Distribution System Restoration With Microgrids Using Spanning Tree Search

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***Abstract*—Distribution system restoration (DSR) is aimed at restoring loads after a fault by altering the topological structure of the distribution network while meeting electrical and operational constraints. The emerging microgrids embedded in distribution systems enhance the self-healing capability and allow distribution systems to recover faster in the event of an outage. This paper presents a graph-theoretic DSR strategy incorporating microgrids that maximizes the restored load and minimizes the number of switching operations. Spanning tree search algorithms are applied to find the candidate restoration strategies by modeling microgrids as virtual feeders and representing the distribution system as a spanning tree. Unbalanced three-phase power flow is performed to ensure that the proposed system topology satisfies all operational constraints. Simulation results based on a modified IEEE 37-node system and a 1069-node distribution system demonstrate the effectiveness of the proposed approach.**

***Index Terms*—Distribution automation, Gridlab-D, microgrids, self-healing, service restoration, spanning tree.**

I. INTRODUCTION

DEVELOPMENT of a self-healing power network to allow resilience and fast recovery of power systems in response to disturbances has been envisioned [1] for the future power grids. Distribution system restoration is intended to promptly restore as much load as possible in areas where electricity service is disrupted following an outage. It plays a critical role in the future Smart Grid to modernize the power grids at the distribution level. The emerging microgrid technology, which enables self-sufficient power systems with distributed energy resources (DERs), provides further opportunities to enhance the self-healing capability. A microgrid can operate in an islanded mode in separation of the system during an outage. Hence, the customers in microgrids may be able to avoid extended outages [2], [3]. When a blackout occurs, microgrids can be controlled to provide an efficient DSR strategy to reduce the restoration time of the distribution system.

The adoption of advanced metering, communication, and automatic control infrastructure in the distribution system, which enables self-healing, is moving rapidly in the U.S. There are 16 on-going smart grid demonstration projects funded by the Department of Energy across the nation [4]. As a part of the Pacific Northwest smart grid demonstration project, automatic switches, capacitors and reclosers are added into the distribution automation systems [4]. The installations and applications of these new devices and technologies will enable remote monitoring, coordination and control of the distribution systems, and ultimately lead to faster response to disturbance and shorter restoration time.

DSR is a multi-objective, non-linear, and combinatorial optimization problem with numerous constraints, including topological and operating constraints. The large number of components in a distribution system adds to the complexity of the problem. Various approaches have been proposed for the DSR problem, including expert systems [5], [6], fuzzy logic [7], [8], multi-agent systems [9], heuristic search [10] and mathematical programming [11]. However, few DSR strategies consider microgrids. Bi-directional power flows, meshed configurations, and limited capacities of DERs resulting from the microgrids are major challenges for the development of DSR strategies. A fully decentralized multi-agent system is proposed [12] to tackle the complex DSR problem incorporating distributed generations. Based on a knapsack problem formulation and graph models, a new DSR procedure is developed using the dispersed generation availability [13].

A distribution network can be modeled by a graph $G(V,E)$ that contains a set of vertices V and a set of edges E. Therefore, DSR can be formulated as a problem of identifying the desired graph topology subject to various constraints. In this paper, a graph-theoretic search algorithm is proposed to identify a post-outage distribution system topology that will achieve a minimum number of switching operations and take full advantage of the resources of microgrids. The proposed algorithm has the following features:

1) The number of switching actions for DSR is minimized. The algorithm is designed to optimize the system topology that restores the maximum amount of load. It also provides a feasible switching sequence leading to the final system topology.
2) Microgrids are modeled as *virtual feeders*. This modeling technique ensures that generation limits of DERs can be

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formulated as electrical constraints of the distribution feeders. In addition, the island configuration of the microgrid can be modeled as a supplemental topology constraint of the distribution system.

3) A three-phase unbalanced power flow model is used to determine whether a proposed solution satisfies the electrical and operating constraints.

The remainder of this paper is organized as follows. Section II describes the formulation of DSR with microgrids as a multi-objective problem with constraints. Section III provides an explanation of the graph-theoretic concepts. This section also describes the proposed graph-theoretic distribution restoration algorithm based on spanning tree search. In Section IV, the GridLAB-D simulation environment together with the taxonomy feeder models is introduced. Section V proposes a two-phase strategy to meet the objectives described in Section II. Section VI presents the simulation results based on a modified IEEE 37-node system and a 4-feeder, 1069-node distribution system. The conclusions are stated in Section VI.

II. PROBLEM FORMULATION

The DSR process must be efficient in order to reduce outage durations and meet the customer expectations. An efficient restoration plan should be established quickly to assist operators in the distribution operating center. Operators take remote control actions or dispatch field crews to implement the restoration plan. Moreover, the restoration plan should provide a feasible system topology and a sequence of switching operations to reach the final configuration. In this study, the reconfiguration of distribution feeders with microgrids is formulated as a constrained multi-objective problem, i.e.,

Objectives:

$\mathrm{Min}\ n_{\mathrm{sw}}$ (Minimizing the total number of switch operations)

$\mathrm{Max}\ \Sigma_{i \in \mathbf{N}_{\mathrm{res}}} S_i$ (Maximizing the amount of total load restored)

Subject to

i) $$|\mathrm{I}_l^{\min}| \le |I_l| \le |\mathrm{I}_l^{\max}|, l \in \mathbf{L}_{\mathrm{en}} \tag{1}$$

ii) $$|\mathrm{V}_k^{\min}| \le |V_k| \le |\mathrm{V}_k^{\max}|, k \in \mathbf{B}_{\mathrm{en}} \tag{2}$$

iii) $$P_j^2 + Q_j^2 \le \left(S_j^{\max \mathrm{T}}\right)^2, j \in \mathbf{F} \tag{3}$$

iv) $$P_m \le \mathrm{P}_m^{\max}, Q_m \le \mathrm{Q}_m^{\max}, m \in \mathbf{M} \tag{4}$$

v) Radial network structure is maintained,

vi) Unbalanced three-phase power flow equations are equality constraints.

n_{sw}	number of switching operations required to reach the final configuration;
$\mathbf{N}_{\mathrm{res}}$	set of restored buses;
S_i	total MVA power of the load at bus i;
I_l	current flows through distribution line l;
$I_l^{\min}$, $I_l^{\max}$	lower and upper limit of the line current of distribution line l, respectively;
$\mathbf{L}_{\mathrm{en}}$	set of lines that are in service;
V_k	bus voltage of load bus k;
$\mathrm{V}_k^{\min}$, $\mathrm{V}_k^{\max}$	lower and upper limit of the bus voltage at load bus k, respectively;
$\mathbf{B}_{\mathrm{en}}$	set of load buses that are in service;
P_j	MW power injected into feeder j;
Q_j	MVar power injected into feeder j;
$S_j^{\max \mathrm{T}}$	maximum capacity of feeder j or the maximum capacity of the transformer at feeder j, whichever is lower;
$\mathbf{F}$	set of all feeders;
P_m	MW power injected into microgrid m;
Q_m	MVar power injected into microgrid m;
$\mathrm{P}_m^{\max}$	maximum MW capacity of DERs;
$\mathrm{Q}_m^{\max}$	maximum MVar capacity of DERs in microgrid m;
$\mathbf{M}$	set of microgrids that are in service.

The objectives of the algorithm include minimizing the total number of switching operations during the restoration process and maximizing the amount of load to be restored in the out-of-service area. Constraints i) and ii) show that the line current and bus voltage should be maintained within acceptable operating limits. Constraint iii) requires that the total load of each feeder should not exceed the maximum capacity of the supplier transformer. Constraint iv) indicates that the total load in each microgrid should not exceed the total generation capability limit of DERs.

III. PROPOSED GRAPH THEORETIC DISTRIBUTION RESTORATION STRATEGY

A. *Representing Distribution Network Using Spanning Tree*

A distribution network is formed by interconnected distribution feeders and microgrids. In this study, each microgrid is modeled as a virtual feeder. The point of common coupling between DERs and the distribution network is viewed as a virtual tie switch. The feeders are connected with each other through normally open tie switches. If a load node is modeled as a vertex and a feeder line section is viewed as an edge in the graph, the distribution network can be represented as a connected graph $G(V, E)$. Here the nodes that denote substations or DERs are referred to as root nodes. The radial structure of a distribution network can be represented by a spanning tree T in G if all root nodes are lumped together into one source node $\mathbf{S}$. All vertices in G are connected through a spanning tree T that does not contain any loop. An example of distribution network with one microgrid (labeled as DERs) and the corresponding spanning tree representation is shown in Fig. 1.

B. *Switching Operation Pairs for DSR*

After the occurrence of a fault, the out-of-service load following the fault isolation should be restored by a sequence of switching operations. Isolating a line segment from the rest of the distribution network due to fault isolation requires opening of one or more (closed) sectionalizing switches. Switching operations during feeder restoration involve opening a normally closed sectionalizing switch and closing a normally open tie switch. Such a pair of switching operations guarantees that the

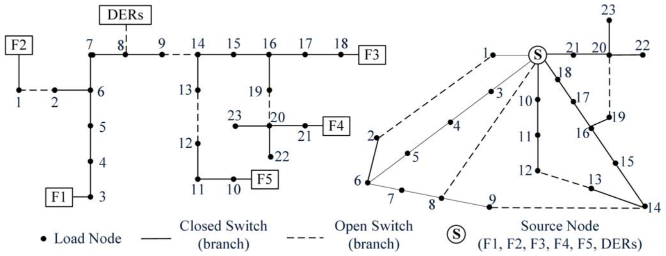


Fig. 1. Spanning tree representation of distribution network.

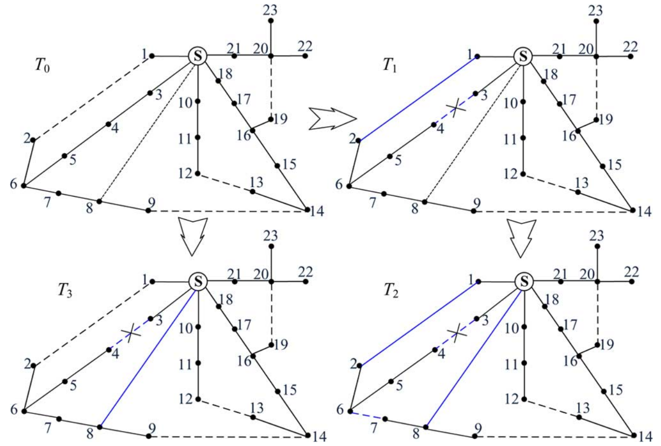


Fig. 2. Example of spanning tree transformations.

radial network structure is maintained during the restoration process. A pair of switching operations corresponds to a *cyclic interchange operation* in graph theory, which consists of adding an edge to the spanning tree to create a cycle and deleting another edge within this cycle for a transition to a new spanning tree. An example of cyclic interchange operations is shown in Fig. 2. The spanning tree T_0 is transformed to spanning tree T_1 by deleting edge e_{3-4} and adding edge e_{1-2}. The spanning tree T_1 is further transformed to spanning tree T_2 by deleting edge e_{6-7} and adding edge $e_{8-\mathrm{S}}$. The cyclic interchange operations transforming the spanning tree T_0 to T_2 represent the switching operations that restore all the out-of-service loads after a failure in e_{3-4}. The feeder F2 picks up the loads at node 2, 4, 5, 6. DERs connecting to node 8 pick up the out-of-service loads at node 7, 8, 9 and form an isolated microgrid.

C. Searching for Spanning Trees for DSR Without Duplication

In a graph G, an initial spanning tree T can be transformed into another spanning tree T' through a single or multiple cyclic interchanges [14]. However, a tree may be generated repeatedly in this process. A graph theoretic algorithm in [15] that generates spanning trees without duplication provides a systematic solution to the problem. The notation and definitions in graph theory that formalize the above algorithm are presented as follows:

Notion: $\bigoplus$ stands for "exclusive or" of two sets; $A - B$ stands for the set containing all elements of A that are not in B.

Definition 1 (Fundamental Cutset): In a connected graph G, a cutset is a set of edges whose removal breaks G into two components [14]. Considering a spanning tree T in G and an edge e in T, a fundamental cutset $S_e(T)$ is a cutset which contains exactly one edge e in T and some other edges in $G - T$ that breaks G into two components if necessary, where $S_e(T)$ stands for a fundamental cutset of e with respect to T. The fundamental cutset $S_e(T)$ can be found by a depth-first-search [16] on T.

Definition 2 (Selective Cyclic Interchange Operation): Starting from a tree T_0, some other trees can be obtained by replacing an edge e of tree T_0 with another edge in the fundamental cutset $S_e(T_0)$, and the resulting trees are distinct. Such an operation is represented by (5), where t represents all trees that are transferred from T_0:

$$\left\{t \middle| t = T_0 \bigoplus \{e, e_i\}, e_i \in S_e(T_0), e_i \neq e\right\}. \tag{5}$$

The DSR problem consists of two major tasks: finding the final optimal network topology and providing a sequence of switching operations leading to this final network topology. If the pre-fault distribution network is represented by a spanning tree T in the graph G, these two tasks can be viewed as a combined task of finding the desired spanning tree T' in graph G and providing a sequence of cyclic interchange operations to transform the initial spanning tree T to T'. An edge in T may be disconnected in T', which indicates that the corresponding branch in the original distribution network is de-energized by sectionalizing switches due to fault isolation.

In this study, the algorithm in [15] is adapted for application to DSR. A sequence of spanning trees can be generated with an increasing number of cyclic interchange operations sequentially without duplication as follows:

1) The original distribution network topology is represented by a reference spanning tree T_0 in graph G. There is a set of edges that represent normally open tie switches (i.e., $G - T_0$).
2) Suppose that one edge e is opened to isolate a fault. Edge e of T_0 is replaced by each edge e_k in the fundamental cutset of e with respect to T_0 (i.e., $e_k \in S_e(T_0)$) to generate a class of spanning trees, i.e., $T_e = \{t | t = T_0 \bigoplus \{e, e_k\}, e_k \in S_e(T_0), e_k \neq e\}$. Then the edges in $T_0 - \{e\}$ are ordered. The number of edges in $T_0 - \{e\}$ is $v - 2$, where v is the number of vertices in G.
3) Starting from T_e, a class of spanning trees T_{e,e_i} are generated by replacing an edge e_i in $T_0 - \{e\}$ with an edge $e_k \in S_{e_i}(T_e) \cap S_{e_i}(T_0)$, where $i = 1, 2, \ldots, v - 2$.
4) Iterating the procedure in step 3) until the number of cyclic interchange operations reaches the number of tie switches. The serial number of the replaced edge at the current iteration should be larger than the one at the last iteration.

The generated spanning trees represent candidate distribution network topologies to restore the load in the out-of-service areas. As the distribution network has a (weakly) meshed graph, the number of tie-switches in the distribution system is limited. The total number of switching operation pairs to restore the out-of-service load is equal to the number of cyclic interchange operations that transform the initial spanning tree to another. This number is less than or equal to the number of tie-switches in the distribution system, depending on the electrical and operational constraints in the system.

The constraint $e_k \in S_{e_i}(T_e) \cap S_{e_i}(T_0)$ in the third step ensures that there is no duplication in the generated spanning trees. The example in Fig. 2 demonstrates this feature as follows. If edge e_{3-4} is disconnected due to a fault, the spanning trees T_1 and T_3 can be generated from T_0 at Iteration 1:

Iteration 1:

$$T_1 = \left\{ t | t = T_0 \bigoplus \{e_{3-4}, e_{1-2}\}, e_{1-2} \in S_{e_{3-4}}(T_0) \right\}$$

$$T_3 = \left\{ t | t = T_0 \bigoplus \{e_{3-4}, e_{8-\mathrm{S}}\}, e_{8-\mathrm{S}} \in S_{e_{3-4}}(T_0) \right\}$$

where $S_{e_{3-4}}(T_0) = \{e_{1-2}, e_{3-4}, e_{8-\mathrm{S}}, e_{9-14}\}$.

Iteration 2: Replacing edge e_{6-7} in T_1 and T_3, two class of spanning trees are calculated as follows:

$$T_{1,e_{6-7}} = \left\{ t | t = T_1 \bigoplus \{e_{6-7}, e_i\}, e_i \in S_{e_{6-7}}(T_0) \cap S_{e_{6-7}}(T_1) \right\}$$

where $S_{e_{6-7}}(T_0) = \{e_{8-\mathrm{S}}, e_{9-14}\}$, $S_{e_{6-7}}(T_1) = \{e_{8-\mathrm{S}}, e_{9-14}\}$:

$$T_{3,e_{6-7}} = \left\{ t | t = T_1 \bigoplus \{e_{6-7}, e_i\}, e_i \in S_{e_{6-7}}(T_0) \cap S_{e_{6-7}}(T_3) \right\}$$

where $S_{e_{6-7}}(T_3) = \{e_{1-2}, e_{3-4}\}$.

It can be seen that $S_{e_{6-7}}(T_0) \cap S_{e_{6-7}}(T_3)$ is an empty set, hence the spanning tree T_2 can only be transformed from T_1. The repeated transformation from T_3 to T_2 is avoided.

D. Graph Simplification

The number of desired spanning trees for DSR increases exponentially with the increasing number of vertices and edges in graph G. The computational complexity can be reduced significantly if the spanning tree search algorithm is performed on a simplified graph, especially for a distribution system network.

In the cyclic interchange operations for DSR, the branches without switches in the original distribution network cannot be used to change the configuration. Hence, all branches without switches of the original distribution network can be removed. Moreover, the distribution network has a large number of degree one vertex. Removal of degree one vertices makes no impact on the spanning tree generation. A graph simplification scheme for spanning tree search is proposed as follows.

1) All edges representing tie-switches and corresponding vertices are retained in the simplified graph.
2) The edge representing sectionalizing switch that is opened due to fault isolation and corresponding vertices are retained in the simplified graph.
3) All degree one vertices and the only edge connecting to it are removed until there is no more degree one vertex in the graph.
4) After removing all degree one vertices, all vertices with three degree or more are retained in the simplified graph.
5) Degree two vertices that are not connected to any edge representing tie-switch are removed.

An example to demonstrate the proposed graph simplification scheme is shown in Fig. 3. In a realistic distribution network, the number of branches and buses is large. After the graph simplification stage, the size of the graph is significantly reduced. As an example, before graph simplification, the graph of the test system in Section VI has 1069 vertices and 1079 edges. After simplification, the number of vertices and edges is reduced to 27 and 37, respectively.

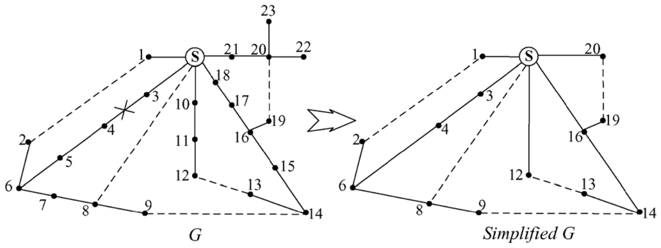


Fig. 3. Example of graph simplification.

The candidate switching operations can be obtained by mapping the cyclic interchange operations on the simplified graph to the switching operations on the original distribution network. Since all tie-switch branches are retained on the simplified graph, the operation to add an edge on the graph is performed by closing a corresponding tie-switch. Removing an edge on the graph can be achieved by opening a sectionalizing switch operation. The candidate sectionalizing switch can be chosen from a corresponding set of sectionalizing switches.

IV. UNBALANCED POWER FLOW FOR DISTRIBUTION SYSTEMS WITH MICROGRIDS

The feasibility of the sequence of switching operations for each spanning tree is evaluated by unbalanced three-phase power flow calculations using GridLAB-D. GridLAB-D [17] is a distribution system modeling and simulation environment developed by the Pacific Northwest National Laboratory (PNNL) for the study of distribution systems and smart grid technologies. The algorithms that are used to calculate unbalanced three-phase power flow on the distribution level include Gauss-Seidel (GS) and Newton Raphson (NR) methods [18]. In this study, the NR method is used. A set of radial distribution test feeders representing those found in various regions of the Continental U.S. are created in a GridLAB-D compatible format [19], [20]. These taxonomy feeder models with detailed load models represent realistic distribution feeders supplied by utilities in the U.S.

This study assumes that microgrids have the capability to provide the local load with their real and reactive power demand, maintain the voltage profile and stabilize the frequency in an islanded operation mode. The capacity of local load should be less than the total generation capacity of all DERs in the islanded microgrid. This assumption is a basic feature of microgrids [2], [3], [21]–[23]. Thus, the bus with DERs is defined as a slack bus in the power flow modules of GridLAB-D. After the unbalanced three-phase power flow is performed, the injected real and reactive power at the slack bus will be compared with the generation limits (4) of DERs in the isolated microgrid. If the constraint in (4) is satisfied, the operation mode is considered feasible.

V. DISTRIBUTION SYSTEM RESTORATION STRATEGY

A. Two-Phase DSR Strategy

A two-phase strategy is proposed to meet the objectives described in Section II. In the first phase, the network configuration with the minimum number of switching operations that can restore all load in the out-of-service area is searched sequentially. The three-phase unbalanced power flow calculation is applied to determine if electrical and operating constraints are met by the candidate network configuration. If a feasible solution cannot be found to restore all loads in the out-of-service area, a partial restoration strategy will be identified in the second phase. A flow chart of proposed algorithm is shown in Fig. 4. The iterative searching procedure is given below:

Step 1) Convert the distribution network to a graph G as described in Section III-A.

Step 2) Simplify the graph G to G' following the proposed strategy in Section III-D.

Step 3) A set of candidate network topologies is searched, starting from the first iteration. The spanning tree search algorithm introduced in Section III-C is applied to calculate all possible spanning trees $(X_1, X_2, \dots X_\mathrm{n})$ at each iteration. The inserted edge in the newly formed spanning tree indicates a closing operation of a tie-switch. The deleted edge in the newly formed spanning tree indicates the opening operation of a sectionalizing switch. The radial structure is maintained during the cyclic interchange operations.

Step 4) If the set of candidate network topologies obtained at the Step 3) is not empty, an unbalanced three-phase power flow is performed and constraints i), ii), iii), and iv) in Section II are evaluated. If no constraint violation exists, the solution is stored as the feasible final configuration. Otherwise, infeasible network configurations are removed in the solution list according to the criterion illustrated in the next subsection. After that, a three-phase power flow calculation is performed again to check the next candidate configuration in the solution list. The infeasible network configurations are checked and removed after every power flow calculation. If there is not a network configuration in the solution list that can restore all the loads in the out-of-service area, go to step 3) to find possible network configurations with an increased number of switching operations and the number of iterations i is incremented by one.

Step 5) If a solution satisfying all the electrical and operating constraints is found, output the result. If all potential network topologies are explored and there is no configuration that satisfies all electrical and operation constraints, the partial load to be restored in the out-of-service area is selected. The minimum amount of load to relieve the overload is identified for the network topology with the least severe overload condition. Based on the previous power flow calculation results, if the lowest node voltage in the system is considered as the minimum node voltage for this network topology, then the network topology with the highest minimum node voltage is considered as the least severe overload topology.

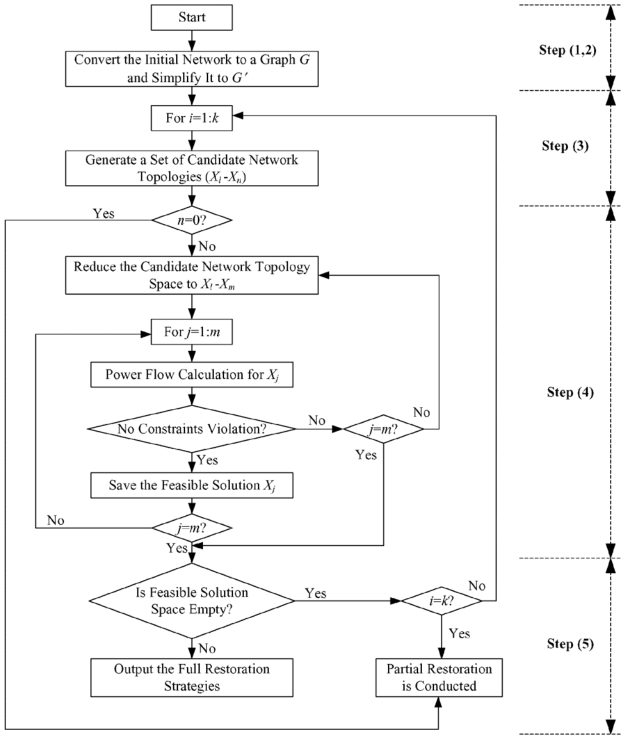


Fig. 4. Proposed distribution restoration algorithm, where K is the number of tie-switches in the system and i is the number of iterations.

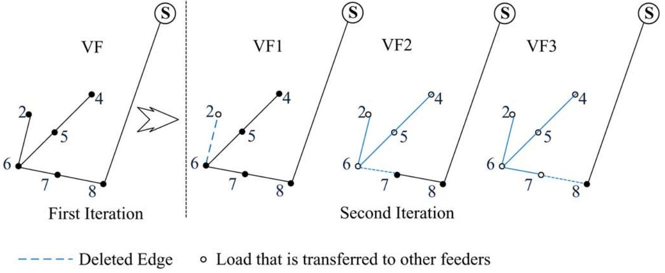
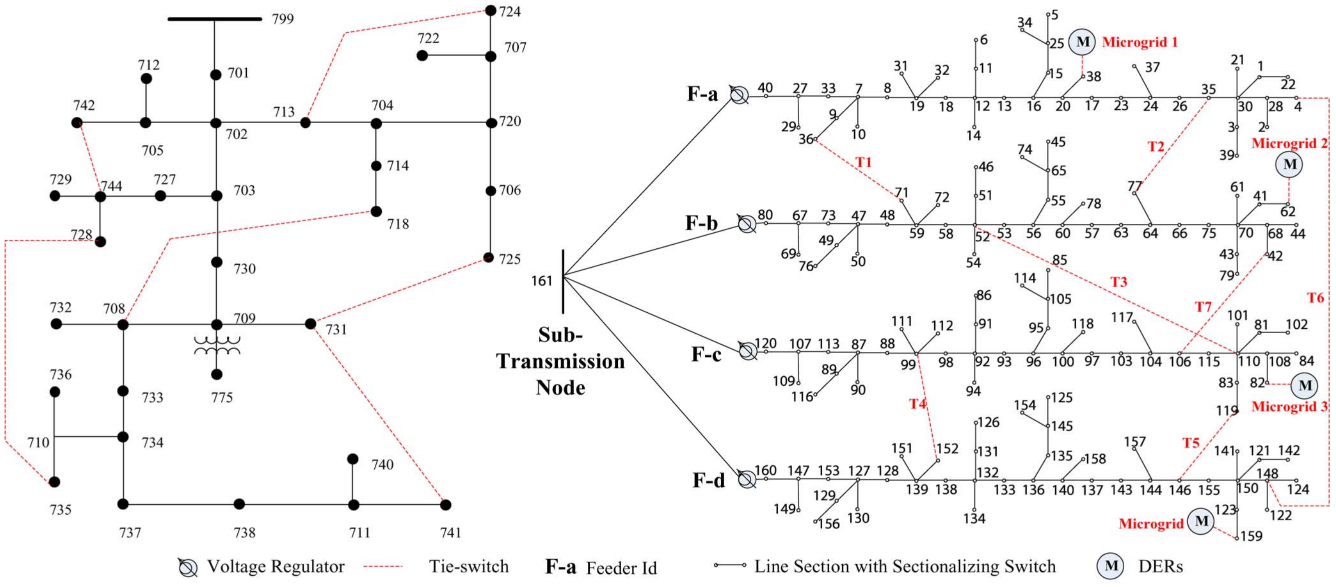


Fig. 5. Sub-tree growth example.

B. Sub-Tree Growth Criterion to Reduce the Number of Candidate Topologies

A sub-tree growth criterion is proposed to avoid unnecessary power flow calculations. In Fig. 5, an example, the virtual feeder in Fig. 1, is used to illustrate this criterion.

Due to a fault on edge e_{3-4} in Fig. 1, the out-of-service loads are transferred to the virtual feeder by deleting edge e_{3-4} and adding edge $e_{8-\mathbf{S}}$ at the first iteration. However, the power flow results indicate that the sub-tree starting from node $\mathbf{S}$ to node 2 and node 4 in Fig. 5 is overloaded. Hence the second iteration is triggered to generate more spanning trees. If the switch at edge $e_{\mathbf{S}-1}$ is opened and tie switch e_{1-2} is closed in Fig. 1, the new, longer sub-tree has a more severe overloading condition since more loads are transferred to it. Similarly, if the switch at edge e_{16-19} is opened and tie switch e_{19-22} is closed in Fig. 1, the overloading condition still exists in the virtual feeder. Hence, the feasible candidate switching operation leading to the new spanning trees is to close the switch at



(a) (b)

Fig. 6. One-line diagrams of test systems. (a) Modified IEEE 37-node system. (b) Simplified one-line diagram of the four-feeder 1069-node distribution system.

edge e_{1-2} and opening the switch at one edge selected from $\{e_{2-6}, e_{6-7}, e_{7-8}\}$ as shown in Fig. 5.

In summary, the overloading sub-tree structure is recorded as a benchmark sub-tree, and updated after each power flow calculation. According to this criterion, only the candidate network topologies that have the potential to alleviate the overload condition will be selected to check the constraints with power flow calculations. Unnecessary power flow calculations on the infeasible candidate network topologies are avoided and the computation time is reduced.

C. Optimality Analysis

An optimal network topology is the one which, among all possible network topologies, has the minimum number of switching operations that transforms the system configuration to the post-restoration configuration. The proposed algorithm applies the spanning tree search algorithm to generate every possible network topology to restore out-of-service load. Starting from a pair of switching operations, a sequence of candidate network topologies with an increasing number of switching operations is generated. If a network topology satisfying the electrical and operating constraints is identified by constraint checking, such a network topology must be optimal. Because all possible network topologies with fewer switching operations have already been explored, and their infeasibility with respect to the constraints has been verified. However, an optimal configuration may not exist. If such an optimal network topology cannot be found, a partial restoration configuration will be pursued instead.

VI. TEST SYSTEMS AND SIMULATION RESULTS

In this study, a modified IEEE 37-node system [24] and a multi-feeder test system consisting of four taxonomy feeder R3-12.47-2 [20] are used to validate the proposed algorithms. Each test system has a GridLAB-D compatible format, with a detailed network model and a combination of constant power, constant current, and constant impedance load models. The proposed algorithm is implemented in MATLAB and GridLAB-D on a PC with an Intel Core i7-3520M 2.9 G CPU and 16 GB RAM.

A. Modified IEEE 37-Node System

The original IEEE 37-Node distribution system [24] is modified following the steps of [11]. There is a sectionalizing switch at every branch of the system. Meanwhile, 6 normally open tie switches are added to the system to create the restoration scenarios as shown in Fig. 6. The allowable voltage at load bus is within 0.95 and 1.05 per unit. The thermal limit of each line is the ampacity of aluminum conductors. The performance of proposed spanning tree search algorithm is compared with a mixed integer non-linear programming (MINLP) algorithm implemented by WSU based on an approach similar to that of [11] and a rule-based system using heuristic search reported in [5]. The results are shown in Table I.

Among three algorithms, the MINLP algorithm requires the longest computation time. Moreover, the switching operations for scenario 3 and 5 provided by the MINLP algorithm lead to a looped network, which violates the radial network structure constraint. The rule-based system method provides good solutions in the shortest time, but cannot guarantee the optimality of the solutions.

Compared with other two methods, the proposed algorithm requires less computing time and the least switch operations. This is because the proposed algorithm searches and detects the possible topologies with fewer number of switching operation pairs, hence the computational burden is reduced and the solution is optimal.

The proposed algorithm incorporates the unbalanced three-phase power flow for each topology candidate. Hence, an infeasible topology with over-current or under-voltage will be removed from the solution candidates. Given a fault on line

TABLE I
COMPARISON OF RESTORATION SOLUTIONS FOR THE MODIFIED IEEE 37-NODE SYSTEM

Scenario	Fault Location	Proposed Spanning Tree Search Algorithms		Mixed Integer Non-linear Programming [11]		Heuristic Search [5]	
		Time (sec.)	Switch Status	Time (sec.)	Switch Status	Time (sec.)	Switch Status
1	713-704	1.335	Close 713-724	41	Close 713-724	0.749	Close 708-718, 713-724 Open 704-720
2	733-734	0.484	Close 731-741	30	Open 711-741 Close 728-735, 731-741	0.834	Close 735-728, 731-741 Open 734-737
3	730-709	0.825	Close 718-708	91	Open 708-733, 738-711 Close 728-735, 725-731, 731-741, 718-708	0.872	Close 708-718, 728-735 Open 708-733
4	703-727	0.805	Close 742-744	3	Close 742-744	0.765	Close 728-735, 742-744 Open 728-744
5	709-708	0.787	Close 718-708	120	Open 734-737 711-741 Close 728-735, 731-741, 718-708	0.967	Close 708-718, 731-741 Open 708-733

702-703 in IEEE 37-node system, at the first iteration of spanning tree search, the out-of-service loads can be restored by one switching operation, i.e., closing line 718-708, line 744-742, or line 731-725. For the action of closing line 718-708, the unbalanced power flow is performed. It can be found that the current at Phase A of line 704-714 is 213.9 amp, which exceeds its current limit, 185 amp for all aluminum conductors (Type 2# AA). The current on Phase A of line 714-718 is 209.8 amp, which is also an over-current. Hence, the scenario of closing line 718-708 is infeasible. After the other two candidates are identified, the procedure is triggered in the second iteration with two switching operation pairs. Finally, it is found that the switching actions, closing 718-708 and 744-742, opening 703-727, are the optimal solution. There is no over-current for this topology. The minimum single-phase system voltage is 0.97 p.u., higher than the low voltage limit. The computational time to calculate the restoration actions for this scenario is 13.8 s.

B. Multi-Feeder Test System With Microgrids

Taxonomy feeder R3-12.47-2 is a representation of a moderately populated urban area, which is composed of single family homes, light commercial loads, and a small amount of light industrial loads. The total load is 17 467.82 kW and 2361.82 kVar. In this multi-feeder system, feeders can interconnect with each other through 7 normally open tie switches. Four nodes are connected with DERs, which can form four isolated microgrids. There are 1069 nodes and 1079 branches in this system. The voltage level for this system is 12.47 kV. Each load node is connected with the system through a step-down transformer (12.47 kV/480 V). The simplified one-line diagram is shown in Fig. 6, in which each edge is a switch, breaker or recloser. This system is assumed to be in a peak load condition. The allowable minimum single phase voltage in this system is 259.698 V, which is 93.72% of the nominal node voltage, i.e., 277.1 V. The allowable maximum transformer capacity at each feeder is 7.5 MVA. The generation capacity limit of each microgrid is shown in Table II.

Table III presents five restoration scenarios. It is assumed that the faulted line is isolated by switching off the sectionalizing switches on both sides of the line. A sequence of switching operations generated by the proposed algorithm is able to restore as much load as possible in the out-of-service area. Power flow calculations are performed to ensure that the post-restoration configuration will not violate the node voltage and transformer capacity constraints. To evaluate the impact of microgrids, the DSR results for the multi-feeder test system without microgrids are also calculated. The results are shown in Table III. In the multi-feeder test system with and without microgrids, the number of tie-switches is 11 and 7, respectively.

TABLE II
GENERATION CAPACITY LIMIT OF EACH MICROGRID

	Microgrid 1	Microgrid 2	Microgrid 3	Microgrid 4
MW capacity	5.15	1.65	2.5	1.00
MVar capacity	2.25	0.95	1.75	0.55

In scenario 1, the system is restored by closing tie-switch T6. The minimum single-phase voltage at the load bus in the post-restoration system is 271.95 V, which is 98.14% of the nominal node voltage.

In scenario 2, the system is restored with the help of microgrids. The injected real and reactive power at bus 38 are 4.11 MW and 1.29 MVar, respectively, which are within the capacity limits of DERs in Microgrid 1. By comparison, if there is no microgrid in the system, it will require more switching operations to pick up the load in the out-of-service area. In other words, a more efficient DSR strategy with fewer switching operations is achieved with microgrids.

In scenario 3, the system is restored by closing 2 tie switches, opening one sectionalizing switch following the operation sequence, as shown in Table III. This scenario is identical to the "load transfer" scenario in [5]. The system is overloaded by connecting all the load in the out-of-service area to feeder F-c, and the overload condition is alleviated by a load transfer operation. Some load is transferred from feeder F-c to feeder F-b by a pair of switching operations, i.e., Opening 115-110 and Closing T3.

In scenario 4, in order to restore the out-of-service loads, it is desirable to close 3 tie-switches and open 2 sectionalizing switches. This scenario is a combination of the "zone restoration" [5] and "load transfer" scenario. The out-of-service area is separated into 2 zones by opening sectionalizing switch between node 150 and 148 first. Then, these two load zones are restored by closing tie-switch T5 and T6. After that, some loads

TABLE III
RESULTS OF DISTRIBUTION SYSTEM RESTORATION WITH MICROGRIDS AND WITHOUT MICROGRIDS

Scenario	Fault Location	Switch Status for DSR with microgrids	Switch Status for DSR without microgrids
1	30-28	Close: 4-148	Close: 4-148
2	16-20	Close: 38-Microgrid 1	Open: 75-70, 83-110 Close: 77-35, 42-106, 146-119
3	56-60	Open: 115-110 Close: 42-106, 52-110	Open: 115-110 Close: 42-106, 52-110
4	140-137	Open: 150-148, 108-82 Close: 4-148, 146-119, 82- Microgrid 3	Open 155-150 Close: 146-119, 4-148 Partial Restoration, 868.11 kVA load should be shed at **F-a**
5	87-88	Open: 115-110, 83-110 Close: 52-110, 119-146, 99-152	Open: 115-110, 83-110 Close: 52-110, 119-146, 99-152

TABLE IV
RESULTS OF DISTRIBUTION SYSTEM RESTORATION WITH MICROGRIDS

Scenario	Fault Location	Switch Status for DSR with microgrids		
		Elapsed time (sec.)	Time for graph search	Time for power flow
1	30-28	11.489	45.99%	55.01%
2	16-20	25.073	30.37%	69.63%
3	56-60	164.852	15.29%	84.71%
4	140-137	354.731	17.97%	82.02%
5	87-88	1133.551	17.86%	82.14%

in feeder F-c are transferred to Microgrid 3 by opening sectionalizing switch 108-82 and closing tie-switch between node 82 and DERs of Microgrid 3 to alleviate the overload condition at feeder F-c.

By comparison, if there is no microgrid in the system, no full load restoration strategy can be found by the proposed algorithm for Scenario 4. The proposed algorithm moves the second phase, i.e., partial restoration. Among all infeasible system topologies, it is found that the partial restoration strategy presented in Table III can pick up as much load as possible. For this post-restoration system, since the capacity limit of the transformer is violated at feeder $\mathbf{F}-\mathbf{a}$, 868.11 kVA load cannot be served at F-a.

Scenario 5 is a restoration scenario with 5 switching operations. This scenario leads to the largest blackout area among all 5 scenarios. After the first 2 switching operations, the out-of-service area is separated into 3 zones. Two zones are connected to feeder F-d and 1 zone is connected to feeder F-b.

Table IV reports the algorithm execution time for the five restoration scenarios tested on the multi-feeder test system that represents the real distribution feeders. On average, the proposed algorithm spends 338 s to find the optimal solution. The average computation time for spanning tree searching is 61 s. The majority of the computation time for the proposed algorithm is calculating the three phase power flow. To accelerate the calculation on large systems, the graph simplification method and the subtree growth algorithm proposed in Section V can be applied to reduce the searching space significantly by removing the infeasible candidate network topologies and avoid the unnecessary power flow calculations. Moreover, if parallel computing techniques are adopted in the power flow calculation, considerable savings in computation time can be achieved.

VII. CONCLUSION AND FUTURE WORK

The proposed graph-theoretic DSR algorithm applies a spanning tree search technique to find the network topologies. The proposed algorithm identifies an optimal solution with minimum switching operations and picks up loads as much as possible without violations of operational constraints. The microgrids in distribution system are modeled as virtual feeders. The simulation results demonstrate that microgrids enhance the recovery capability of a distribution system.

Multiple faults in a number of areas disconnected from one another may arise under severe weather conditions. Further research is needed to evaluate the performance and efficiency of the proposed algorithm to handle multiple faults. This study is concentrated on static constraints such as voltage and current limits. It is desirable to check the feasibility of spanning tree topologies considering the dynamic response in microgrids.

This paper assumes that DERs are normally disconnected with the main grid and an isolated microgrids may pick up the out-of-service loads during the restoration process. Future work should also take into account the interconnection between a microgrid and the main grid. Furthermore, this research can be enhanced by considering the priority of critical loads in microgrids during system restoration, as well as modeling the microgrid as a node in the system during restoration to allow loop topology within the microgrid.

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