{u}_w \le u_w^{rat}$ range of operation in (7). Variations close to the rated speed, therefore, tend to *reduce* the output power from values indicated by (7).

It follows from the above that representation of short duration wind variations would require replacement of (6) by a modified relation as

$$\langle P_w \rangle = \langle P_w(\bar{u}_w) \rangle \triangleq \tilde{\mu}_w(\bar{u}_w, \tau) \cdot P_w^{rat} \qquad (18)$$

where $\tilde{\mu}_w(\bar{u}_w, \tau)$ can be defined as the *output coefficient under turbulence* at the wth WECS installation, given the mean speed $\bar{u}_w$ and turbulence intensity τ for the validity interval of ELD. Proper estimate of $\tilde{\mu}_w(\bar{u}_w, \tau)$ from empirical data is critical for applicability of (18) to ELD problems (3) and (14).

Following the three conclusions mentioned above, one may define the *output coefficient under turbulence* as

$$\tilde{\mu}_w(\bar{u}_w, \tau) \triangleq 1 - \exp\left[-\left(\frac{\bar{u}_w / u_w^{rat}}{\upsilon(\tau)}\right)^{\kappa(\tau)}\right] \qquad (19)$$

where the *scaling parameter* $\upsilon(\tau)$ and *index parameter* $\kappa(\tau)$ are both positive and functions of turbulence intensity τ. It is easy

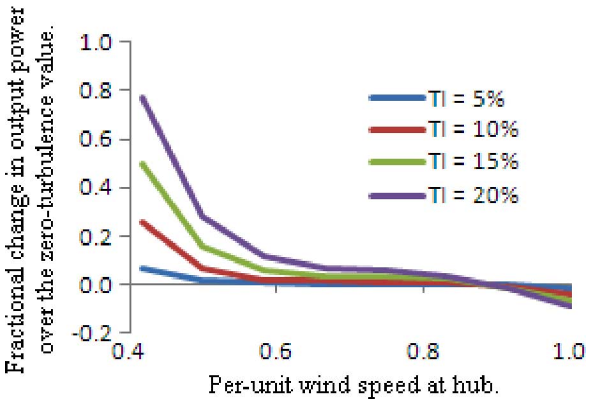

Fig. 1. Effect of different levels of turbulence on power output by a typical wind energy conversion system [13].

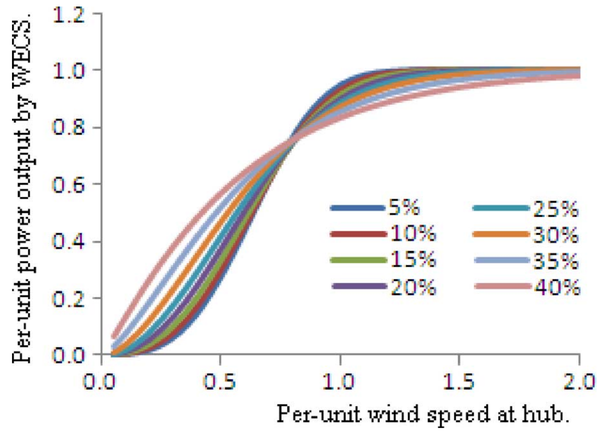

Fig. 2. Power output curve $\tilde{\mu}_w(\bar{u}_w, \tau)$ according to (19) and (20) for a typical wind energy conversion system at different levels of turbulence, as indicated.

to see that (19) approaches the more "sharp" definition of (7) at extreme values of average speed $\bar{u}_w \longrightarrow 0$ and $\bar{u}_w \longrightarrow \infty$, while it transits smoothly in the range $u_w^{in} < \bar{u}_w \leq u_w^{rat}$ as decided by $\upsilon(\tau)$ and $\kappa(\tau)$.

For typical WECS of large capacity, Fig. 1 displays the fractional change in *per-unit output power* $(\langle P_w \rangle / P_w^{rat})$ from its zero-turbulence level $(P_w(\bar{u}_w)/P_w^{rat})$ corresponding to different values of *per-unit wind speed at hub* $(\bar{u}_w/u_w^{rat})$ and turbulence index τ (TI), as reported in [13]. This data is processed to obtain *output coefficient under turbulence* $\tilde{\mu}_w(\bar{u}_w, \tau)$, corresponding to different values of per-unit wind speed and turbulence index. The *scaling* and *index parameters* of (19) can be obtained by suitable curve fit into the $\tilde{\mu}_w(\bar{u}_w, \tau)$ data as

$$\begin{aligned}\kappa(\tau) &\approx 3.49 - 6.01 \cdot \tau \\ \upsilon(\tau) &\approx 0.71 + 0.21 \cdot \tau - 1.26 \cdot \tau^2.\end{aligned} \tag{20}$$

With values of $\kappa(\tau)$ and $\upsilon(\tau)$ known for any τ in the range 0.1–0.4 by (20), (19) can be used to obtain the per unit output power curves $\tilde{\mu}_w(\bar{u}_w, \tau)$ as shown in Fig. 2. Knowing the value of rated power P_w^{rat} for a particular WECS unit, it is then easy to compute the $\langle P_w \rangle$ for validity interval of an ELD.

In many practical cases, judicious choice of the validity interval for ELD can make the range of short duration wind speed variations sufficiently small, so that the corresponding variations in P_w may be low. For such cases, approximations of (13) may be defined in terms of the $\langle P_w \rangle$ average values as

$$\begin{aligned}\tilde{b}_{n,0} &\approx b_{n,0} + \sum_{w}^{W} b_{n,w} \langle P_w \rangle \\ \tilde{b}_{0,0} &\approx b_{0,0} + \sum_{w}^{W} b_{w,0} \langle P_w \rangle + \sum_{w1}^{W} \sum_{w2}^{W} \langle P_w \rangle b_{w1,w2} \langle P_w \rangle.\end{aligned} \tag{21}$$

The approximation for $\{\tilde{b}_{n,0}\}$ follows directly from (13), while that for $\tilde{b}_{0,0}$ requires the output power at different WECS stations to be *statistically independent* [19].

With the approximations (21), optimization of the ELD problem (14) is straightforward. Examples to substantiate this are presented in Sections V and VI.

V. Modification of the Classic Three-Generator ELD Problem

The three-generator ELD problem, as described in [4], has been exhaustively studied and reported over the years. The unit and loss parameters for this classic problem [4] are presented in Tables I and II, respectively ($b_{0,0}$ and $b_{n,0}$ being set to zero), with a total demand of 850 MW. When solved by the Generalized Algebraic Modeling System using the CONOPT nonlinear optimization solver (GAMS-CONOPT [20]), the following base case optima is obtained (in terms of the hypothetical currency unit $\not{R}$ [4]):

$$\begin{aligned}&\text{Demand} : 850\ \text{MW} \\ &\text{Total cost} : C(P_1, P_2, P_3) = \not{R}8344.593 \\ &\text{Generations} : \\ &\quad P_1 = 435.198\ \text{MW}, \quad P_2 = 299.970\ \text{MW}, \\ &\quad P_3 = 130.661\ \text{MW} \\ &\text{Losses} : 15.829\ \text{MW} \\ &\text{System}\ \lambda : \not{R}9.528/\text{MW}.\end{aligned}$$

The unit of lowest capacity (Unit 3) is now replaced by a WECS generator, whose rated output power is identical to the maximum value of 200 MW indicated in Table I, and which follows the output as described by (18)–(20). The maximum possible share of wind power at the above demand level is thereby fixed at 23.53%. The value of $\bar{u}_w$ is to be either obtained from measurements, or by prediction methods on a day-ahead or hour-ahead basis depending on the application [21]. For studies reported in this paper, different values of the short duration mean speed $\bar{u}_w$ are considered between zero to twice the rated value u_w^{rat}. Likewise, for different executions of the ELD, turbulence intensity (TI) is assumed from negligibly small values to a maximum of 0.40.

The running *cost of generation by wind* should apply to Unit 3 in the modified ELD, as represented by (2). This is substantially less than corresponding fossil-fuel units due to the missing "cost-of-fuel" component. Following data published in [22], the cost of kilowatt-hour generated by the wind-based Unit 3 is taken to be 37.55% of the corresponding coal-based steam Unit

TABLE I
GENERATOR PARAMETERS FOR THREE-UNIT ELD PROBLEM [4]

Unit #	Gen. limits (MW)		Generation cost parameters		
	Max.	Min.	$c_{0,n}$ (₨/hr)	$c_{1,n}$ (₨/hr-MW)	$c_{2,n}$ (₨/hr-MW^2)
1	600	150	561	7.92	0.001562
2	400	100	310	7.85	0.001940
3	200	50	78	7.97	0.004820

TABLE II
$b_{n1,n2}$ LOSS PARAMETERS (IN MW^{-1}) FOR THE THREE-UNIT ELD PROBLEM [4]

n_1, n_2	1	2	3
1	3x10^{-5}	0	0
2	0	9x10^{-5}	0
3	0	0	12x10^{-5}

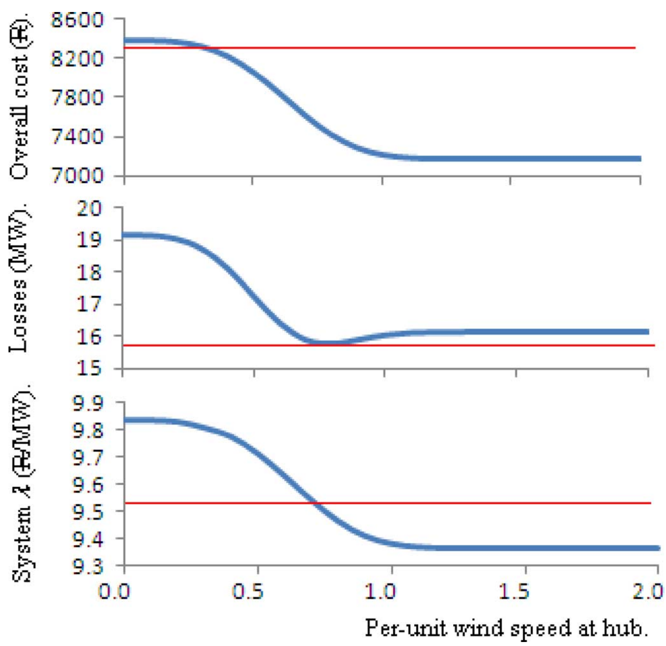

Fig. 3. Optima for the modified three-generator problem [4] at different values of long-duration-stable mean wind speed at WECS hub. Corresponding base case optimal values are indicated in red.

1 (Table I [4]). The WECS-based Unit 3 is accordingly assigned $c_{1,w} = ₨3.68$ per MWh.

For different values of short duration mean wind speed $\bar{u}_w$ and negligible turbulence ($\text{TI} = 1.0 \times 10^{-5}$), Fig. 3 displays the ELD optima in terms of the total cost $C(P_1, P_2, P_3)$, the total transmission losses, and the marginal cost of generation (system-λ). As generation by wind assumes substantial levels at high values of$\bar{u}_w$, the total cost of generation drops from values comparable to the base case to about ₨7200.00. Correspondingly, the losses drop from substantial levels to values close to the base case (minimum of about 15.7 MW) finally settling at about 16 MW. The system-λ is expectedly higher than the base case at low levels of wind-based generation, eventually dropping to values lower than the base case as generation by Unit 3 picks up.

Fig. 4 shows the sensitivity of all three metrics to turbulence intensity. Since high turbulence adds to the output by Unit 3 at low average wind speeds and reduces the same at high average wind speeds, the cost metrics (total cost and system-λ) are accordingly seen to drop at low values, and marginally increase at high values of $\bar{u}_w$. The transmission losses on the other

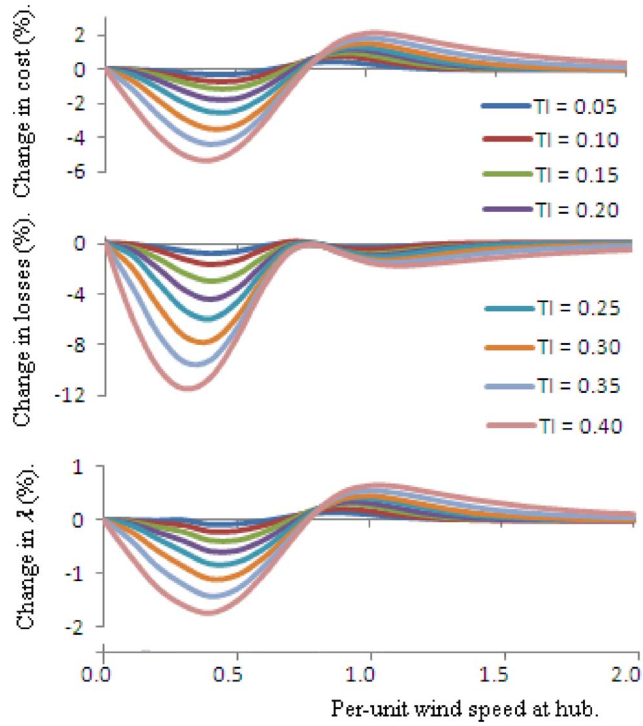

Fig. 4. Sensitivity of the optima (corresponding to Fig. 3) to turbulence for the modified three generator problem [4]. All eight levels of turbulence apply to each of the three plots.

TABLE III
GENERATOR PARAMETERS FOR A 20-UNIT ELD PROBLEM

Unit #	Gen. limits (MW)		Generation cost parameters		
	Max.	Min.	$c_{0,n}$ (\$/hr)	$c_{1,n}$ (\$/hr-MW)	$c_{2,n}$ (\$/hr-MW^2)
1	600	150	1000	18.19	0.00068
2	200	50	970	19.26	0.00071
3	200	50	600	19.80	0.00650
4	200	50	700	19.10	0.00500
5	160	50	420	18.10	0.00738
6	100	20	360	19.26	0.00612
7	125	25	490	17.14	0.00790
8	150	50	660	18.92	0.00813
9	200	50	765	18.27	0.00522
10	150	30	770	18.92	0.00573
11	300	100	800	16.69	0.00480
12	500	150	970	16.76	0.00310
13	160	40	900	17.36	0.00850
14	130	20	700	18.70	0.00511
15	185	25	450	18.70	0.00398
16	80	20	370	14.26	0.07120
17	85	30	480	19.14	0.00890
18	120	30	680	18.92	0.00713
19	120	40	700	18.47	0.00622
20	100	30	850	19.79	0.00773

hand, undergo a reduction both at high and low levels of wind speed, indicating the highest losses when wind-based generation is close to the corresponding base case output by Unit 3 (about 130 MW).

VI. MODIFICATION OF A TWENTY-GENERATOR ELD PROBLEM

The ELD problem, defined by the conventional unit parameters presented in Table III and loss parameters $(b_{n1,n2})$ in Table IV, is to be modified to accommodate wind-based generation up to a maximum of 600 MW. Together all units are expected to meet a demand of 2500 MW (maximum 24% possible share of wind power in each modified ELD), for which

TABLE IV
$b_{n1,n2}$ LOSS PARAMETERS (x10^{-5} MW^{-1}) FOR THE 20-UNIT ELD PROBLEM

Unitτ o	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	8.70	0.43	-4.61	0.36	0.32	-0.66	0.96	-1.60	0.80	-0.10	3.60	0.64	0.79	2.10	1.70	0.80	-3.20	0.70	0.48	-0.70
2	0.43	8.30	-0.97	0.22	0.75	-0.28	5.04	1.70	0.54	7.20	-0.28	0.08	-0.46	1.30	0.80	-0.20	0.52	-1.70	0.80	0.20
3	-4.61	-0.97	9.00	-2.00	0.63	3.00	1.70	-4.30	3.10	-2.00	0.70	-0.77	0.93	4.60	-0.30	4.20	0.38	0.70	-2.00	3.60
4	0.36	0.22	-2.00	5.30	0.47	2.62	-1.96	2.10	0.67	1.80	-0.45	0.92	2.40	7.60	-0.20	0.70	-1.00	0.86	1.60	0.87
5	0.32	0.75	0.63	0.47	8.60	-0.80	0.37	0.72	-0.90	0.69	1.80	4.30	-2.80	-0.70	2.30	3.60	0.80	0.20	-3.00	0.50
6	-0.66	-0.28	3.00	2.62	-0.80	11.80	-4.90	0.30	3.00	-3.00	0.40	0.78	6.40	2.60	-0.20	2.10	-0.40	2.30	1.60	-2.10
7	0.96	5.04	1.70	-1.96	0.37	-4.90	8.24	-0.90	5.90	-0.60	8.50	-0.83	7.20	4.80	-0.90	-0.10	1.30	0.76	1.90	1.30
8	-1.60	1.70	-4.30	2.10	0.72	0.30	-0.90	1.20	-0.96	0.56	1.60	0.80	-0.40	0.23	0.75	-0.56	0.80	-0.30	5.30	0.80
9	0.80	0.54	3.10	0.67	-0.90	3.00	5.90	-0.96	0.93	-0.30	6.50	2.30	2.60	0.58	-0.10	0.23	-0.30	1.50	0.74	0.70
10	-0.10	7.20	-2.00	1.80	0.69	-3.00	-0.60	0.56	-0.30	0.99	-6.60	3.90	2.30	-0.30	2.80	-0.80	0.38	1.90	0.47	-0.26
11	3.60	-0.28	0.70	-0.45	1.80	0.40	8.50	1.60	6.50	-6.60	10.70	5.30	-0.60	0.70	1.90	-2.60	0.93	-0.60	3.80	-1.50
12	0.64	0.98	-0.77	0.92	4.30	0.78	-0.83	0.80	2.30	3.90	5.30	8.00	0.90	2.10	-0.70	5.70	5.40	1.50	0.70	0.10
13	0.79	-0.46	0.93	2.40	-2.80	6.40	7.20	-0.40	2.60	2.30	-0.60	0.90	11.00	0.87	-1.00	3.60	0.46	-0.90	0.60	1.50
14	2.10	1.30	4.60	7.60	-0.70	2.60	4.80	0.23	0.58	-0.30	0.70	2.10	0.87	3.80	0.50	-0.70	1.90	2.30	-0.97	0.90
15	1.70	0.80	-0.30	-0.20	2.30	-0.20	-0.90	0.75	-0.10	2.80	1.90	-0.70	-1.00	0.50	11.00	1.90	-0.80	2.60	2.30	-0.10
16	0.80	-0.20	4.20	0.70	3.60	2.10	-0.10	-0.56	0.23	-0.80	-2.60	5.70	3.60	-0.70	1.90	10.80	2.50	-1.80	0.90	-2.60
17	-3.20	0.52	0.38	-1.00	0.80	-0.40	1.30	0.80	-0.30	0.38	0.93	5.40	0.46	1.90	-0.80	2.50	8.70	4.20	-0.30	0.68
18	0.70	-1.70	0.70	0.86	0.20	2.30	0.76	-0.30	1.50	1.90	-0.60	1.50	-0.90	2.30	2.60	-1.80	4.20	2.20	0.16	-0.30
19	0.48	0.80	-2.00	1.60	-3.00	1.60	1.90	5.30	0.74	0.47	3.80	0.70	0.60	-0.97	2.30	0.90	-0.30	0.16	7.60	0.69
20	-0.70	0.20	3.60	0.87	0.50	-2.10	1.30	0.80	0.70	-0.26	-1.50	0.10	1.50	0.90	-0.10	-2.60	0.68	-0.30	0.69	7.00

the base case optima obtained by GAMS-CONOPT [20] is as follows:

$$\begin{aligned}
&\text{Demand} : 2500\ \text{MW}\\
&\text{Total cost} : C(P_1, \ldots P_{20}) = \$62458.093\\
&\text{Generations} :\\
&\quad P_1 = 513.111\ \text{MW}, \qquad P_2 = 167.385\ \text{MW},\\
&\quad P_3 = 126.986\ \text{MW}, \qquad P_4 = 102.841\ \text{MW},\\
&\quad P_5 = 113.790\ \text{MW}, \qquad P_6 = 73.589\ \text{MW},\\
&\quad P_7 = 114.775\ \text{MW}, \qquad P_8 = 116.541\ \text{MW},\\
&\quad P_9 = 100.612\ \text{MW}, \qquad P_{10} = 106.639\ \text{MW},\\
&\quad P_{11} = 150.468\ \text{MW}, \qquad P_{12} = 292.698\ \text{MW},\\
&\quad P_{13} = 119.190\ \text{MW}, \qquad P_{14} = 31.075\ \text{MW},\\
&\quad P_{15} = 115.841\ \text{MW}, \qquad P_{16} = 36.263\ \text{MW},\\
&\quad P_{17} = 66.966\ \text{MW}, \qquad P_{18} = 87.926\ \text{MW},\\
&\quad P_{19} = 100.913\ \text{MW}, \qquad P_{20} = 54.400\ \text{MW}.\\
&\text{Losses} : 92.009\ \text{MW}\\
&\text{System}\ \lambda : \$20.959/\text{MW}.
\end{aligned}$$

Two variations of the base case are considered so as to realize the 600 MW of wind power capacity. In case ***A***, Unit 1 (Table III) is replaced by a WECS-based unit of 600 MW capacity. In case ***B***, each of the Units 2–4 (Table III) is replaced by a WECS-based unit of 200 MW capacity. For both ***A*** and ***B***, the WECS are assumed to follow the output power curves of Fig. 2 as decided by specific turbulence index.

The cost of generation by wind is computed from the corresponding values for the respective conventional units in a manner similar to that described in Section V, that is, setting $c_{1,w}$ to 37.55% of the $c_{1,n}$ for the conventional unit [22]. The generation cost by WECS units that replace Unit 1 (for case ***A***) and 2–4 (for case ***B***) are thus set at \$6.83/MWh, \$7.23/MWh, \$7.43/MWh, and \$7.17/MWh, respectively.

For the results reported in this paper, both *short duration average wind speed* and *turbulence intensity* are assumed to be identical for all WECS units, thereby implying operation at the same point of per-unit output power curve (Fig. 2). "Unit #1" in case ***A*** represents such a multiunit cluster. Field studies undertaken at various locations of North America [23] and Europe [24] have amply established the mutually "smoothing effect" of output power by aggregation across WECS units within a cluster, which justifies the above assumption. The same may be alternately interpreted as one of *negligible power variability* at the point of connection between a WECS cluster and the network [25]–[27].

For a system that includes distributed wind-based installations, each being a multiunit cluster (case ***B*** has three such clusters, represented by "Unit #2," "Unit #3," and "Unit #4," respectively), different combinations of average wind speed and turbulence intensity are possible between stations. Exhaustive presentation of ELD optima corresponding to all such combinations for the 20-generator system will be cumbersome as well as repetitive in terms of inference. Therefore, for this paper, the assumption of identical *short duration average wind speed* and *turbulence intensity* across all units has been retained for case ***B*** as well. In practice, aggregate wind power between sites is known to have different forms of interdependence [28], and this can significantly affect distributed generation across the utility service area [29].

Fig. 5 compares the optima for both cases ***A*** and ***B*** with the base case at different values of short duration average wind speed, and negligible levels of turbulence. While the generation cost is in general expected to be lower due to $c_{1,w} < c_{1,n}$, the cost reduction is found to be more significant in case ***B***. This is essentially due to the lower losses in ***B*** as compared to ***A*** for similar levels of wind-based output power. Greater reduction in system-λ is accordingly observed throughout case ***B*** in comparison to ***A***, proving a point in favor of distributed generation as against single WECS stations of large capacity.

Variations over the results of Fig. 5 due to significant levels of turbulence, are displayed for both ***A*** and ***B*** in Fig. 6. A comparison shows that other than a consistent reduction of losses at high turbulence in ***A*** (similar to a corresponding observation of the three-generator example of Section V), the sensitivity to the turbulence index is similar between the two cases. Thus

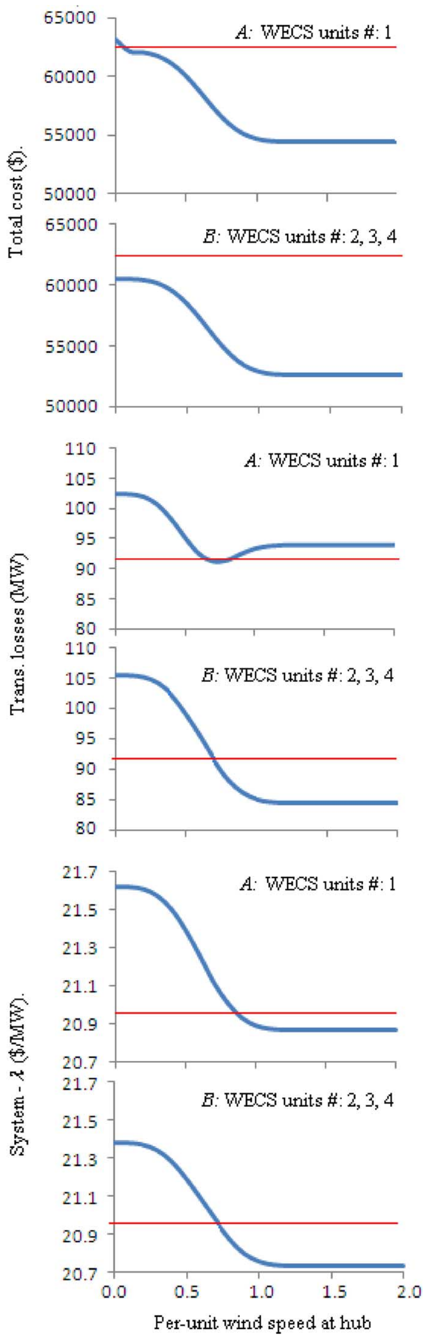

Fig. 5. ELD optima for the modified 20-generator problem (both cases ***A*** and ***B***) at different values of long-duration-stable mean wind speed at WECS hub. Corresponding base case optimal values are indicated in red.

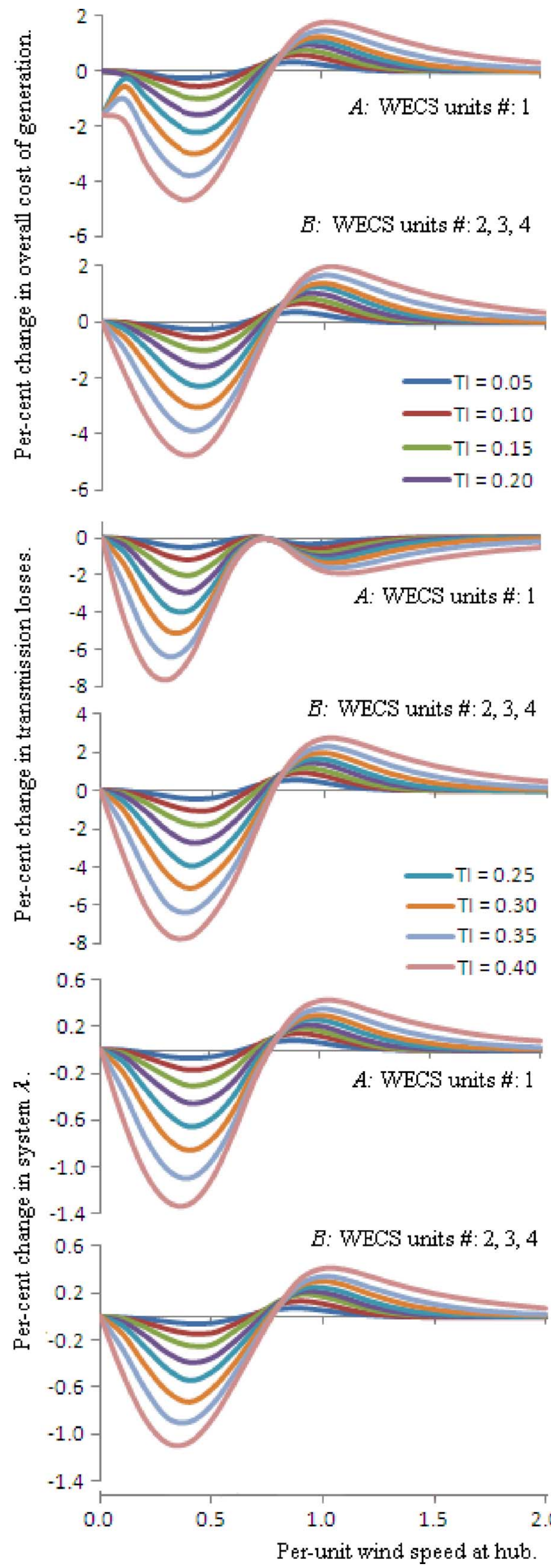

Fig. 6. Sensitivity of the ELD optima for the modified 20-generator problem to turbulence (both cases ***A*** and ***B***, corresponding to Fig. 5). All eight levels of turbulence indicated apply to each of the six plots.

the impact of turbulence is not substantially different between ELD involving large single WECS installations and multiple distributed stations of the same overall capacity, other than in terms of transmission losses.

Fig. 5 shows that at very low values of $\bar{u}_w$ as the WECS at Unit 1 "drops out" (case ***A***), greater dependence on the more expensive generating units pushes the total optimal cost to marginally higher values. As generation by wind picks up to modest levels, the total optimal cost drops and then continues on a smooth trend across increasing range of wind speed. Correspondingly, a small but significant increase in sensitivity is noted for high levels of turbulence in Fig. 6. Elimination at low wind speeds is to be expected of all wind generators in a network, the impact on ELD being decided by the extent of loss in generation capacity.

VII. Conclusion

This paper has attempted to examine the consequence of significant wind-based capacity inclusion on the conventional ELD problem. The effect of short duration wind variations has been considered in terms of two features, namely the *short-duration-stable mean wind speed* and the short duration *turbulence index*.

It is observed that in comparison to the conventional base case, higher levels of wind-based generation reduces the optimal generation cost, transmission losses, and system-λ, all essentially due to the lower running cost of the WECS units. Turbulence has an aggregate effect on all three metrics through distortion of the WECS output power, reducing the same at low values, and enhancing them at high values of wind speed.

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Sanjoy Roy (M'90–SM'97) received the Bachelors degree from the University of Roorkee, Roorkee, the Masters degree from the Indian Institute of Science, Bangalore, and the Doctorate degree from University of Calgary, Calgary, all in electrical engineering, in 1985, 1987, and 1991, respectively.

Between 1992 and 2004, he worked as a member of the faculty at the Indira Gandhi Institute of Development Research, Mumbai, India, and the Indian Institutes of Technology, at Mumbai and then at Delhi, India. Between 2004 and 2010, he was a Professor at the Birla Institute of Technology and Science, Pilani, India, and Coordinator of the *Centre for Renewable Energy and Environment Development* (CREED) at the same institute. He is currently a Professor and Head at the Department of Electrical Engineering, Indian Institute of Technology, Ropar, Punjab, India. His areas of research interest include economics, planning, and grid-integration of renewable energy systems.