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A Method of Self-Recovering Equalization for Multilevel Amplitude-Modulation Systems

YOICHI SATO

***Abstract*—A self-recovering equalization algorithm, which is employed in multilevel amplitude-modulated data transmission, is presented. Such a self-recovering equalizer has been required when time-division multiplexed (TDM) voice or picturephone PCM signals must be transmitted over the existing frequency-division multiplexed (FDM) transmission channel. The present self-recovering equalizer is quite simple, as is a conventional binary equalizer. The convergence processes of the present self-recovering equalizer are shown by computer simulation. Some theoretical considerations on this convergence process are also added.**

I. INTRODUCTION

Recently, in order to carry time-division multiplexed (TDM) high-rate information, for example, digitalized picturephone signals, wide-band digital transmission service has been developed over existing microwave radio and analog coaxial facilities employing a frequency-division multiplexed (FDM) system [1], [2].

In such a ditial communication network, it should be remarked in receiver design that a once-connected transmission route might be reconnected through another route because of transmission troubles. In this situation, the receiver is subject to sudden displacements of all demodulating conditions: carrier phase, timing phase, channel distortion, and so forth. Consequently, it would be necessary for the receiver to recover every demodulating operation so that these operations adapt to the newly connected channel. Also, for adaptive equalization, the self-recovering function must be given. However, in practical cases, where multilevel signaling schemes and pulses of stiffly rolled-off spectrum are employed, conventional adaptive equalization, after route reconnection, fails to readapt, on account of the failure of the data estimation in a multilevel decision circuit.

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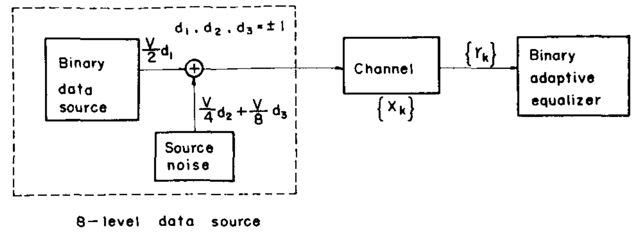

Fig. 1. Conceptual diagram showing self-recovering equalization system.

This concise paper reports the development of the self-recovering equalization employed in a multilevel amplitude-modulation system. The present scheme does not need any particular training signal, but it is obtained by simple modification of the conventional mean-square adaptive equalization based on the following idea.

Presuming that the multilevel signal is decomposed into its polarity signal and the remaining signal, the direct application of the binary mean-square equalizer to the multilevel system treats the remaining signal as random source noise. Hence, the correct tap adjustment will be performed after eliminating the remaining signal according to the averaging effect in the tap-adjusting circuit.

II. SELF-RECOVERING ALGORITHM FOR MULTILEVEL SYSTEM

The present self-recovering algorithm employs a binary decision circuit in place of a multilevel decision circuit, even though the equalizer is applied in a multilevel system. For example, assume an 8-level signal and let the levels be given as

$$V \cdot (\tfrac{1}{2}d_1 + \tfrac{1}{4}d_2 + \tfrac{1}{8}d_3), \tag{1}$$

where d_1, d_2, and d_3 are binary random variables. The self-recovering equalizer estimates only the most significant digit d_1 and, using it, the tap-adjusting quantities are derived. Therefore, remaining digits d_2 and d_3 are treated as random noise superimposed at the data source. Fig. 1 shows its conceptual diagram, where the self-recovering algorithm is written by

$$\begin{bmatrix} c_{-N}^{m+1} \\ \cdot \\ \cdot \\ \cdot \\ c_0^{m+1} \\ \cdot \\ \cdot \\ \cdot \\ c_N^{m+1} \end{bmatrix} = \begin{bmatrix} c_{-N}^{m} \\ \cdot \\ \cdot \\ \cdot \\ c_0^{m} \\ \cdot \\ \cdot \\ \cdot \\ c_N^{m} \end{bmatrix} - \alpha \begin{bmatrix} r_{k+N} \\ \cdot \\ \cdot \\ \cdot \\ r_k \\ \cdot \\ \cdot \\ \cdot \\ r_{k-N} \end{bmatrix} (y_k^m - \gamma \operatorname{sign}(y_k^m)), \tag{2}$$

where the notations indicate

c_i^m ith tap gain of the mth iteration

r_k kth sample of receiving signal

$$r_k = \sum_{i=-\infty}^{\infty} a_i x_{k-i} \tag{3}$$

where x_k is the kth sample of the channel's impulse response, y_k^m is the kth sample of equalizer output,

$$y_k^m = \sum_{i=-N}^{N} r_{k-i} c_i^m = \sum_{l=-\infty}^{\infty} a_l h_{k-l}^m, \tag{4}$$

where h_k^m denotes the kth sample of overall impulse response at equalizer output, α is the tap-adjusting coefficient, and γ is the scaling coefficient. The scaling coefficient γ is to be determined from the level voltage values preset in receiver as follows. For the example of an 8-level signal, let the level voltage values in the receiver be as $V'(\frac{1}{2}d_1 + \frac{1}{4}d_2 + \frac{1}{8}d_3)$, and suppose the situation that the equalization is completely accomplished, then γ specified by $2 \cdot V' \cdot$

$(\frac{1}{4}+\frac{1}{16}+\frac{1}{64}) = \frac{21}{32}V'$ adjusts the equalizer output levels to the preset level voltage values.

In general, it can be assumed that, under small tap-adjusting coefficient α, the mth stochastic tap-gain solution c_i^m $(i = -N,\cdots,N)$ transits closely along its expected value $\bar{c}_i^m$ $(i = -N,\cdots,N)$, which satisfies the following recursive equation:

$$\begin{bmatrix} \bar{c}_{-N}^{m+1} \\ \cdot \\ \cdot \\ \bar{c}_0^{m+1} \\ \cdot \\ \cdot \\ \bar{c}_N^{m+1} \end{bmatrix} = \begin{bmatrix} \bar{c}_{-N}^{m} \\ \cdot \\ \cdot \\ \bar{c}_0^{m} \\ \cdot \\ \cdot \\ \bar{c}_N^{m} \end{bmatrix} - \alpha \begin{bmatrix} \overline{r_{k+N}y_k^m} - \overline{\gamma r_{k+N}\,\mathrm{sign}\,(y_k^m)} \\ \cdot \\ \cdot \\ \overline{r_k y_k^m} - \overline{\gamma r_k\,\mathrm{sign}\,(y_k^m)} \\ \cdot \\ \cdot \\ \overline{r_{k-N}y_k^m} - \overline{\gamma r_{k-N}\,\mathrm{sign}\,(y_k^m)} \end{bmatrix}, \qquad (5)$$

where the bar above each variable denotes the statistical expectation. Therefore, the behavior of the solution of (5) tells us whether convergence to the optimum solution of minimum intersymbol interference is successfully performed or not.

III. RESULTS OF COMPUTER SIMULATION

The computer simulation of the present algorithm (2) was performed based on actual frequency characteristics of a master group band, over which 6.312 Mbit/s digital information employing 8-level vestigial sideband (VSB)-AM was transmitted. The initial peak distortion $\Sigma' \mid x_k \mid / \mid x_0 \mid$ was 1.9736 at the optimum timing phase and its rms eye closure, defined by

$$\frac{(\text{average transmitting signal power} \times \text{mean-square distortion})^{1/2}}{0.5 \times \text{distance between adjacent levels}},$$

was evaluated as 4.0509. The initial tap gain was chosen to be $c_i = 0$ $(i \neq 0)$, $c_0 = 1$. The performances of several self-recovering processes are plotted in Fig. 2, where dotted lines represent the conventional algorithm employing 8-level decision, which shows failure of recovery. Solid lines illustrate the present self-recovering processes. The recovering time for $\alpha = 0.6 \times 10^{-3}$, which is determined by the time when rms eye closure is reduced to 1/3, is read from Fig. 2 as 7000 iterations (approximately 3.3 ms). In practical applications to the wide-band digital transmission systems, this period will be short enough compared to the time interval which will be desired in practical system for a decision on transmission trouble.

IV. THEORETICAL CONSIDERATION

The notable recovering characteristics as shown in Fig. 2 will be theoretically treated when an infinite number of levels is assumed, since the second term in the right-hand side of (5) can be represented explicitly in functional form of the expected solution $\bar{c}_i^m$. The distinction between finite- and infinite-level systems yields only the difference of source noise probability density function as they are discrete and continuous uniform functions, respectively. Such a difference, as long as both source noise models assume uncorrelated sequences, has no significant influence upon the recovering effect. Therefore, we examine the theoretical considerations for an infinite-level system, where convergence to the optimum solution will be shown.

Let the infinite-level signal be

$$a^\infty = V(\tfrac{1}{2}d_1 + \tfrac{1}{4}d_2 + \tfrac{1}{8}d_3 + \tfrac{1}{16}d_4 + \cdots), \qquad (6)$$

whose probability density function is continuous and uniform. Denoting the kth infinite-level signal by a_k, we first prepare the following useful formula (Appendix A):

$$\overline{\Sigma a_k p_k \cdot \mathrm{sign}\,(\Sigma\, a_k q_k)} = V\left(\frac{P_0}{2} - \frac{P_0}{6q_0^2}\Sigma' q_k^2 + \frac{1}{3q_0}\Sigma' p_k q_k\right), \qquad (7)$$

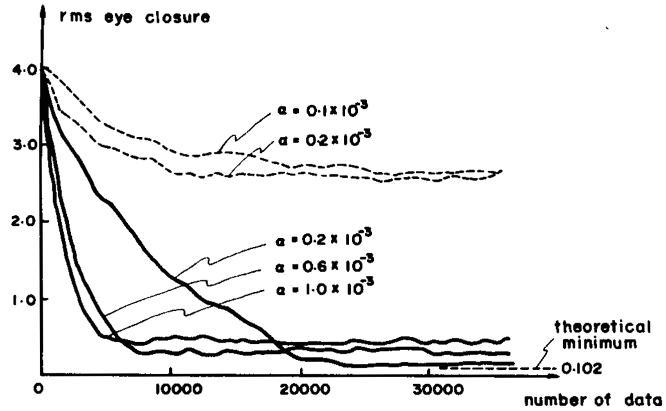

Fig. 2. Self-recovering processes for 8-level PAM system.

where p_k and q_k are any nonrandom constants. The prime on summation denotes the deletion of the $k = 0$ term and the sufficient condition $\Sigma' \mid q_k \mid / \mid q_0 \mid < 1$ has been assumed.

Applying this formula to the jth tap-adjusting term in (5), we obtain

$$\begin{aligned} \overline{r_{k-j}(y_k - \gamma\,\mathrm{sign}\,(y_k))} &= \overline{\Sigma\, a_i x_{k-j-i}(\Sigma\, a_i h_{k-i} - \gamma\,\mathrm{sign}\,(\Sigma\, a_i h_{k-i}))} \\ &= \frac{V^2}{3}\left(1 - \frac{\gamma}{h_0 V}\right)\Sigma\, x_{k-j}h_k \\ &\quad - \frac{\gamma V}{6}\left(1 - \frac{\Sigma' h_k^2}{h_0^2}\right)x_{-j}. \end{aligned} \qquad (8)$$

Furthermore, it is shown by (7) that the above expression is equivalent to partial derivative

$$\frac{\partial}{\partial c_j}\,\overline{(y_k - \gamma\,\mathrm{sign}\,(y_k))^2}. \qquad (9)$$

Therefore, it is said that (5) is the steepest descent minimization algorithm with the following cost function:

$$\begin{aligned} E &= \overline{(y_k - \gamma\,\mathrm{sign}\,(y_k))^2} \\ &= \frac{V^2}{3}\Sigma(h_k)^2 - \gamma V\left(h_0 - \frac{1}{3}\frac{\Sigma' h_k^2}{h_0^2}\right) + \gamma^2. \end{aligned} \qquad (10)$$

On the other hand, for cost function E, the following fundamental properties are satisfied.

1) E is a convex function of tap gains (Appendix B).

2) Numerically, the minimum of E is the same number as the minimum of mean-square distortion $\Sigma' h_k^2/h_0^2$ (Appendix C).

Based on the above properties, we arrive at the following conclusion: the present self-recovering algorithm (2) converges for any large-number-level system provided that the tap-adjusting coefficient α is small enough and initial distortion at the equalizer output satisfies $\Sigma' \mid h_k \mid / \mid h_0 \mid < 1$. While condition $\Sigma' \mid h_k \mid / \mid h_0 \mid < 1$ was introduced as a convenience in derivation of (7), it may be too severe for self-recovering requirement. It is further work to extend this condition by redefining integration limits in (11). In the previous section, it was seen that in a practical example, although it does not satisfy the above condition, self-recovery can be successfully performed.

V. CONCLUSION AND REMARKS

The simple strategy of self-recovering equalization has been presented, which needs no testing pulse inserted in the information sequence. Theoretically, the recovering effect is confirmed under the assumption of infinite level.

The present self-recovering technique can be extended to other modulation schemes, such as quadrature amplitude-modulation system and partial-response signaling scheme.

For Class IV partial-response system (modified duobinary), the

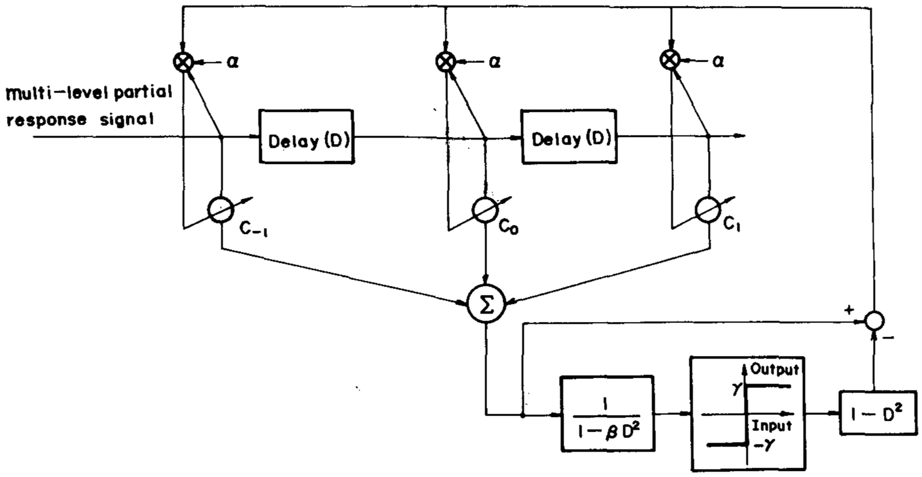

Fig. 3. Self-recovering equalizer for Class IV partial-response system.

self-recovering equalizer must be given as illustrated in Fig. 3. Its self-recovering mechanism can be investigated through the same analysis as in the previous sections. For the same channel as used in previous VSB-AM system, the level number of partial-response signal becomes 15 and initial rms eye closure at equalizer input amounted to 5.7459. In this case, the self-recovering equalization converges successfully, as shown in Fig. 4, where the slowness of convergence is inherent in the partial-response system [3].

APPENDIX A

With each of the $\{a_k\}$ independently and uniformly distributed on the interval from $-V$ to V,

$$\overline{\Sigma a_k p_k \cdot \mathrm{sign}\,(\Sigma a_k q_k)} = \lim_{N\to\infty}\left(\frac{1}{2V}\right)^{2N+1}\int_{-V\le a_{-N},\cdots,a_N\le V}\cdots \cdot\int \Sigma a_k p_k\,\mathrm{sign}\,(\Sigma a_k q_k)\,da_{-N}\cdots da_N.$$

Assuming $q_0 > \sum' |q_k|$, then

$$\begin{aligned}
&\overline{\Sigma a_k p_k \cdot \mathrm{sign}\,(\Sigma a_k q_k)} \\
&= \lim_{N\to\infty}\left(\frac{1}{2V}\right)^{2N+1}\int_{-V\le a_{-N}\cdots a_{-1},a_1,\cdots a_N\le V}\int\left[\int\sum_{\Sigma a_k q_k>0} a_k p_k\,da_0 - \int\sum_{a_k q_k<0} a_k p_k\,da_0\right]da_{-N}\cdots da_{-1}\,da_1\cdots da_N \\
&= \lim_{N\to\infty}\left(\frac{1}{2V}\right)^{2N}\int\cdots\int\left[\frac{1}{2V}\int\sum_{V>a_0>-\Sigma' a_k q_k/q_0} a_k p_k\,da_0 - \frac{1}{2V}\int\sum_{-V<a_0<-\Sigma' a_k q_k/q_0} a_k p_k\,da_0\right]da_N\cdots da_{-1}\,da_1\cdots da_N \\
&= \lim_{N\to\infty}\left(\frac{1}{2V}\right)^{2N}\int\cdots\int\left[\frac{Vp_0}{2} - \frac{p_0(\Sigma' a_k q_k)^2}{2Vq_0^2} + \frac{\Sigma' a_k p_k \Sigma' a_k q_k}{Vq_0}\right]da_{-N}\cdots da_{-1}\,da_1\cdots da_N \\
&= V\left(\frac{P_0}{2} - \frac{P_0}{6q_0^2}\Sigma' q_k^2 + \frac{1}{3q_0}\Sigma' p_k q_k\right).
\end{aligned} \tag{11}$$

APPENDIX B

It is clear that the following factors are satisfied.

1) Cost function E is convex in the surface which satisfies the following constraint:

$$\sum_{i=-N}^{N} x_{-i}c_i = h_0 \qquad (\text{constant}). \tag{12}$$

2) Cost function E is convex along straight line $c = tc^*$, where c^* is an arbitrary tap-gain vector.

From these convexities, the convexity of E is proved by referring to Fig. 5.

From the above first statement,

$$E(c_2'') < \alpha E(c_2) + (1-\alpha)E(c_2'), \qquad 0 < \alpha < 1 \tag{13}$$

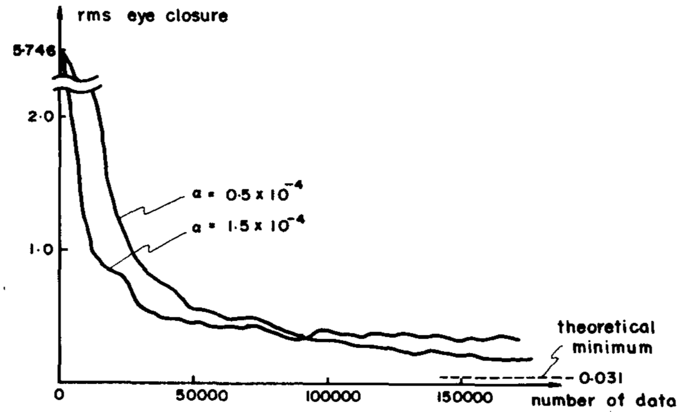

Fig. 4. Self-recovering processes for 15-level Class IV partial-response system.

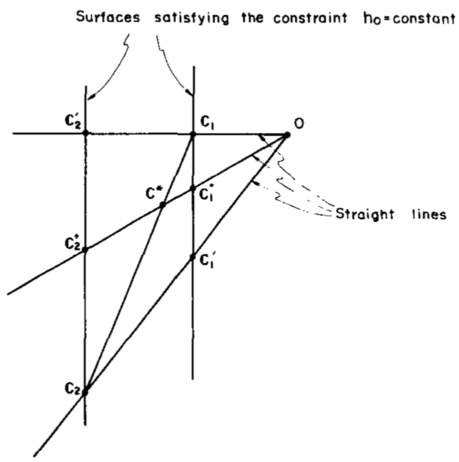

Fig. 5.

and

$$E(c_1'') < \beta E(c_1') + (1-\beta)E(c_1), \qquad 0 < \beta < 1 \qquad (14)$$

are derived for arbitrary c_1, c_2, c_1', c_2', c_1'', and c_2''. Moreover, from the second statement, the following is obtained:

$$E(c^*) < \gamma E(c_2'') + (1-\gamma)E(c_1''), \qquad 0 < \gamma < 1. \qquad (15)$$

Substituting (13) and (14) into (15), we obtain

$$E(c^*) < \gamma(\alpha E(c_2) + (1-\alpha)E(c_2')) + (1-\gamma)(\beta E(c_1') + (1-\beta)E(c_1)). \qquad (16)$$

Therefore, when $\alpha = 1$ and $\beta = 0$, the convexity of E is proved.

APPENDIX C

Denote c, h_0, and $D(=\Sigma' h_k^2/h_0^2)$, when they satisfy gradient (8) being zero for all $j = -N,\cdots,N$, by c^∞, h_0^∞, and D^∞, respectively. Then we obtain

$$2V\left(1 - \frac{\gamma}{h_0^\infty V}\right)\Phi c^\infty = (1 - D^\infty)x, \qquad (17)$$

where (i,j) element of matrix Φ is $\sum_{k=-\infty}^{\infty} x_{k-i}x_{k-j}$ and $x = (x_N,\cdots,x_0,\cdots,x_N)^t$. The proof is accomplished by showing that c^∞ minimizes D. Multiplying Φ^{-1} to both sides of (17) and taking scalar products with x, we obtain

$$2V\left(1 - \frac{\gamma}{h_0^\infty V}\right)h_0^\infty = \gamma(1 - D^\infty)h_0^{0p}, \qquad (18)$$

where h_0^{0p} is the scalar product (x, c^{0p}) and c^{0p} is one of the solutions which gives a minimum D. On the other hand, scalar product $x_j = (x_{N-j},\cdots,x_{-j},\cdots,x_{-N-j})^t$ and both sides of (17) after multiplying by Φ^{-1} give

$$2V\left(1 - \frac{\gamma}{h_0^\infty V}\right)h_{-j}^\infty = \gamma(1 - D^\infty)h_{-j}^{0p}. \qquad (19)$$

In an optimum situation, the relation

$$\frac{1}{h_0^{0p}} - 1 = D^{0p} \qquad (20)$$

is easily derived from [4, eq. (18)]. Substituting this into (18), we obtain

$$2(Vh_0^\infty - \gamma) = \gamma\frac{1 - D^\infty}{1 + D^{0p}} \qquad (21)$$

using (21) to eliminate h_0^∞ from (19), h_{-j}^∞ (for $j \neq 0$) is derived as

$$h_{-j}^\infty = \frac{\gamma}{2V}\{(1 - D^\infty) + 2(1 + D^{0p})\}h_{-j}^{0p} \qquad (22)$$

and substituting this and h_0^∞ satisfying (21) into $\Sigma'(h_k^\infty)^2/(h_0^\infty)^2$, we obtain

$$D^\infty = \Sigma'(h_k^{0p})^2(1 + D^{0p})^2. \qquad (23)$$

Substituting (20) into this result, the resultant relation $D^\infty = D^{0p}$ is derived.

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Determining Repeater Margin with a Stub and Attenuator

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***Abstract*—The margin of a high-speed digital regenerative repeater can be determined easily by adding a misterminated stub at the repeater input. The stub adds an interfering signal which, if large enough, will cause the repeater to make errors. The amplitude of the interfering signal is determined by the termination, so the termination impedance at which errors begin is a good measure of repeater margin. No calibration procedure is required.**

INTRODUCTION

In working with digital regenerative repeaters, it is often desired to determine the margin of a repeater which operates error free. Such a margin measure might be desired as an aid in locating a source of intermittent errors or as part of an installation acceptance test. If the repeater to be tested is located in a manhole, or for other reasons is not accessible, rather complex equipment and procedures are called for.[1] If the repeater is located in terminal equipment, or in the laboratory, a simpler and more accurate procedure, involving injection of noise and observation of the resulting error rate, is possible. Even this simpler procedure can be rather complex, however, due to the necessity for accurate adjustment of the amplitude of the noise source relative to the signal amplitude. This correspondence suggests a very simple technique for margin measurement applicable to higher speed digital repeaters.

TECHNIQUE

To determine the margin, the interfering ("noise") signal to be added to the digital stream at the input to the repeater is derived by delaying and attenuating the stream. This is readily done by bridging a stub of proper delay at the input of the repeater. The delayed re-

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